Chapter 1

Cognitive Radio Networks: An Information Theoretic Perspective

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Abstract

Information-theoretic limits of cognitive radio networks have been under exploration since more than a decade ago. Although such limits are unknown for many networks, including the simplest case with two pairs of transmitter-receiver, there are several cases for which the capacity limits are obtained either exactly or up to a constant gap. The goal of this chapter is to provide insights into the nature of transmission techniques associated with optimal communication when cognitive radio technology is used. Outlining the state of the art in the information-theoretic analysis of different cognitive systems, we highlight the salient features/points of the capacity-achieving or capacity-approaching strategies that should be considered in wireless network design paradigms based on this technology. In particular, we emphasize on the interaction of cognitive radio with emerging technologies for 5G networks.

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1. Introduction

Cognitive radios are intelligent communication devices that exploit information about their environment to increase the spectral efficiency of communication over a given spectrum band. Cognitive radio communication is one of the promising technologies for improving spectrum utilization in the fifth generation (5G) of wireless communication systems. With an eye toward 5G networks, this chapter surveys the fundamental limits of communication and associated transmission techniques for various wireless network design paradigms based on this promising technology.

The idea of cognitive radios was born out of the spectrum shortage in the form of various solutions in which new devices were allowed to exploit the spectrum of coexisting noncognitive devices while impacting noncognitive users’ communication only minimally. Cognitive radios sense their environment, employ advanced radio and signal processing techniques and use novel spectrum allocation policies to improve spectral utilization by concurrently transmitting or interweaving their signals with those of existing users.

From an information-theoretic perspective, “awareness” of a cognitive node about other nodes is abstracted as side information which can be any information about those nodes activity (transmission/reception time), channels state information (CSI), messages, codebooks, etc. Cognitive communication is then referred to a communication system in which each cognitive node can make use of any side information about other nodes with which it has a shared spectrum. Figure 1 models the simplest cognitive radio network in which there is one noncognitive transmitter (Tx1) and one cognitive transmitter (Tx2) as well as their corresponding receivers (Rx1 and Rx2). Note that, in Figure 1, the direction of side information is what differentiates the cognitive and noncognitive users.

It is worth mentioning that, in general, depending on the availability of side information three types of behavior can be defined for the transmitters. (a) Competitive: neither of the transmitters has knowledge of the other transmitter’s side information. (b) Cognitive: only one transmitter (namely, the cognitive transmitter) has knowledge of the other user’s side information (see Figure 1). (c) Cooperative: both transmitters have knowledge of the other user’s side information. Throughout this chapter, we focus on the cognitive behavior.

1.1. Cognitive Radio Network Paradigms

Depending on the type of available network side information and the regulatory constraints, cognitive radio networks can be divided into three main paradigms [1]: interweave, underlay, and overlay. While in the last two cases, the cognitive users concurrently transmit over...
the same spectrum as the primary users, in the first case cognitive users use spectrum holes (temporary space-time frequency voids) for transmission.

- **Interweave (interference avoidance):** In the interweave paradigm, cognitive users ‘opportunistically’ use the spectrum so that their activity does not interfere the activity of noncognitive users. In other words, they only transmit during spectrum holes. To avoid interfering with noncognitive users, cognitive users require knowing the activity information of the noncognitive users in the shared spectrum. This paradigm, which is the simplest yet the most common paradigm was the original motivation for cognitive radio.

- **Underlay (interference control):** In this paradigm, the cognitive users can transmit over the same spectrum as the noncognitive users provided that the interference seen by the noncognitive users is maintained to an acceptable level, i.e., certain QoS should be satisfied. The cognitive users are often called secondary users in this paradigm as they are not allowed to significantly interfere with the communication of noncognitive (primary) users. Thus, they require the knowledge of the “acceptable levels” of interference at the primary users.

- **Overlay (interference mitigation):** Similar to the underlay paradigm, in the overlay paradigm cognitive users can transmit simultaneously with the noncognitive users. The main difference is that the cognitive users have the knowledge of the noncognitive users’ and possibly their messages codebooks in addition to their channel gains. Thus, the cognitive users can allocate part of their power to relay the noncognitive users’ message. This can help boost the information rate at the noncognitive receivers. On the other hand, the interference to cognitive users can be mitigated or even canceled by using this side information (knowledge of codebooks).

It is worth noting that the first paradigm is also be referred to as opportunistic spectrum access and the other two paradigms may also be referred to as concurrent spectrum access [2]. Unless otherwise stated, in this chapter cognitive radio refers to overlay cognitive radio.

### 1.2. Chapter Outline

The chapter is structured as follows. In Section 2., we first define achievable rates and capacity region for the cognitive interference channel. We then give a comprehensive summary of the capacity results established for this channel. Our survey begins with the works on the simplest cognitive network, i.e., a network consisting of two pairs of transmitter-receiver, one cognitive and one noncognitive. We will cover both discrete memoryless and Gaussian channels. This will be followed by stating fundamental results for \( K \)-user and multi-antenna cognitive interference channels. Our goal is not just to show how these capacity regions can be obtained but to get intuition into the optimal communication over this basic channel. In fact, rather than the capacity regions per se, the techniques used to get such regions are important in this study. Such insight can be used to extend the results to more complex networks. In Section 3., we briefly describe the interplay cognitive radio and emerging techniques in wireless communication. This is followed by future research directions in Section 4., which includes open problems. We conclude the chapter in Section 5.
2. Cognitive Radio Channels: Capacity Results and Intuitions

Information theory provides a framework for analyzing the fundamental limits of communication. Fundamental limits can then be used as benchmarks for the operation of the desired communication system (cognitive radio networks here). This, in turns, allows researchers and engineers to measure to what extent a practical network is efficient and also guides them in the design and standardization phases.

The two-user interference channel (IC) is a two-transmitter two-receiver network, in which each transmitter has an independent message for its respective receiver [3–7]. The transmitters do not have side information about the other user’s communication. Since users communicate over a shared channel, they interfere with each other. In the cognitive radio communication setting, one transmitter (cognitive transmitter) is able to sense the environment and obtain side information about the other transmitter (noncognitive or primary transmitter). Such a communication channel is called cognitive interference channel, also known as interference channel with “unidirectional” cooperation, or simply cognitive channel. We formally define this channel and its derivatives in the following.

2.1. Discrete Memoryless Channel

Consider a two-user discrete memoryless cognitive interference channel (DM-CIC), depicted in Figure 2, in which user 1 and user 2 wish to transmit independent messages $M_1$ and $M_2$, respectively, to their corresponding receivers.\(^1\) This channel is defined by a tuple $(X_1, X_2; p(y_1, y_2|x_1, x_2); Y_1, Y_2)$ where $X_1, X_2$ and $Y_1, Y_2$ are input and output alphabets and $p(y_1, y_2|x_1, x_2)$ is channel transition probability function. A $(2^{nR_1}, 2^{nR_2}, n, \epsilon_1^n, \epsilon_2^n)$ code for this channel consists of two independent messages $M_i$, $i \in \{1, 2\}$, two encoding functions $f_i$, two decoding functions $g_i$, and two average probability errors $\epsilon_i^n$, in which

1. $M_i$ is uniformly distributed over $[1, 2, \ldots, 2^{nR_i}]$.
2. encoder $i$ assigns a codeword $x_i^n(m_i)$ to each message $m_i$
3. decoder $i$ assigns an estimate $\hat{M}_i \in [1, 2, \ldots, 2^{nR_i}]$ to each received sequence $y_i^n$, and
4. $\epsilon_i^n = p(\hat{M}_i \neq M_i) = \frac{1}{2^{nR_i}} \sum_{m_i=1}^{2^{nR_i}} p(\hat{M}_i \neq m_i)$.

A rate pair $(R_1, R_2)$ is achievable if there exist a sequence of codes $(2^{nR_1}, 2^{nR_2}, n, \epsilon_1^n, \epsilon_2^n)$ with $\epsilon_1^n \to 0$ and $\epsilon_2^n \to 0$. The capacity region of this channel is the closure of the set of achievable rates.

Motivated by cognitive radio’s promise to increase the spectral efficiency in wireless systems, the study of interference channel with cognitive users has been receiving increasing attention during the past years. Fundamental limits of the cognitive interference channel, in which the cognitive transmitter non-causally knows the full message of the primary user has been studied in [8–17]. This channel was first introduced in [8] where the authors obtained achievable rates by applying Gel’fand-Pinsker coding [18] to the celebrated Han-Kobayashi encoding [6] for the IC. The capacity of this channel remains unknown in

\(^1\)We should highlight that this channel models the overlay paradigm, discussed earlier in this chapter.
Figure 2. The two-user discrete memoryless cognitive interference channel (DM-CIC). Message $M_1$ is known to both Encoder 1 and Encoder 2, indicating that Encoder 2 corresponds to the cognitive user. $M_2$ is known only to the cognitive encoder. $X_1$ and $X_2$ are the channel inputs, $Y_1$ and $Y_2$ are the channel outputs, and $p(y_1,y_2|x_1,x_2)$ is the channel transition probability.

Table 1. The summary of capacity results for the DM-CIC.

<table>
<thead>
<tr>
<th>Label</th>
<th>DM-CIC class</th>
<th>Condition</th>
<th>Capacity region</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_I$</td>
<td>cognitive-less-noisy</td>
<td>$I(U;Y_1) \leq I(U;Y_2)$</td>
<td>$R_1 \leq I(U;Y_1)$</td>
<td>[19]</td>
</tr>
<tr>
<td>$C_{II}$</td>
<td>strong interference</td>
<td>$I(X_1,X_2;Y_1) \leq I(X_1,X_2;Y_2)$</td>
<td>$R_1 \leq I(X_1;Y_1)$</td>
<td>[11]</td>
</tr>
<tr>
<td>$C_{III}$</td>
<td>weak interference</td>
<td>$I(X_1;Y_1) \leq I(X_1;Y_2)$</td>
<td>$R_1 \leq I(U;X_1;Y_1)$</td>
<td>[10]</td>
</tr>
<tr>
<td>$C_{IV}$</td>
<td>better-cognitive-decoding</td>
<td>$I(U,X_1;Y_1) \leq I(U,X_1;Y_2)$</td>
<td>$R_1 + R_2 \leq I(U,X_1;Y_1) + I(X_1;Y_2</td>
<td>U,X_1)$</td>
</tr>
<tr>
<td>$C_{IV}$</td>
<td>cognitive-more-capable</td>
<td>$I(X_1,X_2;Y_1) \leq I(X_1,X_2;Y_2)$</td>
<td>$R_1 \leq I(U,X_1;Y_1)$</td>
<td>[20]</td>
</tr>
</tbody>
</table>

The capacity of the DM-CIC is known for several classes, including the cases in which the cognitive user is less noisy or more capable than the primary user, as well as weak and strong interference regimes. These capacity regions and their corresponding conditions are listed in Table 1. It can be checked that $C_I \subset C_{II} \subset C_{III} \subset C_{IV}$ [20] and $C_{IV} \equiv C_{III}$ [22]. For all of the above cases, the cognitive receiver has a better condition (more information) than the primary one in some sense, as it can be understood from the corresponding conditions in Table 1. It is important to note that the cognitive-more-capable channel (labeled $C_{IV}$) includes all other cases as its subcases (see Fig. 3 and [20]). For this reason, it suffices to discuss the achievable scheme for this case.

The achievability scheme of the capacity region of the cognitive-more-capable is based on superposition coding at the cognitive transmitter. With sophisticated schemes, which combine other techniques such as rate-splitting and Gel'fand-Pinsker coding (binning) with superposition coding, one may enlarge the achievable rate region when $I(X_1,X_2;Y_1) \not\leq I(X_1,X_2;Y_2)$ [13, Theorem 7]. However, it is not clear how much gain this complication.
Figure 3. The class of the DM-CIC. The cognitive receiver is superior to the primary receiver for the cognitive-more-capable and all its subclasses.

brings in. In addition, such techniques (e.g., binning) are too complicated to be used in practical networks.

It is worth pointing out that the capacity region of the cognitive-more-capable DM-CIC, given in Table 1, is the same as the capacity region of the DM-CIC in which the cognitive receiver (Receiver 2) needs to decode both messages. The capacity region in the latter case is obtained in [23, Theorem 4]. Interestingly, this additional constraint, i.e., the constraint that the cognitive receiver must also decode $M_2$, leads to the determination of the capacity region of the DM-CIC for any channel condition. On the contrary, the cognitive-more-capable DM-CIC by definition implies $I(X_1, X_2; Y_1) \leq I(X_1, X_2; Y_2)$; that is, the capacity region of this channel is valid only if the aforementioned condition on the channel holds. Comparing the two capacity results, we conclude that in the cognitive-more-capable DM-CIC channel the cognitive receiver can decode both messages.

Remark 1. Superposition coding at the cognitive transmitter is the capacity-achieving technique in all above cases. Nonetheless, more complicated techniques, such as rate splitting and banning, are reported to result in a larger achievable region, in general.

Remark 2. When the cognitive transmitter and receiver are in the vicinity of the noncognitive transmitter and far away from the noncognitive receiver, there is a high possibility for the cognitive receiver to be more capable the noncognitive receiver; i.e., $I(X_1, X_2; Y_2) \geq I(X_1, X_2; Y_1)$ hold. As discussed earlier, in such a case, superposition coding at the cognitive transmitter is optimal.

2.2. Gaussian Channel

In this subsection, we study the two-user Gaussian cognitive interference channel (GCIC). We first describe the channel model and then summarize the previously known results for the GCIC as well as the one-sided GCIC.
2.2.1. Two-User Gaussian Channel

The two-user Gaussian cognitive interference channel, depicted in Figure 4, is composed of two transmitter-receiver pairs in which each transmitter communicates with its corresponding receivers while interfering with the other receiver. This model is very similar to that of the two-user Gaussian interference channel; the only difference is in that the cognitive transmitter knows the message (and possibly the codewords) of the primary user. This flow of information is shown by the dashed line in Figure 4.

Without loss of generality, we use the standard form of the Gaussian interference channel [24], in which, for a single channel use, the channel is expressed by

\[ Y_1 = X_1 + aX_2 + Z_1, \]
\[ Y_2 = bX_1 + X_2 + Z_2, \]

where \( a \) and \( b \) are two non-negative real numbers representing the crossover gains; and, for \( j \in \{1, 2\} \), \( X_j, Y_j, \) and \( Z_j \), respectively, represent the transmitted signal, received signal, and the channel noise, and \( Z_1 \) and \( Z_2 \) are independent and identically distributed (i.i.d.) Gaussian random variables with zero means and unit variances. Let \( M_1 \) and \( M_2 \) be two independent messages uniformly distributed over \( M_1 = [1, \ldots, 2^{nR_1}] \) and \( M_2 = [1, \ldots, 2^{nR_2}] \), respectively.\(^2\) Transmitter \( j \) wishes to transmit message \( M_j \) to receiver \( j \) in \( n \) channel uses at rate \( R_j \), and \( X_j \) is subject to an average power constraint \( P_j \), i.e.,

\[ \frac{1}{n} \sum_{i=1}^{n} \|X_{ji}\|^2 \leq P_j, \quad j = 1, 2. \]

The capacity region of this channel is defined as the set of all rate pairs \((R_1, R_2)\) for which each receiver is able to decode its own message with arbitrarily small probability of error.

\(^2\)For \( j \in \{1, 2\} \), \( M_j \) is a random variable distributed over set \( M_j \), and \( m_j \) is a realization of \( M_j \).
Capacity region of the Gaussian cognitive interference channel is known at certain interference regimes. All capacity regions know for this channel are based on using a combination of dirty paper coding (DPC) [25] and superposition coding at the cognitive user. Before stating the capacity region, we discuss this achievable region in the following.

As discussed earlier, in the cognitive interference channel, the cognitive user knows the noncognitive user’s messages and codewords. This signifies that the cognitive user can use this knowledge to cancel the interference received from the noncognitive user via DPC. On the other hand, to compensate for the interference the noncognitive transmitters creates on the cognitive receiver and, thus, to improve the achievable rate at the noncognitive receiver, it would be useful if the cognitive user allots part of its power to help send the codewords of the primary user. The latter scenario implies superposition coding.

Not surprisingly, an optimal encoding strategy at the cognitive transmitter is to use DPC to encode $M_2$ while treating $X_1$ as interference and, then, superimpose $M_1$ on top of that to help convey $M_1$ to Receiver 1. Superposition coding implies that the cognitive user partially uses its power to help send the codewords of the primary user. $X_2$ contains two independent Gaussian parts, $X_2 = \sqrt{\alpha P_2}V_1(m_1) + \sqrt{\alpha P_2}V_2(m_2)$, in which $V_1$ and $V_2$ are auxiliary random variables used to encode $m_1$ and $m_2$, respectively, and $0 \leq \alpha \leq 1$ and $\bar{\alpha} = 1 - \alpha$. The primary user, however, does not have a knowledge about the cognitive user’s messages; thus, it uses its whole power to transmit $m_1$, i.e., $X_1 = \sqrt{P_1}V_1(m_1)$.

For decoding, one strategy is to let the noncognitive receiver (Receiver 1) simply decode its own codeword assuming the other codeword as interference. From (1a), it is seen that $Y_1 = \sqrt{P_1}V_1(m_1) + a\sqrt{\alpha P_2}V_1(m_1) + a\sqrt{\alpha P_2}V_2(m_2) + Z_1$. This indicates that $(\sqrt{P_1} + a\sqrt{\alpha P_2})V_1(m_1)$ is the useful signal at Receiver 1 while $a\sqrt{\alpha P_2}V_2(m_2)$ is the interference. Therefore, $R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\sqrt{P_1} + a|\sqrt{\alpha P_2}|^2}{1 + a^2\bar{\alpha} P_2} \right)$ is achievable by treating the interference as noise. On the other hand, in view of (1b), the signal seen by the cognitive receiver can be expressed as $Y_2 = (b\sqrt{P_1} + \sqrt{\alpha P_2})V_1(m_1) + \sqrt{\alpha P_2}V_2(m_2) + Z_2$. Due to the DPC at the cognitive transmitter, the cognitive receiver can cancel the interference $V_1(m_1)$; thus, $R_2 \leq \frac{1}{2} \log (1 + \bar{\alpha} P_2)$ is achievable. Finally, considering the error analysis for the sum rate, the above encoding and decoding result in the following achievable rate region [9]:

**Lemma 1.** The set of rate pairs $(R_1, R_2)$ satisfying

\[
R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\sqrt{P_1} + |a|\sqrt{\alpha P_2}|^2}{1 + a^2\bar{\alpha} P_2} \right), \tag{3a}
\]

\[
R_2 \leq \frac{1}{2} \log (1 + \bar{\alpha} P_2), \tag{3b}
\]

\[
R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + P_1 + a^2 P_2 + 2|a|\sqrt{\alpha P_1 P_2} \right), \tag{3c}
\]

in which $0 \leq \alpha \leq 1$ and $\bar{\alpha} = 1 - \alpha$ is achievable for the cognitive interference channel.

The above rate region simplifies to the capacity region of the cognitive interference channel under certain channel conditions, as listed below.

- **Weak interference** ($|a| \leq 1$) [9, 10]: In this regime, the optimal encoding strategy at the cognitive transmitter is to use DPC and superposition coding, as explained in the achievability of the above rate region. In particular, since the interference channel
gain is small, Receiver 1 does not attempt to decode the interference. It simply treats interference as noise and this turns out to be the optimal solution. Moreover, it can be checked that (3c) is redundant for $|a| \leq 1$ and the capacity region is obtained by

$$R_1 \leq \frac{1}{2} \log \left( 1 + \frac{\sqrt{P_1} + |a| \sqrt{\alpha P_2}}{1 + a^2 \alpha P_2} \right),$$

(4a)

$$R_2 \leq \frac{1}{2} \log (1 + \alpha P_2).$$

(4b)

- **Strong interference** ($|a| > 1$): In this case, since $|a| > 1$, the interference at Receiver 1 is stronger than that in the weak interference case. As a result, depending on the value of $|a|$, decoding $M_1$, or a part of the interference (unwanted message), can be beneficial. In [11, Theorem 6] it is proved that both users can decode both messages when $|a| \geq 1$, $|b\gamma - 1| \geq |a - \gamma|$, and $|b\gamma + 1| \geq |a + \gamma|$ where $\gamma \triangleq \sqrt{P_1/P_2}$. In such a case

$$R_2 \leq \frac{1}{2} \log (1 + \alpha P_2),$$

(5a)

$$R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + P_1 + a^2 P_2 + 2 |a| \sqrt{\alpha P_1 P_2} \right),$$

(5b)

characterize the capacity region.

In [17], it is shown that the encoding and decoding strategy resulting Lemma 1 can be optimal when $|a| \geq 1$. Specifically, it is shown that the above inequalities also give the capacity region when $|a| \geq 1$ and $P_1 |1 - |a||^2 \geq (|a|^2 - 1)(1 + P_2 + |b|^2 P_1) - P_1 P_2 |1 - |a||^2$. Noting that (3a) is redundant for $|a| \geq 1$, we can see that (5) is also the capacity region in this case. The above two set of conditions be both valid under certain channel realizations. This indicates that more than one scheme can be optimal at least at certain channel conditions. Specifically, in the above cases the interference is canceled in two radically different ways, i.e., by decoding and then canceling it versus using DPC. The former does not require any information at the encoder while the latter requires knowing the interference at the encoder and applies a very complex encoding.

- **Cognitive receiver needs to decode both messages**: The capacity region of the Gaussian cognitive interference channel is also known when the cognitive receiver needs to decode both users’ message [23].

### 2.3. Gaussian Z-Channel

A Z-Channel (or one-sided interference channel) models a two-transmit two-receiver scenario in which one of the users does not experience interference. In a cognitive channel, due to asymmetric transmitters in which only one transmitter has information about the other, two different ZICs are conceivable: one with no interference at the noncognitive receiver

\[\text{In this case, the channel becomes a compound multiple access channel (MAC) and the capacity region of compound MAC is applicable.}\]
Consider a two-user cognitive Gaussian interference channel in which \( b = 0 \), as shown in Figure 5. The capacity region of this channel is established in several ranges of interference gain \([14–17]\): these results are summarized in Table 2. While in the low interference regime a combination of dirty paper coding and superposition coding is the capacity-achieving scheme, in the high interference regime superposition coding single-handedly can achieve the capacity region. From this table, it is clear that the capacity region of the cognitive Z-Channel is unknown only when \( \sqrt{1 + P_1 / (1 + P_2)} < |a| < \sqrt{1 + P_1} \).

It is known that time-sharing can increase the achievable rates for the interference channel and one-sided interference channel \([6, 26]\). Similarly, time-sharing can increase the secrecy capacity of the IC, see \([27, \text{Lemma 3}]\), for example. It would be interesting to apply time-sharing to the secrecy capacity of cognitive interference channel.
2.4. Capacity Approximation

Finding the exact capacity region for many multi-user channel networks has appeared to be daunting challenging. Considering this difficulty, one way to get insights into the behavior of different multi-user channels is to resort to approximation. Two approximation metrics have gained significant attention during the past decade. These are degrees of freedom (DoF) and generalized degrees of freedom (GDoF) respectively.

- **degrees of freedom (DoF):** The DoF or the *multiplexing gain* is a means of approximating the sum capacity of a channel/network.\(^4\) It gives the *pre-log* of the sum-rate capacity of a given multi-user channel in the high SNR regime. Although rather coarse, DoF provides an analytically tractable way to characterize the sum capacity in a given multi-user channel in the high SNR regime. For example, the DoF for \(K\)-user Gaussian interference channel is shown to be \(\frac{K}{2}\), and can be achievable through the interference alignment (IA). This means that each user can enjoy half of the spectrum in the high SNR regime.

- **generalized degrees of freedom (GDoF):** The GDoF generalizes the notion of the DoF into different SNR regimes and, thus, is a much more powerful metric. The GDoF is also known as the capacity region to within a constant gap. The insight obtained for the DoF may not hold true for the GDoF. As an important example, it is known that the relay does not increase the DoF of the interference channel with relay whereas it can increase the GDoF of that channel \([28]\).

While the DoF and GDoF approximate the sum-capacity, there are also metrics to determine either an *additive* or a *multiplicative* gap between the inner and outer bounds for a certain channel, rather than only their sum-capacity. An additive gap between the inner and outer bounds is useful at high signal-to-noise power ratios (SNR) because in such a regime the difference between inner and outer bound is small in comparison to the magnitude of the capacity region. A multiplicative gap is useful at low SNR, where the ratio between the inner and outer bounds can be a better indicator of their distance.

Etkin et al. obtained an approximation of the capacity region of the real-valued two-user Gaussian interference channel to within \(\frac{1}{2}\) bits in \([29]\). Rini et al. \([17]\), found the capacity region of the two-user real-valued Gaussian cognitive interference channel to within 1.87 bits/s/Hz. This constant gap was obtained by using insights from the high SNR deterministic approximation of the Gaussian cognitive interference channel. Additive gap on the capacity region of this channel is known to within 1.87 bit/s/Hz \([17]\) while the multiplicative gap is known to within 2 bits/s/Hz. To achieve the multiplicative gap one can use a simple *time-sharing* between the following two achievable points:

\[
A = (R_1^A, R_2^A) = \left( \frac{1}{2} \log \left( 1 + (\sqrt{P_1} + |a| \sqrt{P_2})^2 \right), 0 \right),
\]

\[
B = (R_1^B, R_2^B) = \left( 0, \frac{1}{2} \log (1 + P_2) \right).
\]

It can be seen that to achieve the point \(A\) the cognitive user sacrifice its rate and only transmits the codewords of the noncognitive user. On the other hand, to achieve the point \(B\)

\(^4\)DoF region is a similar metric which studies both individual and sum rates.
the noncognitive user must be silent while the cognitive user transmits only its own code-words. Finally, for different values of $a$, the additive gap is obtained by applying different achievable schemes in [17, Table II].

2.5. Further Capacity Results

2.5.1. Secrecy Capacity

The secrecy capacity of two-user DM-CIC is studied in [23,30,31]. In [23] it is assumed that $M_2$ is confidential and needs to be kept secret from noncognitive receiver (Rx1 in Fig 2); in addition, it is assumed that the cognitive receiver decodes both messages whereas the noncognitive receiver decodes only message 1. This is different from [30] in that it is not assumed that the cognitive receiver decodes both messages; it also does not assume that the cognitive transmitter knows the other user’s message. The DM-CIC with two confidential messages is studied in [31], in which both primary and cognitive messages must be secure at unintended receivers.

2.5.2. Multi-User Channels

So far we have focused on the two-user channels which include one noncognitive and one cognitive user. In general, multiple cognitive and multiple noncognitive users in the overlay network can simultaneously share the same spectrum. The extension of the capacity results of the two-user channels to 3-user and, in general, $K$-user channels is not straightforward. To find fundamental limits of these channels, the techniques used for establishing the capacity results in the previous sections can be used. For example, the rate-splitting approach can be generalized as a way to cope with interference from multiple senders. However, such a scheme becomes extremely complicated when the number of users increases. Interference alignment can be promising approaches for the $K$-user interference channel. Interference alignment in cognitive nodes can reduce the interference at both the noncognitive and the cognitive receivers. Recall that cognitive users can perform relaying of noncognitive messages and precoding against interference. Understanding the interplay between these techniques is an important and interesting research topic. A survey on multi-user cognitive interference channels can be found in [32].

The capacity region of the multicast cognitive interference channel in which each transmitter wishes to transmit an independent message to a set of users is investigated [33]. This channel can be seen as a two-user cognitive interference channel in which user 1 and user 2 wish to transmit independent messages $M_1$ and $M_2$, respectively, to $Y_{11},\ldots,Y_{1N_1}$ and $Y_{11},\ldots,Y_{1N_2}$, where $N_1 \geq 1$ and $N_2 \geq 1$ are arbitrary integers. The paper has interesting capacity results for multi-primary ($N_1 \geq 2$ and $N_2 = 1$) and multi-secondary ($N_1 = 1$ and $N_2 \geq 2$) cognitive interference channels in various interference regimes, including very strong, very weak, and mixed very weak/strong interference regimes. These capacity results are mainly the extensions of the capacity results in [9,10] and are a step forward toward the scenarios where multiple users wish to communicate over the same chunk of spectrum.
2.5.3. MIMO Channel

Multiple-input and multiple-output (MIMO) communication can also be exploited in the cognitive radio networks as a potential method for the spectrum sharing. Multiple antenna techniques can be used for throughput enhancement and interference cancellation. Fundamental limits of MIMO cognitive radio has been studied in the literature. Most of the results, however, discuss the MIMO cognitive interference channel from the DoF perspective, either with perfect or delayed channel state information at transmitter (CSIT) [34, 35].

It is known that cognitive message sharing can increase the sum DOF of the MIMO cognitive interference channel for certain scenarios. Further, in terms of sum DOF, having a cognitive transmitter is more beneficial than having a cognitive receiver. Specifically, for a MIMO Gaussian interference channel with $L_1, L_2$ antennas at transmitters and $N_1, N_2$ antennas at receivers the following DoF results are obtained in [34].

- **cognitive message sharing**: For the case of cognitive message sharing in which only the transmitter of the secondary user (transmitter 2) knows the message of transmitter 1, the sum DOF is given by

$$\min\{L_1 + L_2, N_1 + N_2, \max(L_2, N_1)\}. \quad (8)$$

Note that this is an information-theoretic setting where the message of the primary user is provided by a *genie* to the transmitter of the secondary user *noncausally* and *without noise*.

- **cooperation at transmitters**: User cooperation refers to the case where several distributed nodes can cooperate with each other to form a transmit antenna array or a receive antenna array. The links between cooperating transmitters or cooperating receivers are assumed to be *noisy*. For the case of users’ cooperation (be it at the transmitters side, receivers side, or both sides), the sum DOF is

$$\min\{L_1 + L_2, N_1 + N_2, \max(L_1, N_2), \max(L_2, N_1)\}. \quad (9)$$

Note that (9) is the same as the sum DOF of the channel without cooperation [34]. Thus, cooperation via noisy link cannot increase the sum DoF of the MIMO interference channel whereas message sharing can increase it. Nonetheless, it should be highlighted that both techniques may increase the sum capacity of the MIMO interference channel.

*Remark* 3. Message sharing can increase the sum DoF of the MIMO interference channel as well as its sum capacity.

In [35], the DoF region of MIMO cognitive interference channel is obtained when CSIT is not available. Interestingly, it is shown that CSIT is not necessary for DoF-optimal performance at certain antenna configurations, e.g., when $N_2 \geq N_1 \geq L_2$

3. Cognitive Radio and 5G Technologies

Wireless communication systems have undergone a revolution about once every decade. Such a revolution leads to a completely new standard making a new generation of wireless
networks. Expected to commercialized around 2020, the 5th generation (5G) mobile networks must support about 1000 times higher system capacity than current 4G systems, as well as 10 times less latency, and about 100 times more devices. To provide such a huge system capacity, three key approaches have been suggested: network densification, adding a large quantity of new bandwidth, and increasing spectral efficiency.

Cognitive radio is one of the technologies that can, in conjunction with several other promising technologies, address the spectrum scarcity problem. In this section, we study the interplay between cognitive radio and emerging 5G technologies such as massive MIMO [36], cloud radio access networks (cloud RAN) [37–39], mmWave communication [40], non-orthogonal multiple access (NOMA), full-duplex, etc. The goal is to understand how each potential technology can be combined with the cognitive radio to increase the spectral and energy efficiency of wireless systems.

3.1. Interference Management in 1G-4G

Practical interference management approaches can be divided into two main categories:

• **ignore interference:** When interference is sufficiently weak then it is usually ignored by treating it as noise. Such an approach deals with *signal levels*. Treating interference as noise is proven to be optimal in achieving the sum capacity of the interference channel at very weak interference regime.

• **avoid interference:** To avoid interference, usually orthogonal multiple access methods such as frequency division multiple access (FDMA), time division multiple access (TDMA), code division multiple access (CDMA), and orthogonal frequency division multiple access (OFDMA) is employed. These approaches deal with *signal space*. Strong interferers can also be avoided by decoding and canceling it. Using fractional frequency reuse (FFR) is another way to avoid interference in practical wireless networks. FFR orthogonally allocates frequency at the cell-border regions in which intercell interference is usually high.

In 1G-4G wireless technologies, the above orthogonal strategies have been adopted for interference management. While the underlay and interweave cognitive radio systems can operate with the above mentioned multiple access techniques, the overlay cognitive radio proposes an inherently different approach, as it implies using the same frequency/time for cognitive and non-cognitive users. As such, the overlay cognitive radio requires non-orthogonal multiple access techniques, as described in the following section.

3.2. Cognitive Radio and NOMA

Wireless systems must provide service to multiple users concurrently. Multiple access is a technique that allows multiple users to share an allotted spectrum (a channel) in an effective manner. Multiple access schemes are commonly designed to share the channel *orthogonally*. For example, multiple access schemes in 1G-4G cellular networks, i.e., TDMA, FDMA, CDMA, and OFDMA, all are orthogonal multiple access (OMA) schemes. This is because in these schemes access to the channel is orthogonalized in time, frequency, or
code domain. That is, no two users share the same spectrum at the same time or using the same code. The rationale behind such orthogonal access methods is to avoid inter-user interference which, in turn, makes signal detection simpler. However, due to this resource rationalization, OMA techniques can support a limited number of users and have low spectral efficiency. While exponentially increasing number of devices, mostly Internet of Things (IoT) devices, are being introduced to wireless communication networks, there has been a flurry of research activity on new types of multiple access methods, random access methods, and waveform design that can accommodate such massive number of devices in 5G and beyond networks [41].

Non-orthogonal multiple access (NOMA), in contrast to OMA, is referred to techniques that allow to scheduled multiple users over a single resource. NOMA can be realized in different domains, including in the code and power domains [41–43]. In the code domain, similar to CDMA, each user has its own code (spreading sequences) for sharing the entire resource, but these codes are not orthogonal. In the power domain, NOMA exploits the channel gain differences between the users for multiplexing via power allocation.

From the information-theoretic perspective, power domain NOMA is merely a new name for a well-established theory. The basic theory of NOMA has been around for several decades under the name of the broadcast channel (BC) and multiple access channel (MAC) in a single-cell setting, and interference channel (IC) in a multi-cell network [44–46]. The new name, NOMA, is coined to differentiate it from the conventional multiple access technique in 1G-4G wireless networks such as TDMA, FDMA, CDMA, and OFDMA. It should be, however, mentioned that although the theory of downlink NOMA (BC) has been around since 1960’s, it has not been implemented mainly due to the complexity associated with successive interference cancellation (SIC) required at the mobile handsets. Today, with the advance of processors it is possible to implement SIC at the user equipment. This has stimulated a large body of research in academia and industry on NOMA for 5G.

Similar to IC and BC, by definition, overlay cognitive radio networks imply non-orthogonal transmission, as they let noncognitive and cognitive users use the same resource concurrently. NOMA cognitive radio may, however, refer to the case where there are multiple noncognitive users or cognitive users. In any case, the theory of overlay cognitive radio networks, discussed in Section 2., can be used to design effective transmit/receive strategies when power domain NOMA is in place. Combination of these two technologies can bring further spectral efficiency in addition to other benefits of NOMA. NOMA can be also applied to underlay cognitive radio networks to improve the outage probability [47].

3.3. Cognitive Radio and Other 5G Technologies

During past several years, a number of other technologies have been considered for inclusion in in 5G in academia, industry, standardization bodies. This includes, but is not limited to, massive MIMO [36], cloud radio access networks (cloud RAN) [37–39], mmWave [40] and full-duplex [48] communication. These technologies can be combined with the cogni-

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5 Although optimal uplink and downlink transmit/receive strategies are unknown for multi-cell networks, in general, a combination of NOMA and OMA results in the largest achievable region [46].

6 It is worth mentioning that complex user terminal capabilities, such as network assisted interference cancelation and suppression (NAICS), has been included in 3GPP LTE-A.
ative radio to increase the spectral and energy efficiency of wireless systems.

While there has been significant attention to combine these technologies with cognitive radio, in most of these works cognitive radio operate either in the interweave or underlay paradigms. Then, there is a big gap in combining these technologies with cognitive radios that operate in overlay paradigm. We believe, the introduction of NOMA to practical wireless networks will pave the road for the implementation of overlay cognitive networks in future wireless networks.

4. Future Research Directions

Cognitive radio has rendered many traditional problems in information and communication theory. It has also uncovered new problems that need research. It is a gold mine of research problems, in particular, in terms of fundamental limits. In this section, we list some of those problems. We also discuss the challenges in bringing those results into practice.

4.1. Open Fundamental Limits

Here, we outline the open information-theoretic problems for the two-user, K-user, and MIMO cognitive interference channels.

- **Two-user cognitive interference channel:** In the discrete memoryless case, as discussed in Section 2.1., the capacity of this channel is known when the cognitive user is more capable than the noncognitive user, i.e., \( I(X_1, X_2; Y_1) \leq I(X_1, X_2; Y_2) \). Otherwise, when \( I(X_1, X_2; Y_1) > I(X_1, X_2; Y_2) \), the capacity region is open. For the Gaussian case, the capacity region is fully characterized for weak interference (\(|a| \leq 1\)). Besides, the capacity region is known for part of the strong interference regime, as discussed in Section 2.5.2. In the Z-interference case, the capacity is open only for \( \sqrt{1 + P_1/(1 + P_2)} < |a| < \sqrt{1 + P_1} \), as can be seen from Table 2. We believe, in the above Gaussian cases, time-sharing can increase the achievable region similar to that of the interference channel described in [7, Lemma 3] and [26, Lemma 1], and the references therein. This technique has been applied to enlarge secrecy achievable region of the Z-interference channel in [27, Lemma 3], and it give a better region compared to the TDM/FDM region, too. Time-sharing is expected to improve the gap between the inner and outer bounds and improve or theoretical knowledge about this channel.

- **K-user cognitive interference channel:** There are very few capacity results for the K-user channels, with \( K > 2 \), including the 3-user channel. Characterizing new capacity results and/or obtaining any insight into the optimal solution for these channels would be very valuable. Another possible direction is to find personably low gaps between the inner and outer bounds if it is not possible to find the capacity region. This can help gain insight on achievable schemes that are not far away from the capacity region.

- **MIMO cognitive interference channel:** In Section 2.5.3., we indicated that capacity region of the MIMO cognitive interference channel is open, even with full CSI. In
contrast, the DoF region of this channel is known both with and without CSI, see [34] and [35], respectively. A bridging step would be to work on GDoF of this channel.

4.2. More Practical Setting

Apart from the DoF region with no CSI in the MIMO setting [35], the results we discussed in the previous subsection depend on both the non-causal knowledge of noncognitive users’ message at the cognitive transmitter and perfect CSI at both transmitters. When this is not the case, dirty paper coding, and interference mitigation techniques in general, may suffer in terms of rate. Cooperative relaying [49] is an interesting direction in relieving the non-causal message knowledge. In the case of imperfect CSI, other notions of capacity such as ergodic capacity or outage capacity can be studied. Most of such results in the literature are either for underlay or interweave cognitive radio, but not overlay cognitive radio systems. In addition, it is very important to understand the tradeoff between the schemed that need learning the channel and interference that should be mitigated and more practical interference management techniques mentioned in Section 3.1.

5. Conclusion

This chapter has provided a high-level overview of recent information-theoretic results for overlay cognitive radio networks, which we believe will become one of the technologies driving the evolution to future cellular systems. The capacity region of the cognitive interference channel has been established under several channel conditions, including the case where the cognitive receiver is more capable than the noncognitive receiver. These results collectively demonstrate that when cognitive users know the noncognitive user’s messages in a non-causal fashion, achievable rates largely increases for both users. These results, however, depend on both the non-causal knowledge as well as having perfect CSI at the transmitters. It would be very interesting to understand the fundamental limits of this channel under delayed CSI or no CSI, and use the insight in the design of practical wireless networks. Specifically, the capacity region is open in most $K$-user cognitive channels, including the MIMO case, even with perfect CSI assumption.

We have also highlighted the large potential of combining cognitive radio with 5G specific technologies such as NOMA, massive MIMO, and cloud RAN in terms of spectral efficiency, energy efficiency, and low latency. There are still several challenges ahead to realize the full potential of the technology, both in theory and practice. This gives researchers a rich research area to work.

References


