Distributed Deployment Algorithms in a Network of Nonidentical Mobile Sensors Subject to Location Estimation Error

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***Abstract*—In this paper, we study sensor deployment algorithms in the presence of location estimation error. Existing Voronoi-based mobile sensor deployment algorithms require location awareness to guarantee a simple coverage detection, and they miss the mark if the location information is inaccurate. But, it is often expensive to include a GPS receiver in each node, and location information is inaccurate as sensors estimate locations from the messages they receive. We propose a new Voronoi-based diagram, named *guaranteed additively weighted Voronoi diagram (GAWVD)*, that guarantees the coverage hole detection for each cell individually, provided that upper bounds on localization errors are assumed. Although location inaccuracy would appear to deteriorate the total coverage, our simulation results demonstrate that the proposed method can exploit this inaccuracy to improve the network coverage.**

I. Introduction

Area coverage is a key design parameter in any wireless sensor network (WSN) as it is required for fulfilling the sensing tasks. Hence, when the network topology changes, maintaining or increasing the total sensing coverage is very important since each sensor can obtain a limited view of the environment, both in range and accuracy, as it can only cover a limited physical area. Mobile sensors can be used to increase the area coverage (reduce the *coverage holes*) by moving toward the correct places, i.e., by adapting the network topology.

To reduce the coverage holes, movement-assisted sensor deployment protocols based on *Voronoi diagrams* are proposed in the literature, both for *regular sensors* [1] and *irregular sensors* (i.e., when the sensing radii of the sensors can be different) [2]. Voronoi-based diagrams facilitate finding the coverage holes by dividing the field into regions such that in each region any point out of the *sensing disk* of the corresponding sensor is a coverage hole, i.e., it cannot be covered by other sensors. That is, a *single-cell-based* coverage analysis is sufficient. The existing Voronoi-based diagrams, and corresponding deployment algorithms, assume that the exact locations of all nodes are known by the other nodes, and they miss the mark if the location information is not accurate. However, practically it is often too expensive to include a GPS receiver in a sensor network node. Instead, each sensor estimates the location of its neighboring nodes by using a localization technique [3]. Therefore, the location information of neighboring sensors is not usually exact at the other nodes.

We are interested in mobile sensor deployment in a general network environment where the prior locations of nodes are not known, the nodes distribution is irregular, their sensing radii are non-identical, and their locations are estimated in the other nodes, and thus may not be exact. We introduce a Voronoi-based diagram called the *guaranteed additively weighted Voronoi diagram (GAWVD)* which is appropriate for finding the coverage holes in the presence of localization error. We then elaborate on how the construction of regions in the GAWVD is simpler than other guaranteed weighted Voronoi diagrams. The development of the GAWVD diagram is such that the coverage hole detection can be carried out in a cell-based manner; this is used in the deployment algorithms which exploit farthest point and minmax point strategies for sensor movements.

The rest of this paper is organized as follows. In Section II we discuss the basics of Voronoi-based diagrams in sensor networks and set the stage to introduce the GAWVD in Section III. The shape of regions in Voronoi-based diagrams is investigated in Section IV. The GAWVD is then used for sensor deployment, along with two movement strategies, in Section V. We evaluate the performance of the proposed algorithms in Section VI. Section VII concludes the paper.

II. Preliminaries and Background

Let $\mathcal{F} \subset \mathbb{R}^2$ be a field containing n sensors s_i, $1 \leq i \leq n$, with sensing ranges r_i, $r_i > 0$, located at planar coordinate $p_i = (x_i, y_i)$. We denote this set of sensors by $\mathcal{S} = \{s_i\}$, where $i \in \mathcal{N} := \{1, 2, \ldots, n\}$. Throughout this paper we assume $i, j \in \mathcal{N}$ and $i \neq j$, unless otherwise stated. We use $d(a, b)$ or $\| b - a \|$ to denote the *Euclidean distance* between two points a and b. The *coverage area of a sensor* (s_i, r_i) located at p_i is a *disk* of radius r_i centered at p_i. A point q is covered by this sensor if and only if $d(q, p_i) \leq r_i$.

A main question in the mobile sensor network deployment is to come up with algorithms that can maximize the total coverage of a desired field ($\mathcal{F}$) for a given number of sensors (n). To this end, it is easier to divide the field into n *cells (regions)* in a proper way, each corresponding to one sensor, and try to maximize the coverage in each cell, which is called the *local coverage*, separately. Voronoi-based diagrams [4] are popular in discovering the coverage hole in the sensor deployment context. However, a basic property of many applied WSNs is that the sensors are non-identical [5], i.e., the sensing radii of the sensors can be different. For such networks, the Voronoi partitioning does not necessarily imply that coverage hole in one cell can be found only based on the coverage of its corresponding sensor. This is because, under such a circumstance, a point that is not covered by the sensor belonging to its own cell may be covered

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by a neighboring sensor. To benefit from the simplicity of single-cell-based coverage hole detection, generalized versions of the Voronoi diagram, known as weighted Voronoi diagrams are proposed in the literature. Some commonly-used weighted Voronoi diagrams, in the context of WSNs, are multiplicatively weighted Voronoi diagram (MWVD) [2], additively weighted Voronoi diagram (AWVD) [6], and power diagram (PD) [7].

III. Guaranteed Additively Weighted Voronoi Diagram

Localization, i.e., determining the location of sensors, is one of the fundamental problems of real-world sensor networks as it is required for the completion of many basic tasks, including packet routing and coverage detection. In practice, it is often too expensive to equip each sensor with localization hardware, e.g. a GPS. Instead, usually a small fraction of nodes are equipped with GPS, and other nodes use a localization technique to find the sensors locations. As a result, the locations of sensors are not perfectly known in the network. This is correct even if the sensors' positions are broadcasted through a central unit, as communication is subject to errors, delay, and other imperfections. The conventional and weighted Voronoi diagrams, discussed in the previous section, require the perfect locations of each sensor and its neighboring sensors; otherwise, these diagrams cannot guarantee a single-cell-based coverage hole detection. The GMWVD and GPD are proposed to tackle this problem [7]. In this paper, we propose a new method based on the *additively weighted Voronoi diagram*. Compared to the GMWVD, this method offers a simpler region construction.

Consider a sensor network in which the locations of sensors are estimated at different nodes and thus are subject to error. Let p_i (the exact location of s_i) be within a disk of radius ϵ_{ij}, $\epsilon_{ij} \geq 0$, centered at p_{ij} (the estimated position of the ith sensor at sensor j); i.e., let ϵ_{ij} denote the maximum error in the estimation of the location of s_i at s_j. Similarly, suppose each sensor measures its own location with some errors; this is represented by p_{ii} and the error bound is assumed to be ϵ_{ii}.

We modify the AWVD in a way that even in the presence of estimation errors a single-cell-based search for coverage holes detection is guaranteed to work within each cell. Note that, if p_{ij} is the estimated position of the ith sensor at sensor j for any $i, j \in \mathcal{N}$, then the exact location of s_i is somewhere within a disk of radius ϵ_{ij} centered at p_{ij}. Hence, the set of all points $q \in \mathcal{F}$ which are surely closer to s_i, in the additively weighted distance sense, than to s_j is characterized by

$$\max_{q_1 \in \mathrm{C}(p_{ii}, \epsilon_{ii})} d(q, q_1) - r_i \leq \min_{q_2 \in \mathrm{C}(p_{ji}, \epsilon_{ji})} d(q, q_2) - r_j. \quad (1)$$

where $\mathrm{C}(a, r)$ denotes a circle of radius r centered at a. Since the maximum possible distance between q and a point in $\mathrm{C}(p_{ii}, \epsilon_{ii})$ is $d(q, p_{ii}) + \epsilon_{ii}$ and the minimum distance between q and a point in $\mathrm{C}(p_{ji}, \epsilon_{ji})$ is $d(q, p_{ji}) - \epsilon_{ji}$, the ith region can be written as

$$\Pi_i^{\mathrm{GAWVD}} = \{q \in \mathcal{F} \mid d(q, p_{ii}) + \epsilon_{ii} - r_i \leq d(q, p_{ji}) - \epsilon_{ji} - r_j\}. \quad (2)$$

Using this diagram, it is guaranteed that, even with estimation error, if a point inside a region is not sensed by its corresponding sensor, no other sensor can sense it either. In other words,

Proposition 1. *In GAWVD, for any $q \in \Pi_i$, $d(q, p_i) > r_i \Rightarrow d(q, p_j) > r_j$.* [1]

Proposition 2. *The regions defined by* (2) *are disjoint.*

IV. Shape of the Regions

A. With perfect Location information

In order to realize to which cell a given point belongs, it is important to find out the borders between cells in Voronoi-based diagrams. To find the locus of points which are on the cell borders of different diagrams, the "$\leq$" needs to be satisfied with "equality" in those regions. Then, (1) in [7] becomes $d(q, p_i) = d(q, p_j)$ which implies q to be on the *perpendicular bisector* of the segment connecting p_i to p_j; i.e., the locus is a line. Next, on the borders of the MWVD ((2) in [7]) we get $\frac{d(q,p_i)}{r_i} = \frac{d(q,p_j)}{r_j}$, or equivalently, $\frac{d(q,p_i)}{d(q,p_j)} = \frac{r_i}{r_j} \triangleq \alpha$, and we have

Proposition 3. *The border between s_i and s_j in the MWVD is an arc of an Apollonian circle of radius $R = \frac{\alpha d(p_i, p_j)}{(1-\alpha^2)}$.*

Similarly, for the AWVD, and PD it can be seen that

Proposition 4. *The borders of regions in the AWVD are composed of arcs of different hyperbolas.*

Proposition 5. *The borders of regions in the PD are composed of lines perpendicular to the segment connecting the neighboring sensors.*

When location information is perfect, the power diagram is preferred to the other weighted Voronoi diagrams from the region construction point of view, because computing the cell boundaries and constructing the regions is simpler. This is particularly important in the context of sensor networks, as a less computationally complex algorithm implies a better energy efficiency. However, when estimation error comes in, this argument is not valid.

B. With Location Estimation Error

The main advantage of the GAWVD to the GMWVD and GPD proposed in [7], is the fact that the borders of region for each sensor can be determined analytically; in fact, similar to the AWVD diagram, these borders are hyperbolic arcs, except that $r_i - r_j$ is replaced with $r_i - r_j - \epsilon_{ii} - \epsilon_{ji}$, as from (2) we obtain $d(q, p_{ii}) - d(q, p_{ji}) = r_i - r_j - \epsilon_{ii} - \epsilon_{ji}$. In contrast, the characteristics of the regions defined by the GMWVD and GPD are different from that of the MWVD and PD. Moreover, it is not easy to find the borders of regions in the GMWVD and GPD. For instance, (4) in [7] does not result in a constant $\frac{d(q,p_{ii})}{d(q,p_{ij})}$ i.e., the locus is not an Apollonian circle. Likewise, with (5) in [7] the regions shape is not known, while in the GAWVD the regions have the same shape as the AWVD. This is the main advantage of the GAWVD to the GMWVD and GPD.

There are many different algorithms for constructing various types of Voronoi diagrams, but in all of them it is required to find the boundaries of the Voronoi regions [9]. Since energy consumption is a major concern in any WSN, it is important to keep this process as simple as possible and the GAWVD seems to be the best option in this sense. Figure 1 shows an example of

[1] The proofs of propositions 1-5 are omitted due to space restrictions, and can be found in the longer version of the paper [8].

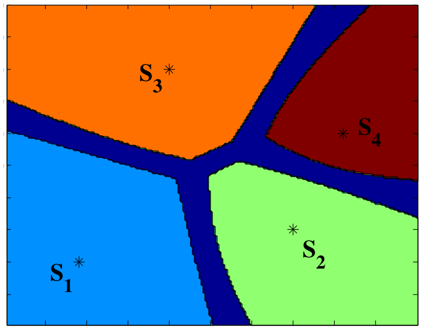


Fig. 1. An example of the GAWVD for 4 sensors with different sensing radii.

the GAWVD for four sensors with different sensing radii and subject to localization error. As it can be seen in this figure, the GAWVD does not partition the field. In fact, the points in the dark blue area are not assigned to any sensor. Note that, depending on the values of localization errors, in practice this region can be covered by sensors, at least partially.

V. Deployment Protocols

The GAWVD introduced in the previous section is used to develop deployment algorithms for a heterogeneous MSN where sensors' locations are known with certain errors. We use two movement strategies, namely, farthest point (FP) and minmax point (MP) strategies [2], to find the new candidate location for each sensor. These movement strategies are used in combination with the GAWVD, which was introduced in Section III. As a result, the below two deployment methods are possible:

- Farthest point based GAWVD (FPGAW)
- Minmax point based GAWVD (MPGAW)

The deployment algorithms are iterative and in each iteration it is tried to improve the total coverage, at least as much as a threshold $\delta > 0$. The algorithm is stopped if no improvement is possible or a certain number of iterations ($I_{\max}$) has passed. Algorithm 1 briefly describes the FPGAW method. The deployment algorithm for the MPGAW is exactly the same, except that instead of the FP strategy the MP strategy is used for movement. Figure 2 shows an operational example of the MPGAW algorithm. In this example, 27 sensors are randomly placed in a $50\text{m} \times 50\text{m}$ flat field: 15 with a sensing radius of 6m, 9 with a sensing radius of 6.5m, and 3 with a sensing radius of 7m. Furthermore, the measurement errors ϵ_{ii} and ϵ_{ij} are assumed to be 0 and 1m, respectively, for all sensors. Three snapshots are provided in this figure, where sensing disks of the sensors (filled circles) are depicted in each snapshot. Using this algorithm with the initial setting of Fig. 2(a), the coverage is improved from 62.14% to 81.27% after the first round, and it finally reaches 95.30%.

Algorithm 1 Deployment Algorithm for the FPGAW algorithm

$k \leftarrow 0,\ iterations \leftarrow 0,\ \mathcal{D} \leftarrow 1$
while $\mathcal{D} = 1$ and $iterations < I_{max}$ **do**
  for $i = 1$ to n **do**
  - construct Π_i according to (2) and set $\Pi_i^k \leftarrow \Pi_i$
  - find π_i^k (the area of the ith region that is covered by s_i)

  end for
  - $iterations \leftarrow iterations + 1$
  - $k \leftarrow k + 1$
  - $\mathcal{C} \leftarrow 0$

  for $i = 1$ to n **do**
  - calculate a new location (p_i^k) for s_i, based on the FPGAW movement strategy
  - evaluate π_i^k based on the new location for the current region (Π_i^{k-1})

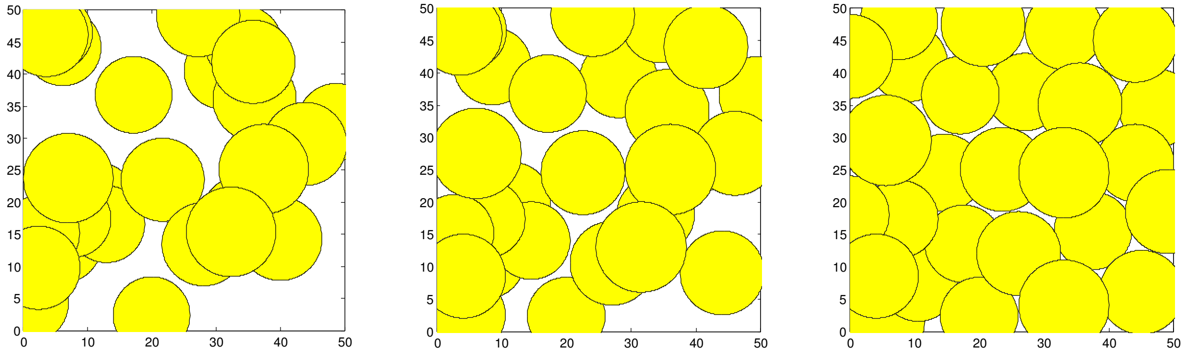
    if $\pi_i^k > \pi_i^{k-1} + \delta$ **then**
      move s_i to p_i^k
      $\mathcal{C} \leftarrow \mathcal{C} + 1$
    end if
  end for
  if $\mathcal{C} = 0$ **then**
    $\mathcal{D} = 0$
  end if
end while

VI. Simulation Results

To evaluate the performance of the proposed algorithms we carry out simulations for 18, 27, 36, and 45 sensors in a $50m \times 50m$ field, and using 20 random initial arrangements for the sensors. The minimum coverage improvement threshold δ is set to be $0.1m^2$, meaning that the algorithm stops if the local coverage improvement of no sensor is more than $0.1m^2$. We use three types of sensors with sensing radii equal to $6m, 6.5m$ and $7m$; the percentage of each type of sensors is 55.56%, 33.33%, and 11.11%, respectively (for any $n = 18, 27, 36, 45$). Also, it is assumed that the values of ϵ_{ii} and ϵ_{ij} are 0 and 1m, respectively, for all sensors.

Total coverage of the field after certain number of iterations is the most important parameter we are interested in. We compare the results of the proposed techniques based on the GAWVD with those based on the GMWVD and GPD, proposed by the authors in [7]. In Table I we represent the final coverage for each algorithm and different number of sensors. In general, the MP-based algorithms provide a better coverage than the FP-based algorithms; this is, however, achieved at the expense of a higher energy consumption, which is caused by a larger number of movements and travel distances for the sensors, as represented in Tables II and III. In short, the GAWVD method performs as well as the GMWVD and GPD, while it benefits from simpler cell border construction.

Our other result is counterintuitive: localization error can increase the total coverage. Our simulation experiments reveal that the FPGAW technique can result in a better coverage even when the localization errors are relatively large, although this varies for different number of sensors in the field. Table IV presents the coverage results for a range of ϵ_{ij} starting from 0 to $1.5m$, when $\epsilon_{ii} = 0.1$m. Based on these results, we conclude that using the proposed GAWVD, location estimation error can come in handy to improve the system performance, when the number of sensors is relatively small. However, the amount of improvement depends on other parameters such as the density of sensors in the field and is subject to further studies.



(a) (b) (c)

Fig. 2. Snapshots of the execution of the MPGAW strategy: (a) initial coverage (b) field coverage after the first round (c) final coverage.

TABLE IV
THE FINAL COVERAGE PERCENTAGE FOR DIFFERENT MEASUREMENT ERRORS UNDER FPGAW ALGORITHM

n	exact	$\epsilon_{ij} = 10cm$	$\epsilon_{ij} = 25cm$	$\epsilon_{ij} = 50cm$	$\epsilon_{ij} = 75cm$	$\epsilon_{ij} = 1m$	$\epsilon_{ij} = 1.5m$
18	72.41%	72.70%	72.23%	72.98%	72.33%	72.37%	72.02%
27	89.52%	88.95%	89.43%	89.79%	88.32%	87.21%	85.56%
36	96.95%	96.94%	96.62%	96.02%	95.27%	94.55%	92.57%
45	98.00%	97.96%	97,67%	97.47%	96.64%	96.34%	94.34%

TABLE I
NETWORK COVERAGE VERSUS NUMBER OF SENSORS FOR DIFFERENT ALGORITHMS

	$n = 18$	$n = 27$	$n = 36$	$n = 45$
Initial Coverage	56.13%	70.58%	79.55%	86.56%
MPGAW	75.62%	92.36%	97.28%	98.44%
MPGMW	75.73%	92.23%	97.09%	98.16%
MPGP	75.89%	92.23%	97.26%	98.43%
FPGAW	72.01%	88.03%	94.92%	96.25%
FPGMW	73.00%	87.93%	94.68%	96.65%
FPGP	72.77%	88.14%	94.62%	96.40%

TABLE II
AVERAGE TRAVEL DISTANCE PER SENSOR FOR THE PROPOSED ALGORITHMS GIVEN DIFFERENT NUMBER OF SENSORS IN THE FIELD

Algorithm	$n = 18$	$n = 27$	$n = 36$	$n = 45$
MPGAW	5.17	4.36	2.68	1.61
MPGMW	5.26	4.27	2.61	1.57
MPGP	5.28	4.31	2.70	1.64
FPGAW	4.10	2.85	1.82	1.01
FPGMW	4.24	2.81	1.80	1.05
FPGP	4.29	2.91	1.82	1.02

TABLE III
NUMBER OF MOVEMENTS REQUIRED TO REACH THE TERMINATION CONDITION USING THE PROPOSED ALGORITHMS WITH DIFFERENT NUMBER OF SENSORS

Algorithm	$n = 18$	$n = 27$	$n = 36$	$n = 45$
MPGAW	2.71	2.95	1.59	0.90
MPGMW	2.83	2.84	1.51	0.85
MPGP	2.73	2.88	1.60	0.91
FPGAW	1.44	1.94	1.48	0.88
FPGMW	1.45	2.01	1.48	0.90
FPGP	1.54	1.97	1.49	0.88

VII. CONCLUSIONS

A modified version of Voronoi diagram, named the guaranteed additively weighted Voronoi diagram (GAWVD), has been introduced to guarantee a cell-based search for coverage hole detection in a mobile sensor network with non-identical sensing radii where sensors' locations are estimated at each node and thus can be inaccurate. The proposed diagram is then used by the farthest point and minmax point movement strategies, to iteratively relocate the sensors in order to improve the network's sensing coverage. Numerical results show the effectiveness of the proposed diagram and deployment algorithms in increasing the network's coverage in the presence of localization error. In addition, it appears that a reasonably small error in location information improves the network coverage when compared to the case where the exact location information is available.

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