On the Number of Users Served in MIMO-NOMA Cellular Networks

(Invited Paper)

Wonjae Shin[†*], Mojtaba Vaezi[*], Jungwoo Lee[†], and H. Vincent Poor[*]
[†]Department of Electrical and Computer Engineering, Seoul National University, Korea
[*]Department of Electrical Engineering, Princeton University, Princeton, NJ, USA
Email: {wonjae.shin, junglee}@snu.ac.kr,{mvaezi, poor}@princeton.edu

***Abstract*—Envisioned to be a component of 5G cellular networks, *non-orthogonal multiple access* (NOMA) can accommodate more users and has superior spectral efficiency compared to the conventional *orthogonal* multiple access schemes. NOMA, however, intensifies inter-cell interference since its power allocation strategy is biased toward cell-edge users. To deal with this problem in a multi-cell multiple-input multiple-output (MIMO) communication network, a new *interfering channel alignment* based NOMA technique is developed in this paper. The *maximum number of users* supported by the proposed scheme in multi-cell MIMO networks is characterized for any given system parameter values such as the number of transmit and receive antennas and the number of cells.**

I. Introduction

Non-orthogonal multiple access (NOMA) [1] has recently attracted considerable interest, both in academia and industry. NOMA can improve spectral efficiency and support massive connectivity, compared to the conventional orthogonal multiple access schemes. As such, NOMA is envisioned to be a component of 5G cellular networks and it has been proposed for the 3rd Generation Partnership Project (3GPP) Long Term Evolution–Advanced (LTE–A) standards [2].

By superimposing multiple users' signals in the *downlink* cellular network, NOMA allows them to concurrently use the same frequency band. To separate these superimposed signals, it exploits the path loss differences amongst the users. With this rationale, commonly, users with very different channel conditions (good and poor) are selected to form a cluster in NOMA. A cluster can have multiple users but it usually includes two users in the literature [1], [3], [4]. Naturally, a user close to the cell-center experiences very different channel conditions compared to one close to the cell-edge; thus, such users make a good pair, as shown in Fig. 1. At the receiver side, in each cluster, the far away user (user 2 and user 4 in Fig. 1) treats the other user's signal (user 1 and user 3, respectively) as noise, whereas a user closer to the base station (BS) decodes and removes the interfering signal before decoding its own signal [1], [3]–[5].

To improve the overall spectral efficiency and user fairness, in NOMA, users with low channel gain get more transmission power, unlike conventional power allocation strategies, such as *water filling*. This implies that transmission power should be increased for users close to the cell-edge, which, in turn, increases inter-cell interference (ICI). Then, ICI, which is always an issue in multi-cell networks, is intensified in a NOMA-based scheme. That is, cell-edge users will suffer from severe interference from the neighboring cells.

This research was supported in part by the U.S. National Science Foundation under Grant CMMI-1435778.

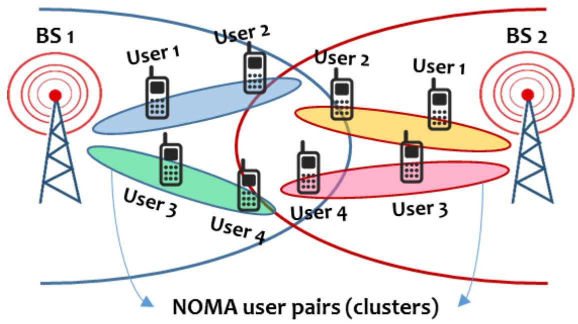

Fig. 1. Illustration of a two-cell MIMO-NOMA communication network, in which each cell includes two clusters and each cluster has two users.

While NOMA was originally introduced for systems based on single-input single-output (SISO) nodes, incorporation of multiple-input multiple-output (MIMO) nodes has expectedly further boosted the spectral efficiency [3]–[5]. In single-antenna nodes, it is easy to order the users based on their channel gains which are scalars. However, MIMO channels are matrices which makes it difficult to order users and design optimal beamforming/decoding strategies. In the MIMO-NOMA context, the ICI problem has been studied in several works [6]–[8]. All of these works, however, consider a two-cell network. In this paper, we consider a more general setting in which there can be an arbitrary number of cells.

The contributions of this paper are twofold: 1) we generalize the *interfering channel alignment* (ICA)-based approach, recently introduced in [8] for a two-cell MIMO-NOMA network, to the general case of $L \geq 2$ cells; and, 2) we characterize the maximum number of users that can be supported by the proposed scheme for given system parameters such as the number of transmit/receive antennas and the number of cells.

For notation, we use uppercase boldface letters (e.g., $\mathbf{A}$) for matrices, lowercase boldface letters (e.g., $\mathbf{a}$) for vectors, and $(.)^{\dagger}$ for the conjugate transpose. $\mathbf{I}_M$ denotes the identity matrix of size M and $\mathbf{a} \perp \mathbf{A}$ denotes that $\mathbf{a}$ is orthogonal to the subspace spanned by the columns of $\mathbf{A}$.

978-1-5090-2061-4/16/$31.00 ©2016 IEEE

II. System Model

We consider a multi-cell MIMO cellular system with $L \geq 2$ cells,[1] each consisting of K clusters of users. Each user is equipped with N antennas and each BS has M antennas. In each cluster, a two-user NOMA scheme with superposition coding is adopted. Let $\ell \in \mathcal{L} \triangleq \{1, \ldots, L\}$ be the cell index and $k \in \mathcal{K} \triangleq \{1, \ldots, K\}$ represent the cluster index within each of the cells. Also, let $j \in \mathcal{J} \triangleq \{1, \ldots, J\}$ be the user index in each cluster. We assume each cluster has two users, one close to the cell-center and the other one close to the cell-edge; these are denoted by $j=1$ and $j=2$, respectively. Let $\mathbf{H}_{j,k}^{[\ell]} \in \mathbb{C}^{N\times M}$ denote the serving channel matrix for the jth user at the kth cluster of cell ℓ. Also, let $\mathbf{G}_{j,k}^{[\ell,\bar{\ell}]} \in \mathbb{C}^{N\times M}$ denote the interfering channel matrix for the jth user at the kth cluster of cell ℓ coming from cell $\bar{\ell}$, where $\bar{\ell} \in \mathcal{L}\backslash\{\ell\}$. Since the cell-center users are usually far from the other BSs, it is reasonable to assume that they do not suffer from inter-cell interference. Thus, received signals at the kth *cell-center* ($j=1$) and *cell-edge* ($j=2$) users of cell ℓ, is then given by

$$\mathbf{y}_{1,k}^{[\ell]} = \mathbf{H}_{1,k}^{[\ell]} \sum_{m=1}^{K} \mathbf{x}_m^{[\ell]} + \mathbf{z}_{1,k}^{[\ell]}, \tag{1}$$

$$\mathbf{y}_{2,k}^{[\ell]} = \mathbf{H}_{2,k}^{[\ell]} \sum_{m=1}^{K} \mathbf{x}_m^{[\ell]} + \sum_{\bar{\ell}\in\mathcal{L}/\{\ell\}} \left(\mathbf{G}_{2,k}^{[\ell,\bar{\ell}]} \sum_{m=1}^{K} \mathbf{x}_m^{[\bar{\ell}]}\right) + \mathbf{z}_{2,k}^{[\ell]}, \tag{2}$$

where $\mathbf{x}_k^{[\ell]} \in \mathbb{C}^{M\times 1}$ is the superimposed signal for the kth cluster in BS ℓ and is given by

$$\mathbf{x}_k^{[\ell]} = \mathbf{v}_k^{[\ell]} s_k^{[\ell]} = \mathbf{v}_k^{[\ell]} \left(\sqrt{\lambda_{1,k}^{[\ell]}} s_{1,k}^{[\ell]} + \sqrt{\lambda_{2,k}^{[\ell]}} s_{2,k}^{[\ell]}\right), \tag{3}$$

in which $s_{j,k}^{[\ell]}$ is a data symbol, $\lambda_{j,k}^{[\ell]}$ is the power allocation coefficient for the superimposed signal, and $\mathbf{v}_k^{[\ell]} \in \mathbb{C}^{M\times 1}$ is a *transmit beamforming* vector. Without loss of generality we can assume $\lambda_{1,k}^{[\ell]} + \lambda_{2,k}^{[\ell]} = 1$ and $\left\|\mathbf{v}_k^{[\ell]}\right\|_2 = 1$. We also assume that all entries of $\mathbf{H}_{j,k}^{[\ell]}$ and $\mathbf{G}_{j,k}^{[\ell,\bar{\ell}]}$ are independently drawn from continuous distributions, and global channel knowledge is available at each BS.

III. ICA-Based Multi-Cell MIMO-NOMA

Recently, an ICA-based approach was introduced in [8] for a two-cell MIMO-NOMA network to align multiple interfering channels within one effective channel after applying receive beamforming. The algorithm includes designing transmit and receive beamforming vectors. For cell-edge users, this algorithm first designs receive beamformers to effectively align multiple interfering channels into one effective channel. Owing to this channel alignment, each BS is able to treat these interfering channels as one channel. Next, it designs transmit beamformers to remove both inter-cluster interference and inter-cell interference. To this end, transmit beamforming vectors need to be orthogonal to the subspace spanned by inter-cell and inter-cluster interference vectors.

[1]The case $L = 1$ corresponds to single-cell MIMO-NOMA [1]–[5] and is not considered in this paper.

In this work, we generalize this algorithm so that it works for $L \geq 2$ cells. In fact, we propose an ICA-based NOMA technique that aligns multiple inter-cell interference channels into a reduced number of channels. In addition, based on the proposed ICA-based NOMA scheme, we establish the *maximum* number of users served in multi-cell MIMO networks. The general results will be introduced in Section IV. In the remainder of this section, we provide two examples to familiarize the reader with the way we design transmit and receive beamformers. In both examples the number of cells is assumed to be three ($L = 3$). Thus, for clarity of notation, we use α, β, and γ as cell indices; i.e., $\ell \in \{\alpha, \beta, \gamma\}$.

A. Example 1: $\Delta = 1$

Let $L = 3$, $K = 2$, $M = 3$, and $N = 5$. Then, we can align four interfering channels into one ($\Delta = 1$) effective channel if the *receive beamformers* $\mathbf{w}_{2,1}^{[\ell]}, \mathbf{w}_{2,2}^{[\ell]} \in \mathbb{C}^{N\times 1}$ and the aligned interfering channel $\boldsymbol{\tau}_1^{[\ell]} \in \mathbb{C}^{M\times 1}$ satisfy

$$\begin{aligned}\boldsymbol{\tau}_1^{[\alpha]} &= \mathbf{G}_{2,1}^{[\beta,\alpha]^\dagger}\mathbf{w}_{2,1}^{[\beta]} = \mathbf{G}_{2,2}^{[\beta,\alpha]^\dagger}\mathbf{w}_{2,2}^{[\beta]} \\ &= \mathbf{G}_{2,1}^{[\gamma,\alpha]^\dagger}\mathbf{w}_{2,1}^{[\gamma]} = \mathbf{G}_{2,2}^{[\gamma,\alpha]^\dagger}\mathbf{w}_{2,2}^{[\gamma]},\end{aligned} \tag{6a}$$

$$\begin{aligned}\boldsymbol{\tau}_1^{[\beta]} &= \mathbf{G}_{2,1}^{[\alpha,\beta]^\dagger}\mathbf{w}_{2,1}^{[\alpha]} = \mathbf{G}_{2,2}^{[\alpha,\beta]^\dagger}\mathbf{w}_{2,2}^{[\alpha]} \\ &= \mathbf{G}_{2,1}^{[\gamma,\beta]^\dagger}\mathbf{w}_{2,1}^{[\gamma]} = \mathbf{G}_{2,2}^{[\gamma,\beta]^\dagger}\mathbf{w}_{2,2}^{[\gamma]},\end{aligned} \tag{6b}$$

$$\begin{aligned}\boldsymbol{\tau}_1^{[\gamma]} &= \mathbf{G}_{2,1}^{[\alpha,\gamma]^\dagger}\mathbf{w}_{2,1}^{[\alpha]} = \mathbf{G}_{2,2}^{[\alpha,\gamma]^\dagger}\mathbf{w}_{2,2}^{[\alpha]} \\ &= \mathbf{G}_{2,1}^{[\beta,\gamma]^\dagger}\mathbf{w}_{2,1}^{[\beta]} = \mathbf{G}_{2,2}^{[\beta,\gamma]^\dagger}\mathbf{w}_{2,2}^{[\beta]}.\end{aligned} \tag{6c}$$

To satisfy the above alignment condition, we can find the receive beamformers for all three cells by solving the matrix equation (4) (at the top of next page). Since the size of the unified matrix in the left-hand side of (4) is $12M\times(3M + 6N)$ and all elements of $\mathbf{G}_{2,k}^{[\ell,\bar{\ell}]}$ are independently drawn from a continuous distribution, one can easily show that this matrix has a null space almost surely. So, it is possible to find the vectors $\mathbf{w}_{2,1}^{[\ell]}$ and $\mathbf{w}_{2,2}^{[\ell]}$ that satisfy the above alignment conditions.

The second step is to design the *transmit beamformers* $\mathbf{v}_k^{[\ell]}$. These are designed independently for each cell. Hence, without loss of generality, we focus on cell α. $\mathbf{v}_k^{[\alpha]}$ can be designed to ensure zero inter-cell and inter-cluster interference at the cell-edge users if

$$\mathbf{v}_k^{[\alpha]} \perp \begin{bmatrix} \boldsymbol{\tau}_1^{[\alpha]^\dagger} \\ \bar{\mathbf{H}}_{2,k}^{[\alpha]} \end{bmatrix}^\dagger, \tag{7}$$

where $\bar{\mathbf{H}}_{2,k}^{[\alpha]}$ is a $(K-1)\times M$ matrix obtained by removing the kth row of $\mathbf{H}_2^{[\alpha]} = \left[\mathbf{H}_{2,1}^{[\alpha]^\dagger}\mathbf{w}_{2,1}^{[\alpha]} \cdots \mathbf{H}_{2,K}^{[\alpha]^\dagger}\mathbf{w}_{2,K}^{[\alpha]}\right]^\dagger$. Recall from (4) that the receive beamformer $\mathbf{w}_{2,k}^{[\alpha]}$ is designed independently of the channel matrix $\mathbf{H}_{2,k}^{[\alpha]}$. Also, the elements of the effective channel vectors $\mathbf{H}_{2,k}^{[\alpha]^\dagger}\mathbf{w}_{2,k}^{[\alpha]}$ and $\mathbf{H}_{2,\bar{k}}^{[\alpha]^\dagger}\mathbf{w}_{2,\bar{k}}^{[\alpha]}$ for $\bar{k} \in \mathcal{K}\backslash\{k\}$ are statistically independent random variables. Moreover, the aligned interfering channel $\boldsymbol{\tau}_1^{[\alpha]}$ is not a function of $\mathbf{H}_{2,k}^{[\alpha]}$ or $\mathbf{G}_{2,k}^{[\alpha,\bar{\alpha}]}$. This implies that all the column vectors of

$$\left[\begin{array}{ccc|cccccc} -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\beta,\alpha]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\beta,\alpha]^\dagger} & \mathbf{0} & \mathbf{0} \\ -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\gamma,\alpha]^\dagger} & \mathbf{0} \\ -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\gamma,\alpha]^\dagger} \\ \hline \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{G}_{2,1}^{[\alpha,\beta]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\alpha,\beta]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\gamma,\beta]^\dagger} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\gamma,\beta]^\dagger} \\ \hline \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{G}_{2,1}^{[\alpha,\gamma]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{G}_{2,2}^{[\alpha,\gamma]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\beta,\gamma]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\beta,\gamma]^\dagger} & \mathbf{0} & \mathbf{0} \end{array}\right] \left[\begin{array}{c} \boldsymbol{\tau}_1^{[\alpha]} \\ \boldsymbol{\tau}_1^{[\beta]} \\ \boldsymbol{\tau}_1^{[\gamma]} \\ \hline \mathbf{w}_{2,1}^{[\alpha]} \\ \mathbf{w}_{2,2}^{[\alpha]} \\ \mathbf{w}_{2,1}^{[\beta]} \\ \mathbf{w}_{2,2}^{[\beta]} \\ \mathbf{w}_{2,1}^{[\gamma]} \\ \mathbf{w}_{2,2}^{[\gamma]} \end{array}\right] = \mathbf{0}, \tag{4}$$

$$\left[\begin{array}{cccccc|cccccc} -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\beta,\alpha]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\beta,\alpha]^\dagger} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\gamma,\alpha]^\dagger} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\gamma,\alpha]^\dagger} \\ \hline \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\alpha,\beta]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\alpha,\beta]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\gamma,\beta]^\dagger} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\gamma,\beta]^\dagger} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{G}_{2,1}^{[\alpha,\gamma]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\alpha,\gamma]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,1}^{[\beta,\gamma]^\dagger} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}_M & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G}_{2,2}^{[\beta,\gamma]^\dagger} & \mathbf{0} & \mathbf{0} \end{array}\right] \left[\begin{array}{c} \boldsymbol{\tau}_1^{[\alpha]} \\ \boldsymbol{\tau}_2^{[\alpha]} \\ \boldsymbol{\tau}_1^{[\beta]} \\ \boldsymbol{\tau}_2^{[\beta]} \\ \boldsymbol{\tau}_1^{[\gamma]} \\ \boldsymbol{\tau}_2^{[\gamma]} \\ \hline \mathbf{w}_{2,1}^{[\alpha]} \\ \mathbf{w}_{2,2}^{[\alpha]} \\ \mathbf{w}_{2,1}^{[\beta]} \\ \mathbf{w}_{2,2}^{[\beta]} \\ \mathbf{w}_{2,1}^{[\gamma]} \\ \mathbf{w}_{2,2}^{[\gamma]} \end{array}\right] = \mathbf{0}, \tag{5}$$

$\left[\boldsymbol{\tau}_1^{[\alpha]} \;\; \bar{\mathbf{H}}_2^{[\alpha]^\dagger}\right]$ are linearly independent almost surely, and it is possible to find the precoding vector $\mathbf{v}_k^{[\alpha]}$ in the null space of the matrix, for any $k \in \mathcal{K}$. The same approach can be applied to find $\mathbf{v}_k^{[\beta]}$ and $\mathbf{v}_k^{[\gamma]}$.

Since the cell-center users are assumed not to suffer from interference caused by the neighboring cells, for these users we only need to remove the inter-cluster interference caused by other $K-1$ clusters within that cell. This can be accomplished by a zero-forcing (ZF) decoder with K receive antennas, i.e.,

$$\mathbf{w}_{1,k}^{[\ell]} \perp \mathbf{H}_{1,k}^{[\ell]}\left[\mathbf{v}_1^{[\ell]} \cdots \mathbf{v}_{k-1}^{[\ell]} \; \mathbf{v}_{k+1}^{[\ell]} \cdots \mathbf{v}_K^{[\ell]}\right]. \tag{8}$$

Thus, with the help of ICA we can equivalently decompose the three-cell MIMO-NOMA channels into $3K$ pairs of single-antenna NOMA channels. In particular, the received signals at a cell-center user ($j = 1$) and cell-edge user ($j = 2$) of the kth cluster in cell ℓ are given by

$$\mathbf{w}_{j,k}^{[\ell]^\dagger}\mathbf{y}_{j,k}^{[\ell]} = \tilde{h}_{j,k}^{[\ell]}\left(\sqrt{\lambda_{1,k}^{[\ell]}}s_{1,k}^{[\ell]} + \sqrt{\lambda_{2,k}^{[\ell]}}s_{2,k}^{[\ell]}\right) + \tilde{z}_{j,k}^{[\ell]}, \tag{9}$$

where $\tilde{h}_{j,k}^{[\ell]} = \mathbf{w}_{j,k}^{[\ell]^\dagger}\mathbf{H}_{j,k}^{[\ell]}\mathbf{v}_k^{[\ell]}$ and $\tilde{z}_{j,k}^{[\ell]} = \mathbf{w}_{j,k}^{[\ell]^\dagger}\mathbf{z}_{j,k}^{[\ell]}$ are, respectively, the effective serving channel coefficient and effective noise term after applying transmit/receive beamformings. Therefore, for the cell-center users, intra-cluster interference can be decoded and canceled using successive interference cancellation so that the desired message is decoded free of interference. The cell-edge users, however, decode their messages by treating the intra-cluster interference as noise [1].

B. Example 2: $\Delta = 2$

Consider a three cell network with two users ($L = 3$, $K = 2$) with $M = 4$ and $N = 5$. With this configuration, we can align four interfering channels into two effective channels. As an example, this can be accomplished if

$$\boldsymbol{\tau}_1^{[\alpha]} = \mathbf{G}_{2,1}^{[\beta,\alpha]^\dagger}\mathbf{w}_{2,1}^{[\beta]} = \mathbf{G}_{2,2}^{[\beta,\alpha]^\dagger}\mathbf{w}_{2,2}^{[\beta]}, \tag{10a}$$

$$\boldsymbol{\tau}_2^{[\alpha]} = \mathbf{G}_{2,1}^{[\gamma,\alpha]^\dagger}\mathbf{w}_{2,1}^{[\gamma]} = \mathbf{G}_{2,2}^{[\gamma,\alpha]^\dagger}\mathbf{w}_{2,2}^{[\gamma]}, \tag{10b}$$

$$\boldsymbol{\tau}_1^{[\beta]} = \mathbf{G}_{2,1}^{[\alpha,\beta]^\dagger}\mathbf{w}_{2,1}^{[\alpha]} = \mathbf{G}_{2,2}^{[\alpha,\beta]^\dagger}\mathbf{w}_{2,2}^{[\alpha]}, \tag{10c}$$

$$\boldsymbol{\tau}_2^{[\beta]} = \mathbf{G}_{2,1}^{[\gamma,\beta]^\dagger}\mathbf{w}_{2,1}^{[\gamma]} = \mathbf{G}_{2,2}^{[\gamma,\beta]^\dagger}\mathbf{w}_{2,2}^{[\gamma]}, \tag{10d}$$

$$\boldsymbol{\tau}_1^{[\gamma]} = \mathbf{G}_{2,1}^{[\alpha,\gamma]^\dagger}\mathbf{w}_{2,1}^{[\alpha]} = \mathbf{G}_{2,2}^{[\alpha,\gamma]^\dagger}\mathbf{w}_{2,2}^{[\alpha]}, \tag{10e}$$

$$\boldsymbol{\tau}_2^{[\gamma]} = \mathbf{G}_{2,1}^{[\beta,\gamma]^\dagger}\mathbf{w}_{2,1}^{[\beta]} = \mathbf{G}_{2,2}^{[\beta,\gamma]^\dagger}\mathbf{w}_{2,2}^{[\beta]}. \tag{10f}$$

The above alignment conditions can be equivalently represented by the matrix equation (5) (at the top of this page). The size of this unified matrix is $12M \times (3\Delta M + 6N)$. Again, since all elements of $\mathbf{G}_{2,k}^{[\ell,\bar{\ell}]}$ are independently drawn from a continuous distribution, this matrix will have a null space almost surely. Therefore it is possible to find the beamforming vectors that satisfy the alignment conditions in (10).

Similar to Example 1, the next step is to design the *transmit beamformers* $\mathbf{v}_k^{[\ell]}$. Again, without loss of generality, we only discuss how to design $\mathbf{v}_k^{[\alpha]}$. To ensure zero inter-cell and inter-cluster interferences at the cell-edge users $\mathbf{v}_k^{[\alpha]}$ must salsify

$$\mathbf{v}_k^{[\alpha]} \perp \begin{bmatrix} \boldsymbol{\tau}_1^{[\alpha]^\dagger} \\ \boldsymbol{\tau}_2^{[\alpha]^\dagger} \\ \bar{\mathbf{H}}_{2,k}^{[\alpha]} \end{bmatrix}^\dagger, \tag{11}$$

where $\bar{\mathbf{H}}_{2,k}^{[\alpha]}$ is similar to that in (7). Then again, with the same argument as that in Example 1, all the row vectors of $\left[\boldsymbol{\tau}_1^{[\alpha]} \;\; \boldsymbol{\tau}_2^{[\alpha]} \;\; \bar{\mathbf{H}}_2^{[\alpha]^\dagger}\right]^\dagger$ are linearly independent almost surely, and we can find $\mathbf{v}_k^{[\alpha]}$ in the null space of the matrix, for any $k \in \mathcal{K}$. The same approach can be used to find $\mathbf{v}_k^{[\beta]}$ and $\mathbf{v}_k^{[\gamma]}$. The design of receive beamformers $\mathbf{w}_{1,k}^{[\ell]}$ for the cell-center users is exactly the same as those in Example 1.

As it would be understood from the above examples, having different numbers of aligned (effective) interference channels (different Δs) can change the required system parameters.

IV. Main Results

In this section, we introduce the key results for the proposed ICA-based NOMA technique.

Lemma 1. *For an L-cell MIMO network, to simultaneously support K clusters per cell we must have*

$$M \geq K + \Delta, \tag{12a}$$

$$N \geq \max\left\{\frac{(L-1)K - \Delta}{K} M + \epsilon, K\right\}, \tag{12b}$$

where Δ is an integer between 1 and $\min\{(L-1)K, M-1\}$ *and ϵ is an arbitrarily small positive number, i.e.,* $0 < \epsilon \ll 1$.

Proof. In order to confine all $Q \triangleq (L-1)K$ interfering channels of each BS within Δ-dimensional signal space, we must have

$$\begin{aligned} &\operatorname{span}\left[\boldsymbol{\tau}_1^{[\ell]} \quad \boldsymbol{\tau}_2^{[\ell]} \quad \cdots \quad \boldsymbol{\tau}_\Delta^{[\ell]}\right] \\ &= \operatorname{span}\left\{[\,\mathbf{G}_2^{[1,\ell]} \; \mathbf{G}_2^{[2,\ell]} \cdots \; \mathbf{G}_2^{[L,\ell]}\,]\backslash \mathbf{G}_2^{[\ell,\ell]}\right\}, \end{aligned} \tag{13}$$

in which

$$\mathbf{G}_2^{[\ell',\ell]} = \left[\mathbf{G}_{2,1}^{[\ell',\ell]^\dagger} \mathbf{w}_{2,1}^{[\ell']} \cdots \mathbf{G}_{2,K}^{[\ell',\ell]^\dagger} \mathbf{w}_{2,K}^{[\ell']}\right]. \tag{14}$$

We only need to consider the case

$$\Delta \leq Q = (L-1)K, \tag{15}$$

as otherwise the solution is trivial. This is as if we have Δ bins and we want to distribute Q items in those bins such that each bin gets at least one item. Let $n_j^{[\ell]}$, $j \in \{1, \ldots, \Delta\}$ be the number of interfering channels aligned to channel $\boldsymbol{\tau}_j^{[\ell]}$. Then, it is clear that $n_j^{[\ell]} \geq 1$. Further, $\sum_{j=1}^{\Delta} n_j^{[\ell]} = Q$. The mapping of $n_j^{[\ell]}$ interfering channels to one channel ($\boldsymbol{\tau}_j^{[\ell]}$) creates $n_j^{[\ell]} - 1$ constraints, like those in (6). Hence, in total, we will have $\sum_{j=1}^{\Delta}(n_j^{[\ell]} - 1) = Q - \Delta$ constraints. Now, considering all cells in the network, we can unify a system of equations by aggregating all interfering channel alignment constraints. Two examples of such a matrix can be seen in (4) and (5). The size of this unified matrix in general is $L(L-1)KM \times (L\Delta M + LKN)$. Since all elements of $\mathbf{G}_2^{[\ell',\ell]}$ in (14) are independently drawn from a continuous distribution, the unified matrix will be full rank almost surely. Thus, it will have a null space and it is possible to jointly construct the receive beamforming vectors that satisfy all the alignment conditions if the number of rows is less than the number of columns; i.e., when $N > \frac{(L-1)K-\Delta}{K}M$. On the other hand, $N \geq K$ receive antennas are required to cancel all the inter-cluster interference at the cell-center users. These two conditions together amount to (12b).

To ensure zero inter-cell and inter-cluster interference at the cell-edge users, the transmit beamforming vectors of the cell-edge users in the kth cluster of ℓ cell have to satisfy

$$\mathbf{v}_k^{[\ell]} \perp \left[\boldsymbol{\tau}_1^{[\ell]} \cdots \boldsymbol{\tau}_\Delta^{[\ell]}\right], \tag{16}$$

and

$$\mathbf{v}_k^{[\ell]} \perp \begin{bmatrix} \mathbf{w}_{2,1}^{[\ell]^\dagger} \mathbf{H}_{2,1}^{[\ell]} \\ \vdots \\ \mathbf{w}_{2,\ell-1}^{[\ell]^\dagger} \mathbf{H}_{2,\ell-1}^{[\ell]} \\ \mathbf{w}_{2,\ell+1}^{[\ell]^\dagger} \mathbf{H}_{2,\ell+1}^{[\ell]} \\ \vdots \\ \mathbf{w}_{2,K}^{[\ell]^\dagger} \mathbf{H}_{2,K}^{[\ell]} \end{bmatrix}^\dagger. \tag{17}$$

From this constraint, the feasibility condition on the number of transmit antennas in (12a) is obtained.

Note that the number of clusters per cell should be greater than or equal to 1, which leads to $M \geq \Delta + 1$ from (12a). This constraint together with (15) imposes that the number of aligned interfering channels Δ from the perspective of each BS cannot be larger than $\min\{(L-1)K, M-1\}$. This completes the proof. □

In Lemma 1, Δ is the dimension of the aligned interfering channels for each cell. As can be seen, Δ can take more than one value. The choice of Δ will affect the number of required transmit and receive antennas. Specifically, as Δ increases M needs to be larger while N can decrease based on (12). Due to this trade-off, Δ needs to be appropriately chosen according to the needs of the communication system, which will be provided in the following theorem.

Theorem 1. *The maximum number of users supported by the proposed scheme in an L-cell MIMO network with M transmit* *antennas at each BS and N receive antennas at each user is given by*

$$2\min\left\{\max\left\{M-\lceil\delta^*\rceil,\lfloor f(\lfloor\delta^*\rfloor)-\epsilon\rfloor,g(1)\right\},N\right\} \quad (18)$$

where

$$\delta^*=\frac{(L-1)M^2-MN}{LM-N}, \quad (19)$$

$$f(x)=\frac{N-(L-2)x+\sqrt{[N-(L-2)x]^2+4(L-1)x^2}}{2(L-1)}, \quad (20)$$

$$g(x)=\min\{M-x,\lfloor f(x)-\epsilon\rfloor\}, \quad (21)$$

and ϵ is an arbitrarily small positive number.

Proof. See Appendix A. □

V. Conclusion

We proposed an interfering channel alignment based NOMA technique for multi-cell MIMO networks. Based on the proposed scheme, we further derived the maximum number of users that can be supported within each cell for a given system parameter.

Appendix A
Proof of Theorem 1

First, from Lemma 1 the following three inequalities have to be satisfied:

$$M\geq K+\Delta, \quad (22a)$$

$$N>\frac{(L-1)K-\Delta}{K}M, \quad (22b)$$

$$N\geq K. \quad (22c)$$

By combining (22a) and (22b), we have

$$\frac{NK}{(L-1)K-\Delta}>M\geq K+\Delta, \quad (23)$$

which results in

$$K<\frac{N-(L-2)\Delta+\sqrt{[N-(L-2)\Delta]^2-4(L-1)\Delta^2}}{2(L-1)}, \triangleq f(\Delta). \quad (24)$$

Therefore, considering (22a) and (22c), the number of clusters per cell is bounded as

$$K\leq\min\left\{M-\Delta,f(\Delta)-\epsilon,N\right\}. \quad (25)$$

Because Δ and K must be integers, we formulate the following optimization problem:

$$\max_{\Delta}2K=2\max_{\Delta}\min\left\{M-\Delta,\lfloor f(\Delta)-\epsilon\rfloor,N\right\} \quad (26a)$$

$$\text{s.t.}\quad L,M,N\geq 2 \quad (26b)$$

$$\Delta\in\left\{1,\ldots,\min\{(L-1)K,M-1\}\right\}. \quad (26c)$$

Note that $M-\Delta$ is linearly decreasing with Δ, and N is a constant with respect to Δ. Moreover, $f(\Delta)$ is strictly convex. To prove the latter, we show that the second derivative of $f(\Delta)$ is larger than 0 since

$$\frac{\partial^2 f(x)}{\partial x^2}=\frac{L^2-[L^2x-(L-2)N]^2A(x)^{-1}}{2(L-1)A(x)^{\frac{1}{2}}}\overset{(a)}{>}0 \quad (27)$$

where $A(x)\triangleq[N-(L-2)x]^2+4(L-1)x^2>0$. The inequality (a) comes from the fact that

$$\begin{aligned}&L^2-\Big[L^2x-(L-2)N\Big]^2A(x)^{-1}\\&=\frac{1}{A(x)}\Big[A(x)L^2-L^4x^2+2(L-2)L^2Nx-(L-2)^2N^2\Big]\\&=\frac{1}{A(x)}\Big[4\underbrace{(L-1)}_{>0,\text{ if }L\geq2}N^2\Big]>0.\end{aligned} \quad (28)$$

Next, let us relax (26) by assuming that Δ can take non-integer values and replacing $\lfloor f(x)-\epsilon\rfloor$ with $f(x)-\epsilon$. Then, one can check that if $\max\{\min\{M-\Delta_1^\star,f(\Delta_1^\star)\},M-\Delta_2^\star\}<N$, the optimal solution should be either $\Delta_1^\star=1$ or $\Delta_2^\star$, where $\Delta_2^\star$ is derived from equating $M-\Delta_2^\star=f(\Delta_2^\star)$, and is given by

$$\Delta_2^\star=\frac{(L-1)M^2-MN}{LM-N}. \quad (29)$$

We note that $\Delta_1^\star=1$ corresponds to the case where $f(x)$ is not strictly increasing with respect to x while $\Delta_2^\star$ corresponds to the case where $f(x)$ is strictly increasing. On the other hand, when $\max\{\min\{M-\Delta_1^\star,f(\Delta_1^\star)\},M-\Delta_2^\star\}\geq N$ the optimal solution can be any numbers between 1 and $\min[(L-1)K,M-1]$, which reveals that the number of served clusters is simply bounded by N.

Going back to the original problem (26) in which Δ must be an integer, the maximum number of clusters can be achieved at $\lfloor\Delta_2^\star\rfloor$, $\lceil\Delta_2^\star\rceil$, or $\Delta_1^\star$. As a result, the optimal solution to the original problem is obtained by

$$\begin{aligned}K^\star&=\min\left\{\max\left\{g(\lceil\Delta_2^\star\rceil),g(\lfloor\Delta_2^\star\rfloor),g(\Delta_1^\star),\right\},N\right\} \quad (30)\\&=\min\left\{\max\left\{M-\lceil\Delta_2^\star\rceil,\lfloor f(\lfloor\Delta_2^\star\rfloor)-\epsilon\rfloor\right\},g(\Delta_1^\star),N\right\},\end{aligned}$$

where $g(x)$ is defined in (21).

References

[1] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in *Proc. IEEE 77th VTC Spring*, pp. 1–5, 2013.

[2] 3GPP RP-150496: "Study on Downlink Multiuser Superposition Transmission," Mar. 2015.

[3] Q. Sun, S. Han, C.-L. I, and Z. Pan, "On the ergodic capacity of MIMO NOMA systems," *IEEE Wireless Commun. Lett.*, vol. 4, no. 4, pp. 405–408, 2015.

[4] Z. Ding, R. Schober, and H. V. Poor, "A general MIMO framework for NOMA downlink and uplink transmission based on signal alignment," *IEEE Trans. Wireless Commun.*, vol. 15, no. 6, pp. 4438–4454, 2016.

[5] Z. Ding, F. Adachi, and H. V. Poor, "The application of MIMO to non-orthogonal multiple access," *IEEE Trans. Wireless Commun.*, vol. 15, no. 1, pp. 537–552, 2016.

[6] S. Han, C.-L. I, Z. Xu, and Q. Sun, "Energy efficiency and spectrum efficiency co-design: From NOMA to network NOMA," *IEEE Multimedia Commun. Tech. Comm. E-Lett.*, vol. 9, no. 5, pp. 21–24, 2014.

[7] J. Choi, "Non-orthogonal multiple access in downlink coordinated two-point systems," *IEEE Commun. Lett.*, vol. 18, no. 2, pp. 313–316, 2014.

[8] W. Shin, M. Vaezi, B. Lee, D. J. Love, J. Lee, and H. V. Poor, "Coordinated beamforming for multi-cell MIMO-NOMA," submitted to *IEEE Commun. Lett.*, July 2016.