Homework 7

1. **Quantization** (20 points)
   An analog signal is quantized and transmitted by using a PCM system. If each sample at the receiving end of the system must be known to within ±0.5% of the peak-to-peak full-scale value, how many binary digits must each sample contain?

2. **TDM** (15 points)
   A signal $m_1(t)$ is bandlimited to 3.6 kHz, and there are three other signals — $m_2(t), m_3(t)$, and $m_4(t)$— bandlimited to 1.2 kHz each. These signals are to be transmitted by means of time-division multiplexing.

   (a) Set up a scheme for accomplishing this multiplexing requirement with each signal sampled at its Nyquits rate.

   (b) What must be the speed of the commutator (in samples per second)?

   (c) If the commutator output is quantized with L=1024 (i.e., it has 1024 different values) and the result is binary-coded, what is the output bit rate?

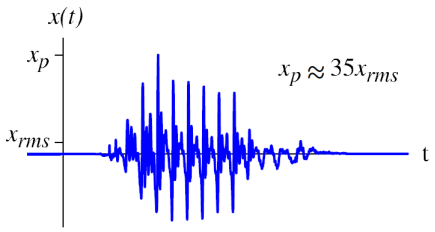

Figure 1: Speech signal in Q3.

3. **PCM with nonuniform quantizer** (25 points)
   Consider a speech signal with spectral components in the range of 300 to 3000 Hz. Recall from lecture notes 5 that speech has a high crest factor as shown in Fig. 1. Assume that a sampling rate of 8000 samples per second will be used to generate a PCM signal. The required signal-to-quantization ratio is 30 dB.

   (a) What is the minimum number of uniform quantization levels needed and what the minimum number of bits per sample needed.

   (b) calculate the minimum system bandwidth requirement.

   (c) repeat parts (a) and (b) when a μ-law compander with $\mu = 255$ is used.

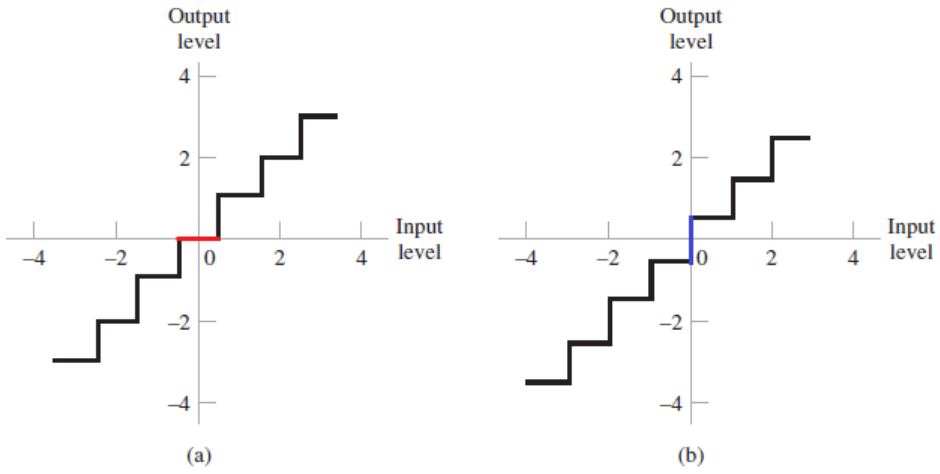

Figure 2: Two types of uniform quantization: (a) midtread (b) midrise

4. **PCM with uniform quantizer** (15 points)
A PCM system uses a uniform quantizer followed by a 7 bit encoder. The system bit rates 50 Mbits/sec.

   (a) Calculate the maximum bandwidth of the message signal for which this system operates satisfactorily

   (b) Determine the output SNR when a full load sinusoidal modulating wave of frequency 1 MHz is applied to the input.

5. **Uniform quantizer with nonuniform input** (25 points)
Consider a uniform midtread quantizer characterized by the input-output relation illustrated in the Fig. 2 (a). Assume that a Gaussian-distributed random variable with zero mean and unit variance is applied to this quantizer input.

   (a) What is the probability that the amplitude of the input lies outside the range -4 to +4. (Hint: you may use the Matlab command `qfunc` to find this)

   (b) Using the result of part (a), show that the output signal-to-noise ratio of the quantizer is given by

   $$SNR_o = 6b - 7.2 \ dB,$$

   where b is the number of bits per sample. Specifically, you may assume that the quantizer input extends from -4 to +4.

   (c) compare the result of part (b) with that obtained in Example 7.2 of the textbook (or, see page 20 in the slides)