Homework 8

NOTE: When needed, you may use the MATLAB command `qfunc(x)` to evaluate $Q(x)$. You can also find the table of Q-function on the last page of this document.

1. In a baseband binary transmission, binary digits are transmitted

$$\begin{cases} Ap(t), & 0 \le t \le T, \quad \text{for } 1 \\ -Ap(t), & 0 \le t \le T, \quad \text{for } 0 \end{cases}$$

The bit duration is T and the pulse shape is $p(t) = 1 - \frac{|T-2t|}{T}, \quad 0 \le t \le T$. 0 and 1 are equiprobable and the channel is AWGN with power spectral density $\frac{N_0}{2}$.

(a) find and sketch the optimum receiver filter $h(t)$

(b) find the probability of error P_e as a function of E_b/N_0

(c) compare the results in part (a) and (b) for the case when the pulse shape is $p(t) = 1 - \frac{t}{T}, \quad 0 \le t \le T$.

2. **Binary Detection** (30 points)

In a polar NRZ line code symbols 1 and 0 are represented by positive and negative rectangular pulses with amplitude $A = 10mV$ and equal duration. Assuming equiprobable transmitted bits with bit rate 10^4 bits/s and an AWGN with zero mean and power spectral density $\frac{N_0}{2} = 10^{-9}$ Watts/Hz

(a) draw the block diagram of an optimal detector using a matched filter

(b) show that an integrate-and-dump circuit can be used instead of matched filter with the same performance

(c) find the probability of error P_e at the output of matched filter

(d) if the bit rate increases to 10^5 bits/s what value of A is required to get the same P_e as in the previous case

3. **Binary Detection** (20 points)
In a polar binary signaling $s_i(t), i = 1, 2$, are $+1$ and -1 Volts signal during the interval $(0, T)$. An AWGN with zero mean and power spectral density $\frac{N_0}{2} = 10^{-5}$ Watts/Hz is added to the signal.

(a) determine the maximum bit rate that can be sent with a BER of $P_e \leq 10^{-4}$

(b) repeat the previous question for the unipolar $s_i(t)$ with +1 and 0 Volts

(c) supposed you are asked to keep the bit rate and BER of the unipolar signaling the same as that in the polar case in part (a). What parameter would you change? and to what extent?

4. **Orthogonal Signaling** (20 points)

A binary signaling with $s_1(t)$ and $s_2(t)$ is called orthogonal if $\int_0^T s_1(t)s_2(t)dt = 0$, where T is the duration of the pulses. Assuming equally likely signals and AWGN with power spectral density $\frac{N_0}{2}$,

(a) show that BER can be written as $P_e = Q\left(\sqrt{\frac{E_1+E_2}{2N_0}}\right)$ where E_1 and E_2 are energy of $s_1(t)$ and $s_2(t)$.

(b) what is the BER in terms of E_b/N_0?

(c) show that $s_1(t) = \sin m\omega_0 t$ and $s_2(t) = \sin n\omega_0 t$, $\omega_0 = 2\pi/T$, are orthogonal for integer m and n, and $m \neq n$.

(d) find BER for part (c) in terms of T and N_0.

TABLE A4.1 *Table of the values of Q-function*[a]

x	$Q(x)$	x	$Q(x)$
0.0	0.50000	2.2	0.01390
0.1	0.46017	2.3	0.01072
0.2	0.42074	2.4	0.00820
0.3	0.38209	2.5	0.00621
0.4	0.34458	2.6	0.00466
0.5	0.30854	2.7	0.00347
0.6	0.27425	2.8	0.00256
0.7	0.24196	2.9	0.00187
0.8	0.21186	3.0	0.00135
0.9	0.18406	3.1	0.00097
1.0	0.15866	3.2	0.00069
1.1	0.13567	3.3	0.00048
1.2	0.11507	3.4	0.00034
1.3	0.09680	3.5	0.00023
1.4	0.08076	3.6	0.00016
1.5	0.06681	3.7	0.00011
1.6	0.05480	3.8	7.24×10^{-5}
1.7	0.04457	3.9	4.81×10^{-5}
1.8	0.03593	4.0	3.17×10^{-5}
1.9	0.02872	4.30	0.85×10^{-5}
2.0	0.02275	4.65	0.17×10^{-5}
2.1	0.01786	5.00	0.03×10^{-5}

[a] Table A4.1 is adapted from Abramowitz and Stegun (1964), pp. 966–972. This handbook tabulates the Gaussian (normal) probability density function

Figure 1: Table of the values of Q-function