Trust Degree Based Beamforming for Multi-Antenna Cooperative Communication Systems

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***Abstract*—In this paper, beamforming design is investigated for a multi-antenna cooperative communication system in which both physical links and social connections (trust degrees) between nodes are taken into account. An optimal beamformer aims to balance between the direct link and the cooperating link as well as respecting the trust degree. The resulting optimization problem is nontrivial to solve, even numerically, as it is not convex. The complexity of the problem is largely reduced by showing that a linear combination of the direct and cooperating links' channel vectors maximizes the achievable rate. Then, a computationally efficient numerical solution is used to maximize the rate. Numerical results demonstrate that significant gains in communication rates can be obtained with the proposed optimal beamforming design.**

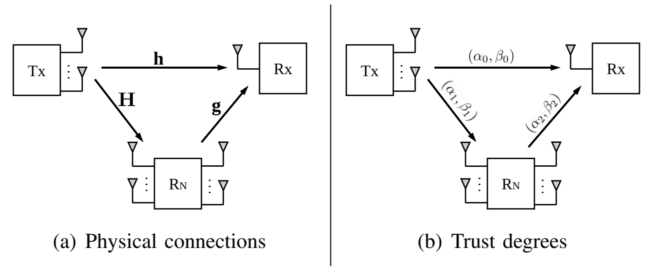

Fig. 1. A multi-antenna cooperative communication network with physical and social connections (trust degrees).

I. Introduction

A salient characteristic of mobile data networks is the dominant operating mode of one person behind each device, which differentiates them from other technological networks such as sensor networks. A direct consequence of this feature of mobile data networks is the existence of both *physical coupling* between mobile devices through shared communication resources such as wireless spectrum, and *virtual coupling* among the users behind these devices in the social domain. These virtual ties, in many ways, shape the data traffic flows and quality-of-service (QoS) requirements in the physical domain. Social interactions could be positive (representing friendship) or negative (representing antagonism), which yield non-trivial impact on users' decision-making processes (e.g., in terms of cooperation) for data transmission and reception. We refer to this two-layered network structure as a mobile/social network.

A number of recent works, e.g., [1]–[6], have studied the integration of mobile data networks with social networks in a variety of settings. In [1], a game theoretic framework called social group utility maximization is developed to maximize a weighted sum of the individual utilities rather than a totally selfish utility. In [2]–[5], social community-aware resource allocation is studied for device-to-device (D2D) networks. A social-aware peer discovery scheme for D2D systems is proposed in [6]. It exploits social network characteristics to assist ad hoc peer discovery and enhance data transmission performance.

This research was supported by the U.S. Army Research Office under Grant W911NF-16-1-0448.

Most of the above works develop communication strategies based on single-antenna nodes. In a recent study, Ryu et al. [7] proposed the use of social connections for beamforming in a cooperative communication system with a single-antenna relay. This scheme uses trustworthiness between nodes to design beamforming for efficient data relaying. It is proved that a linear combination of channel vectors makes an optimal beamformer to maximize expected achievable rate. To find a closed-form solution for the optimal weights of the linear combination, a high signal-to-noise ratio (SNR) approximation is used which explicitly shows the effect of trust degrees on the beamforming design.

In this paper, we improve the achievable rate of [7] and extend it to the case where the relay has multiple antennas. The rate improvement is based on two observations. First, we show that maximum trust degree does not always result in the best rate performance. Thus, we consider optimization over trust degree as well as the beamforming vectors at the source and relay nodes. Second, unlike [7], our design is for the exact achievable rate, not just in the high SNR regime. We use a computationally practical numerical solution that achieves near optimal beamformer performance. The resulting beamformer improves the rate performance when compared with the closed-form solution of the approximated rate given in [7]. Another contribution of the current paper is the extension of the beamforming design in [7] to the case in which the relay has multiple antennas.

The paper is organized as follows. We describe the system model in Section II and formulate an optimization problem to maximize the achievable rate at the destination in Section III.

In Section IV, we prove that optimal beamforming is achieved through a linear combination of the channel vectors for direct and cooperating links, respecting the trust degree. Numerical results are presented in Section V and the paper is concluded in Section VI.

A. Notation

We use $(\cdot)^t$, $(\cdot)^*$, $(\cdot)^\dagger$, and $\mathrm{tr}(\cdot)$ to denote the transpose, conjugate, conjugate transpose, and trace of a matrix, respectively. Matrices are written in bold capital letters and vectors are written in bold small letters. $\mathbf{I}$ represents an identity matrix. For scalar x, $0 \leq x \leq 1$, we define $\bar{x} = 1 - x$. $\|\mathbf{x}\|$ is the Euclidean norm of $\mathbf{x}$.

II. System Model

In this paper, we consider a two-layered network structure, which we call a mobile/social network. As shown in Fig. 1, the network has *physical* connections (Fig. 1(a)) as well as *social* connections represented by trust degrees (Fig. 1(b)). We describe the details of this two-layered network model below, starting with the physical system model.

A. Physical System Model

Consider a multiple-input-multiple-output (MIMO) decode-and-forward (DF) full-duplex two-hop relay network as shown in Fig. 1(a). We assume that the source (S) and relay (R) have N_s and N_r antennas, respectively, and the destination (D) has one antenna only. Let $\mathbf{h} \in \mathbb{C}^{N_\mathrm{s}\times 1}$, $\mathbf{H} \in \mathbb{C}^{N_\mathrm{s}\times N_\mathrm{r}}$, and $\mathbf{g} \in \mathbb{C}^{N_\mathrm{r}\times 1}$ represent the channel coefficients for S-D, S-R, and R-D links, respectively. We assume that channel state information (CSI) is known at the transmitters, e.g., through training/feedback and channel reciprocity, so that they can steer their beamforming vectors to maximize their transmission performance.

In this setting, information transmission can be divided into two phases, as described below.

1) ***Source Phase****:* In this phase, the information symbol x_s is first multiplied by a beamforming vector $\mathbf{w}_\mathrm{s}$ before being transmitted at S. The complex baseband signal received at D and R can be represented as

$$y_\mathrm{SD} = \mathbf{h}^t \mathbf{w}_\mathrm{s} x_\mathrm{s} + n_\mathrm{SD}, \tag{1}$$

$$\mathbf{y}_\mathrm{SR} = \mathbf{H}^t \mathbf{w}_\mathrm{s} x_\mathrm{s} + \mathbf{n}_\mathrm{SR}, \tag{2}$$

where $n_\mathrm{SD} \sim \mathcal{CN}(0,1)$ and $\mathbf{n}_\mathrm{SR} \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$ represent complex additive Gaussian noise at D and R, respectively. The received SNRs at D and R are given by

$$\gamma_\mathrm{SD} = |\mathbf{h}^t \mathbf{w}_\mathrm{s}|^2 P_s, \tag{3}$$

$$\gamma_\mathrm{SR} = \|\mathbf{H}^t \mathbf{w}_\mathrm{s}\|^2 P_s. \tag{4}$$

Thus, the maximum achievable rates of communication from S to D and S to R are given by

$$\mathcal{C}_\mathrm{SD} = \log_2(1 + \gamma_\mathrm{SD}), \tag{5}$$

$$\mathcal{C}_\mathrm{SR} = \log_2(1 + \gamma_\mathrm{SR}). \tag{6}$$

2) ***Relay Phase****:* In this phase, R forwards its decoded symbol x_r to D with power P_r through transmission over N_r antennas. The complex baseband signal received at D is given by

$$y_\mathrm{RD} = \mathbf{g}^t \mathbf{w}_\mathrm{r} x_\mathrm{r} + n_\mathrm{RD}, \tag{7}$$

where $n_\mathrm{RD} \sim \mathcal{CN}(0,1)$. The received SNR at D, in this phase, is given by

$$\gamma_\mathrm{RD} = |\mathbf{g}^t \mathbf{w}_\mathrm{r}|^2 P_\mathrm{r}. \tag{8}$$

Then, D can combine the signal received from S and R to improve the SNR. The achievable rate for the composite link is given by [8]

$$\mathcal{C}_\mathrm{D} = \log_2(1 + \gamma_\mathrm{SD} + \gamma_\mathrm{RD}). \tag{9}$$

Finally, the achievable rate for the DF system is obtained as

$$\begin{aligned} \mathcal{C}_\mathrm{DF} &= \min\{\mathcal{C}_\mathrm{SR}, \mathcal{C}_\mathrm{D}\} \qquad (10) \\ &= \min\Big\{\log_2(1+\gamma_\mathrm{SR}), \log_2(1+\gamma_\mathrm{SD}+\gamma_\mathrm{RD})\Big\}. \end{aligned}$$

B. Trust Degrees

Similar to [7], we model the social system by trust degrees. Defined for each link between every two nodes, *trust degree* is a level of belief that one node can help the other node for a specific action such as relaying [7], [9]. Trust degree can be quantified based on the previous interactions between the nodes or can be estimated from the history of past interactions in a social platform such as in Facebook or Twitter [7], [9]–[11]. Specifically, in our cooperative communication system (see Fig. 1), the relay would be willing to participate in cooperation with probability α_1 and would use a fraction β_1 of its power for this purpose, with $\alpha_1, \beta_1 \in [0\ 1]$.[1] Without loss of generality, and for simplicity of presentation, we absorb β_1 in the relay power. That is, we assume P_r is the fraction of relay power used for cooperation. Then, the trust degree based rate will be

$$\begin{aligned} \mathcal{R}_\mathrm{T} &= \alpha_1 \mathcal{C}_\mathrm{DF} + (1-\alpha_1)\mathcal{C}_\mathrm{SD} \\ &= \alpha_1 \min\Big\{\log_2(1+\gamma_\mathrm{SR}), \log_2(1+\gamma_\mathrm{SD}+\gamma_\mathrm{RD})\Big\} \\ &\quad + \bar{\alpha}_1 \log_2(1+\gamma_\mathrm{SD}) \end{aligned} \tag{11}$$

in which $\mathcal{C}_\mathrm{DF}$ and $\mathcal{C}_\mathrm{SD}$ are from (10) and (5), and $\bar{\alpha}_1 = 1-\alpha_1$.

It is worth noting that for $\alpha_1 = 1$ we get $\mathcal{R}_\mathrm{T} = \mathcal{C}_\mathrm{DF}$ and our system simplifies to a DF MIMO relay beamforming problem which is a non-convex problem and has been extensively studied, e.g., see [8], [12], [13] and the reference therein. On the other hand, for $\alpha_1 = 0$ we get $\mathcal{R}_\mathrm{T} = \mathcal{C}_\mathrm{SD}$ and the system simplifies to multiple-input-single-output (MISO) channel which is a well-understood problem.

[1] In general, for link i we can define the pair (α_i, β_i), $\alpha_i, \beta_i \in [0\ 1]$, as a virtual (social) tie in which α_i denotes the probability of cooperation and β_i denotes the fraction of power that the node uses for collaboration [7]. Here, trust degrees are assumed to be bidirectional.

III. Problem Formulation and Transformation

In this section, we formulate a beamforming design problem to maximize the objective function in (11). We then transform this problem into some equivalent problems that can be tackled more easier. Our goal is to jointly optimize the beamforming vectors $\mathbf{w}_{\rm s}$ and $\mathbf{w}_{\rm r}$ as well as α to maximize (11). Mathematically, this can be expressed as

$$\begin{aligned} \max_{\mathbf{w}_{\rm s},\mathbf{w}_{\rm r},\alpha} \quad & \mathcal{R}_{\rm T} \\ \text{s.t.} \quad & \|\mathbf{w}_{\rm s}\|^2 = 1, \\ & \|\mathbf{w}_{\rm r}\|^2 = 1, \\ & 0 \le \alpha \le \alpha_1, \end{aligned} \tag{12}$$

where $\mathcal{R}_{\rm T}$ is defined in (11).

This is a non-convex optimization problem,[2] for which standard convex optimization techniques cannot be applied directly. As a first step to simplifying (12), we observe that $\gamma_{\rm SD}$ and $\gamma_{\rm SR}$ are independent of $\mathbf{w}_{\rm r}$, the beamforming vector at the relay. In other words, $\mathbf{w}_{\rm r}$ merely affects $\gamma_{\rm RD}$. Hence, the maximal ratio transmission (MRT) beamformer [8], [13], [14] is optimal, i.e.,

$$\mathbf{w}_{\rm r}^* = \frac{\mathbf{g}}{\|\mathbf{g}\|}. \tag{13}$$

With this, the problem reduces to

$$\begin{aligned} \max_{\mathbf{w}_{\rm s},\alpha} \quad & \big[\alpha \min\{\log_2(1+\gamma_{\rm SR}), \log_2(1+\gamma_{\rm SD}+\gamma_{\rm RD}^\star)\} \\ & \quad + \bar{\alpha}\log_2(1+\gamma_{\rm SD})\big] \\ \text{s.t.} \quad & \|\mathbf{w}_{\rm s}\|^2 = 1, \\ & 0 \le \alpha \le \alpha_1, \end{aligned} \tag{14}$$

in which $\gamma_{\rm RD}^\star = \|\mathbf{g}\|^2 P_{\rm r}$ is obtained from (13) and (8).

A. An Upper Bound

Lemma 1. *An upper bound on the achievable rate in* (11) *is given by*

$$\begin{aligned} \mathcal{R}_{\rm T} &\le \alpha_1 \log_2(1+\gamma_{\rm SD}^\star+\gamma_{\rm RD}^\star) + \bar{\alpha}_1 \log_2(1+\gamma_{\rm SD}^\star) \quad (15) \\ &= \alpha_1 \log_2(1+\|\mathbf{h}\|^2 P_{\rm s} + \|\mathbf{g}\|^2 P_{\rm r}) + \bar{\alpha}_1 \log_2(1+\|\mathbf{h}\|^2 P_{\rm s}) \end{aligned}$$

in which $\gamma_{\rm SD}^\star$ is obtained from (3) *for $\mathbf{w}_{\rm s}^* = \frac{\mathbf{h}}{\|\mathbf{h}\|}$.*

Proof. Using (14), we can write

$$\begin{aligned} \mathcal{R}_{\rm T} &= \alpha \min\Big\{\log_2(1+\gamma_{\rm SR}), \log_2(1+\gamma_{\rm SD}+\gamma_{\rm RD}^\star)\Big\} \\ &\quad + \bar{\alpha}\log_2(1+\gamma_{\rm SD}) \\ &\le \alpha \log_2(1+\gamma_{\rm SD}+\gamma_{\rm RD}^\star) + \bar{\alpha}\log_2(1+\gamma_{\rm SD}) \\ &\le \alpha_1 \log_2(1+\gamma_{\rm SD}+\gamma_{\rm RD}^\star) + \bar{\alpha}_1\log_2(1+\gamma_{\rm SD}) \\ &\le \alpha_1 \log_2(1+\gamma_{\rm SD}^\star+\gamma_{\rm RD}^\star) + \bar{\alpha}_1\log_2(1+\gamma_{\rm SD}^\star) \end{aligned} \tag{16}$$

where the first inequality is due to the fact that $\min\{a,b\} \le b$, the second inequality follows from $1+\gamma_{\rm SD}+\gamma_{\rm RD}^\star \ge 1+\gamma_{\rm SD}$ and the fact that $\alpha \le \alpha_1$, and the third inequality is due to a simple optimization over $\gamma_{\rm SD}$ in which $\gamma_{\rm SD}^\star$ is obtained from (3) for $\mathbf{w}_{\rm s}^* = \frac{\mathbf{h}}{\|\mathbf{h}\|}$. □

[2] $\mathcal{R}_{\rm T}$ includes logarithmic functions of quadratic functions of $\mathbf{w}_{\rm s}$; thus, it is a combination of concave and convex functions which is non-convex.

It is worth noting that the above upper bound is achievable if the link between S and R is strong enough so that it is not the bottleneck of the system. This is expected if the relay has multiple antennas and/or is close enough to the transmitter. More accurately, we have the following result.

Theorem 1. *If $\|\mathbf{H}^t\mathbf{w}_{\rm s}\|^2 P_{\rm s} \ge \|\mathbf{h}\|^2 P_{\rm s} + \|\mathbf{g}\|^2 P_{\rm r}$ for $\mathbf{w}_{\rm s}^* = \frac{\mathbf{h}}{\|\mathbf{h}\|}$, then $\mathbf{w}_{\rm s}^* = \frac{\mathbf{h}}{\|\mathbf{h}\|}$ is an optimal beamformer of* (14)*, and*

$$\mathcal{R}_{\rm T} = \alpha_1 \log_2(1+\|\mathbf{h}\|^2 P_{\rm s} + \|\mathbf{g}\|^2 P_{\rm r}) + \bar{\alpha}_1 \log_2(1+\|\mathbf{h}\|^2 P_{\rm s})$$

is achievable for the system depicted in Fig. 1.

Proof. The proof follows from Lemma 1 because the upper bound above is achievable for $\mathbf{w}_{\rm s}^* = \frac{\mathbf{h}}{\|\mathbf{h}\|}$ if $\|\mathbf{H}^t\mathbf{w}_{\rm s}\|^2 P_{\rm s} \ge \|\mathbf{h}\|^2 P_{\rm s} + \|\mathbf{g}\|^2 P_{\rm r}$. □

B. Transforming the Optimization Problem

The optimization problem (12) is a non-convex problem, even for a fixed α. In the following, we aim at simplifying it by converting it into more tractable problems. More specifically, we break (12) into three optimization problems, as described below.

1) $\gamma_{\rm SR} \le \gamma_{\rm SD}$: In such a case, the relay link is not strong enough to be helpful. Then, from (14), it is straightforward to check that the optimal α is zero ($\alpha^\star = 0$) and the problem simplifies to

$$\begin{aligned} \max_{\mathbf{w}_{\rm s}} \quad & \log_2(1+\gamma_{\rm SD}) \\ \text{s.t.} \quad & \|\mathbf{w}_{\rm s}\|^2 = 1, \\ & \gamma_{\rm SR} \le \gamma_{\rm SD}. \end{aligned} \tag{17}$$

Then, without the last constraint, $\mathbf{w}_{\rm s}$ can be easily designed as an MRT beamformer; i.e., $\mathbf{w}_{\rm s}^* = \frac{\mathbf{h}}{\|\mathbf{h}\|}$. Thus, an optimal beamformer is given by $\mathbf{w}_{\rm s}^* = \frac{\mathbf{h}}{\|\mathbf{h}\|}$ provided that the inequality $\gamma_{\rm SR} \le \gamma_{\rm SD}$ holds. The optimal $\mathbf{w}_{\rm s}$ is not trivial in general.

2) $\gamma_{\rm SR} \ge \gamma_{\rm SD} + \gamma_{\rm RD}^\star$: This happens, for example, when the relay is very close to the transmitter. In this case, $\mathcal{R}_{\rm T} = \alpha\log_2(1+\gamma_{\rm SD}+\gamma_{\rm RD}^\star) + \bar{\alpha}\log_2(1+\gamma_{\rm SD})$. Now, since the argument of the first logarithm is always greater than or equal to that of the second one, we have $\alpha^\star = \alpha_1$, and (14) reduces to

$$\begin{aligned} \max_{\mathbf{w}_{\rm s}} \quad & \alpha_1\log_2(1+\gamma_{\rm SD}+\gamma_{\rm RD}^\star) + \bar{\alpha}_1\log_2(1+\gamma_{\rm SD}) \\ \text{s.t.} \quad & \|\mathbf{w}_{\rm s}\|^2 = 1, \\ & \gamma_{\rm SR} \ge \gamma_{\rm SD} + \gamma_{\rm RD}^\star. \end{aligned} \tag{18}$$

Then again, $\mathbf{w}_{\rm s}^* = \frac{\mathbf{h}}{\|\mathbf{h}\|}$ is optimal if the inequality $\gamma_{\rm SR} \ge \gamma_{\rm SD} + \gamma_{\rm RD}^\star$ holds. In such a case, we get $\mathcal{R}_{\rm T} = \alpha_1\log_2(1+\|\mathbf{h}\|^2 P_{\rm s} + \|\mathbf{g}\|^2 P_{\rm r}) + \bar{\alpha}_1\log_2(1+\|\mathbf{h}\|^2 P_{\rm s})$. Note that this is the best achievable rate we can expect from the optimization problem in (12).

3) $\gamma_{\rm SD} < \gamma_{\rm SR} < \gamma_{\rm SD} + \gamma_{\rm RD}^{\star}$: In this case, since $\gamma_{\rm SD} < \gamma_{\rm SR}$, it can be checked from (14) that $\alpha^{\star} = \alpha_1$ is optimal. Physically, this means that the relay is strong enough and the social connection is fully exploited. Further, in view of $\gamma_{\rm SR} < \gamma_{\rm SD} + \gamma_{\rm RD}^{\star}$, (14) simplifies to

$$\begin{aligned} \max_{\mathbf{w}_{\rm s}} \quad & \alpha_1 \log_2(1+\gamma_{\rm SR}) + \bar{\alpha}_1 \log_2(1+\gamma_{\rm SD}) \\ \text{s.t.} \quad & \|\mathbf{w}_{\rm s}\|^2 = 1, \\ & \gamma_{\rm SD} < \gamma_{\rm SR}, \\ & \gamma_{\rm SR} < \gamma_{\rm SD} + \gamma_{\rm RD}^{\star}. \end{aligned} \tag{19}$$

The new optimization problems defined by (17), (18), and (19) are simpler than (14) in that the optimization over the variable α is already carried out. With this divide and conquer approach, we readily obtain the following useful lemmas.

Lemma 2. *The optimal value of α in* (14) *is either zero or α_1.*

Lemma 3. *Let $\mathcal{R}_{\rm T1}$, $\mathcal{R}_{\rm T2}$ and $\mathcal{R}_{\rm T3}$ be the solutions to the optimization problems in* (17)*,* (18) *and* (19)*, respectively. Then, the optimum rate achieved by the solution of* (14) *is equal to* $\max\{\mathcal{R}_{\rm T1}, \mathcal{R}_{\rm T2}, \mathcal{R}_{\rm T3}\}$*. Further, the optimum beamforming vector at the source $\mathbf{w}_{\rm s}$ is the one corresponding to the maximum $\mathcal{R}_{\rm Ti}$ for $i = 1, 2, 3$*

C. Semidefinite Relaxation

By converting the original optimization problem into three subproblems, we fixed the value of α in the previous subsection. However, the new optimization problems are still nonconvex and difficult to solve. A computationally efficient way to tackle the optimization problem in (17) is to use *semidefinite relaxation* (SDR) [15], as described below. It can be checked that the same $\mathbf{w}_{\rm s}$ that maximizes (17) is also optimal for the following problem:

$$\begin{aligned} \min_{\mathbf{w}_{\rm s}} \quad & -\gamma_{\rm SD} \\ \text{s.t.} \quad & \|\mathbf{w}_{\rm s}\|^2 = 1, \\ & \gamma_{\rm SR} \le \gamma_{\rm SD}. \end{aligned} \tag{20}$$

Since $\gamma_{\rm SD}$ and $\gamma_{\rm SR}$ are quadratic in $\mathbf{w}_{\rm s}$, this new problem is a *quadratically constrained quadratic program* (QCQP), and can be solved by the SDR technique [15]. To see this, let us write

$$\begin{aligned} \gamma_{\rm SR} &= \|\mathbf{H}\mathbf{w}_{\rm s}\|^2 P = \mathbf{w}_{\rm s}^{\dagger}\mathbf{H}^{\dagger}\mathbf{H}\mathbf{w}_{\rm s} P \\ &= \operatorname{tr}(\mathbf{H}^{\dagger}\mathbf{H}\mathbf{w}_{\rm s}\mathbf{w}_{\rm s}^{\dagger}) P = P \operatorname{tr}(\mathbf{A}\mathbf{W}), \end{aligned} \tag{21}$$

where we have defined $\mathbf{A} = \mathbf{H}^{\dagger}\mathbf{H}$ and $\mathbf{W} = \mathbf{w}_{\rm s}\mathbf{w}_{\rm s}^{\dagger}$. Then, we can transform the above optimization problem to

$$\begin{aligned} \min_{\mathbf{w}_{\rm s}} \quad & -t \\ \text{s.t.} \quad & P \operatorname{tr}(\mathbf{A}\mathbf{W}) \le t \\ & \operatorname{tr}(\mathbf{W}) = 1 \\ & \operatorname{rank}(\mathbf{W}) = 1, \\ & \mathbf{W} \succeq 0, \end{aligned} \tag{22}$$

in which $\mathbf{W} \succeq \mathbf{0}$ means that $\mathbf{W}$ is a positive semidefinite matrix. Now, the objective function is obviously a convex function. Also, the trace constraints are linear in $\mathbf{W}$. However, the rank constraint is non-convex.[3] We can relax this optimization problem by dropping the rank constraint, which is called SDR. The resulting optimization problem is convex and can be solved efficiently using interior point methods, in polynomial time. We note that SDR converts an NP-hard problem to a polynomial-time solvable problem. But this analytical convenience comes at the expense of conversion of the globally optimal solution of the relaxed problem into a feasible solution of (22). Randomization is an effective method for converting the solution of the relaxed problem to an approximate solution of the original problem [15], [16]. Unfortunately, even SDR is not applicable to (18) and (19), as their objective functions can be neither quadratic nor convex.

IV. An Efficient Heuristic Approach for Beamforming

In the previous section, we explained that the optimization problem (14) is hard to solve even when the trust degree is fixed. The main problem lies in the fact that the resulting optimization formulation is not convex, which does not lend itself to an analytically or numerically efficient solution. Likewise, the SDR method is not applicable to this problem in general. In order to alleviate this problem, we propose an efficient heuristic methodology to construct the beamforming vector and reduce the dimension of the optimum beamforming problem in this section. This approach will lead to a numerically efficient solution for trust degree based beamforming in cooperative mobile data and social networks.

The optimization problem formulated in (14) clearly indicates that the beamforming vector should alter the values of $\gamma_{\rm SD}$ and $\gamma_{\rm SR}$ in a way to maximize the transmission rates. On the other hand, from (3) and (4), we observe that $\gamma_{\rm SD}$ and $\gamma_{\rm SR}$ are functions of $\mathbf{h}$ and $\mathbf{H}$, respectively. Thus, it is intuitive to expect an efficient construction of the beamforming vector based on the knowledge of $\mathbf{h}$ and $\mathbf{H}$. More concretely, the optimal $\mathbf{w}_{\rm s}$ should be a vector in the *span* of the space generated by the columns of $\mathbf{h}$ and $\mathbf{H}$.

One immediate solution to this end would be the normalization of $c\mathbf{h} + \sum_{i=1}^{N_{\rm r}} c_i \mathbf{h}_i$, where $\mathbf{h}_i$ is the ith column of $\mathbf{H}$. However, such a $\mathbf{w}_{\rm s}$ leads to an intractable form when substituted back into (14), as $\mathbf{h}$ and the $\mathbf{h}_i$'s are not necessarily orthogonal. A more amenable form would be obtained if an orthogonal basis of the subspace defined by these vectors is used. This is the approach we follow below.

The orthogonal projection of $\mathbf{H}$ onto the column space of $\mathbf{h}$ is given by $\mathbf{\Pi_h H}$, where $\mathbf{\Pi_h} \triangleq \mathbf{h}(\mathbf{h}^{\dagger}\mathbf{h})^{-1}\mathbf{h}^{\dagger}$ is the standard orthogonal projection matrix [17].[4] Since $\mathbf{\Pi_h}$ is a projection onto $\mathbf{h}$, it is easy to check that each column of $\frac{\mathbf{\Pi_h H}}{\|\mathbf{\Pi_h H}\|}$ is equal to $\frac{\mathbf{h}}{\|\mathbf{h}\|}$. Also, the orthogonal projection of $\mathbf{H}$ onto the

[3] The sum of two rank-one matrices has generic rank two.

[4] Let $\mathbf{\Pi_X} \triangleq \mathbf{X}(\mathbf{X}^{\dagger}\mathbf{X})^{-1}\mathbf{X}^{\dagger}$ be the orthogonal projection onto the column space of $\mathbf{X}$. Then, $\mathbf{\Pi_X^{\perp}} \triangleq \mathbf{I} - \mathbf{\Pi_X}$ represents the orthogonal projection onto the orthogonal complement of the column space of $\mathbf{X}$ [17].

orthogonal complement column space of $\mathbf{h}$ is given by $\mathbf{\Pi}_{\mathbf{h}}^{\perp}\mathbf{H}$, where $\mathbf{\Pi}_{\mathbf{h}}^{\perp} \triangleq \mathbf{I} - \mathbf{\Pi}_{\mathbf{h}}$.

Next, let $\mathbf{\Pi}_{\mathbf{h}}^{\perp}\mathbf{H} \triangleq [\bar{\mathbf{h}}_1|\bar{\mathbf{H}}_1]$ where $\bar{\mathbf{h}}_1$ is the first column of $\mathbf{\Pi}_{\mathbf{h}}^{\perp}\mathbf{H}$ and $\bar{\mathbf{H}}_1$ contains the remaining $N_{\rm r} - 1$ columns of $\mathbf{\Pi}_{\mathbf{h}}^{\perp}\mathbf{H}$. We can find the orthogonal projection of $\bar{\mathbf{H}}_1$ onto the column space of $\bar{\mathbf{h}}_1$ by $\mathbf{\Pi}_{\bar{\mathbf{h}}_1}\bar{\mathbf{H}}_1$, where $\mathbf{\Pi}_{\bar{\mathbf{h}}_1} \triangleq \bar{\mathbf{h}}_1(\bar{\mathbf{h}}_1^{\dagger}\bar{\mathbf{h}}_1)^{-1}\bar{\mathbf{h}}_1^{\dagger}$. Likewise, the orthogonal projection of $\bar{\mathbf{H}}_1$ onto the orthogonal complement column space of $\bar{\mathbf{h}}_1$ is given by $\mathbf{\Pi}_{\bar{\mathbf{h}}_1}^{\perp}\bar{\mathbf{H}}_1$. We now denote $\mathbf{\Pi}_{\bar{\mathbf{h}}_1}^{\perp}\bar{\mathbf{H}}_1 \triangleq [\bar{\mathbf{h}}_2|\bar{\mathbf{H}}_2]$ and find orthogonal projections of $\bar{\mathbf{H}}_2$ onto the column space and orthogonal complement column space of $\bar{\mathbf{h}}_2$. We repeat this procedure until $\bar{\mathbf{H}}_i$ becomes empty. Note that $i \leq N_{\rm r}$ as $\mathbf{\Pi}_{\mathbf{h}}^{\perp}\mathbf{H}$ has $N_{\rm r}$ (independent) columns at most.[5] At this point, the orthogonalization process is completed and we can write

$$\mathbf{w} \triangleq \frac{\mathbf{h}}{\|\mathbf{h}\|} \tag{23}$$

$$\mathbf{w}_i \triangleq \frac{\bar{\mathbf{h}}_i}{\|\bar{\mathbf{h}}_i\|}, \quad i \in \{1, \ldots, N_{\rm r}\}. \tag{24}$$

By definition, we have $\mathbf{w} \perp \mathbf{w}_i^{\perp}$, i.e., $\mathbf{w}^{\dagger}.\mathbf{w}_i^{\perp} = 0 \ \forall i \in \{1, \ldots, N_{\rm r}\}$. The optimal structure of the beamforming vector is then given by the following lemma.

Theorem 2. *The transmit beamformer that maximizes the achievable rate in* (10) *can be represented as*

$$\mathbf{w}_{\rm s} = \sqrt{\gamma_0}\mathbf{w} + \sum_{i=1}^{N_{\rm r}} \sqrt{\gamma_i}\mathbf{w}_i \tag{25}$$

in which $\mathbf{w}$ *and the* $\mathbf{w}_i$*'s are orthonormal bases spanning the column space of* $\mathbf{h}$ *and* $\mathbf{H}$*, respectively, as defined in* (23) *and* (24)*, and* $\gamma_0 + \gamma_1 + \ldots + \gamma_{N_{\rm r}} = 1$.

Proof. We prove this lemma by contradiction, similar to [7]. To simplify the proof, let us rewrite (25) as

$$\mathbf{w}_{\rm s} = \sqrt{\gamma_0}\mathbf{w} + \sqrt{\bar{\gamma}_0}\mathbf{w}^{\perp} \tag{26}$$

in which we define $\mathbf{w}^{\perp} \triangleq \frac{1}{\sqrt{\bar{\gamma}_0}} \sum_{i=1}^{N_{\rm r}} \sqrt{\gamma_i}\mathbf{w}_i$. Next, suppose that $\tilde{\mathbf{w}}_{\rm s} \in \mathbb{C}^{N_{\rm s}\times 1}$ is an optimal beamformer. Obviously, $\tilde{\mathbf{w}}_{\rm s}$ can be represented as $\tilde{\mathbf{w}}_{\rm s} = \sqrt{\eta_1}\mathbf{f}_1 + \sqrt{\eta_2}\mathbf{f}_2 + \ldots + \sqrt{\eta_{N_{\rm s}}}\mathbf{f}_{N_{\rm s}}$ where the $\mathbf{f}_i$'s form an *orthonormal basis*. Since $\mathbf{w}$ and $\mathbf{w}^{\perp}$ are orthonormal vectors, without loss of generality, let $\mathbf{f}_1 = \mathbf{w}$ and $\mathbf{f}_2 = \mathbf{w}^{\perp}$. Then, we get

$$\tilde{\mathbf{w}}_{\rm s} = \sqrt{\eta_1}\mathbf{w} + \sqrt{\eta_2}\mathbf{w}^{\perp} + \sqrt{\eta_3}\mathbf{f}_3 + \ldots + \sqrt{\eta_{N_{\rm s}}}\mathbf{f}_{N_{\rm s}} \tag{27}$$

in which $\mathbf{f}_i^{\dagger} \cdot \mathbf{w} = 0$ and $\mathbf{f}_i^{\dagger} \cdot \mathbf{w}^{\perp} = 0$ for $i \geq 3$. Next, we argue that η_i should be zero for any $i \geq 3$. To this end, we show that using $\tilde{\mathbf{w}}_{\rm s}$ as a beamformer, $\gamma_{\rm SD}$ and $\gamma_{\rm SR}$ do not depend on η_i, $i \geq 3$. The former is rather simple as $\gamma_{\rm SD} = |\mathbf{h}^t\tilde{\mathbf{w}}_{\rm s}|^2 P_{\rm s} = \eta_1\|\mathbf{h}\|^2 P_{\rm s}$, in view of (27) and (23). It is also easy to verify that $\gamma_{\rm SR} = \|\mathbf{H}^t\tilde{\mathbf{w}}_{\rm s}\|^2 P$ is independent of η_i, $i \geq 3$. To see this, recall from the orthogonalization process in (24) that the $\mathbf{w}_i$'s form an orthonormal basis spanning the column space of $\mathbf{H}$. Hence, each column of $\mathbf{H}$ is a linear combination of the $\mathbf{w}_i$'s, and thus, is orthogonal to $\mathbf{f}_i$ for $i \geq 3$ due to the construction of the $\mathbf{f}_i$'s in (27) and that of $\mathbf{w}^{\perp}$ in (26). This means that $\hat{\mathbf{w}}_{\rm s} = \sqrt{\eta_1}\mathbf{w} + \sqrt{\hat{\eta}_2}\mathbf{w}^{\perp}$ with $\hat{\eta}_2 = \eta_2 + \eta_3 + \ldots + \eta_{N_{\rm s}}$ can strictly increase the optimum value of the data rate in (14) when compared with $\tilde{\mathbf{w}}_{\rm s}$. But, this contradicts the assumption that $\tilde{\mathbf{w}}_s$ is an optimal beamformer and not equal to $\mathbf{w}_s$. Therefore, any optimum beamformer must be equal to $\mathbf{w}_{\rm s}$ given in (26). This completes the proof. □

[5]Here, we have assumed $N_{\rm r} \leq N_{\rm s}$. In general, $i \leq \min\{N_{\rm r}, N_{\rm s}\}$.

Remark 1. Note that the problem has been largely simplified in Theorem 2 as only $N_{\rm r}$ real numbers are to be determined whereas in the original problem we needed to find $N_{\rm s}$ complex numbers. Note that, in practical systems, $N_{\rm s} \gg N_{\rm r}$ and $N_{\rm r}$ is usually 1, 2, or 4 as typical relay nodes are assumed to be network users rather than being extra middle-boxes to help for cooperative communication.

Remark 2. For the MISO case ($N_{\rm r} = 1$), from (26), it is easy to see that $\mathbf{w}^{\perp} = \mathbf{w}_1$. Then, to determine $\mathbf{w}_{\rm s}$ in (25), we only need to find γ_0. We should highlight that this case was studied in [7], where the authors found a closed-form solution for γ_0, based on an approximated achievable rate. However, the optimization problem in [7] is solved for a fixed α, i.e., $\alpha = \alpha_1$, whereas in Lemma 2 we proved that the optimal α can be 0 or α_1. In addition, the solution obtained for γ_0 is only for an approximated achievable rate in the high SNR regime, which tends to lose its accuracy when SNR values become smaller.

V. Numerical Results

We evaluate the performance of the proposed approach for generating beamforming vectors and compare it with the existing schemes in this section. For the purposes of simulation, we will assume that the channels $\mathbf{h}, \mathbf{H}$ and $\mathbf{g}$ are complex Gaussian vectors whose entries are independent and identical distributed (i.i.d.) Gaussian random variables with zero means and variances $\sigma_{\mathbf{h}}^2$, $\sigma_{\mathbf{H}}^2$, and $\sigma_{\mathbf{g}}^2$, respectively. Unlike [7], we optimize the exact achievable rate given by (14), not just a high SNR approximation of that. All graphs are based on averaging over 10,000 channel realizations.

We first focus on the MISO case and compare our results with those of [7], for $N_{\rm s} = 4$ and $(\sigma_{\mathbf{h}}^2, \sigma_{\mathbf{H}}^2, \sigma_{\mathbf{g}}^2) = (-5, -4, 10)$dB. In Fig. 2, we show the effect of optimizing over α rather than fixing it at $\alpha = \alpha_1$, which is the case in [7]. Recall from Lemma 2 that $\alpha^{\star} \in \{0, \alpha_1\}$. Hence, we choose between $\alpha = 0$ or $\alpha = \alpha_1$, whichever choice gives a better rate. In Fig. 2, we take $\alpha_1 = 0.7$ and direct transmission refers to the case with $\alpha = 0$. This figure clearly shows that using the maximum trust degree (α_1) is not always the best approach to optimize data rates, and, depending on the channel conditions, the transmission rate may increase by using a fine-tuned α smaller than α_1. As can be seen in Fig. 3, there is a visible gap between our scheme and the one proposed in [7] for different values of $N_{\rm s}$ at all SNRs, although this gap reduces as $N_{\rm s}$ increases or the relay link becomes stronger.

In Fig. 4, we consider the MIMO case and compare the achievable rates for different values of $N_{\rm r}$. Expectedly, as the

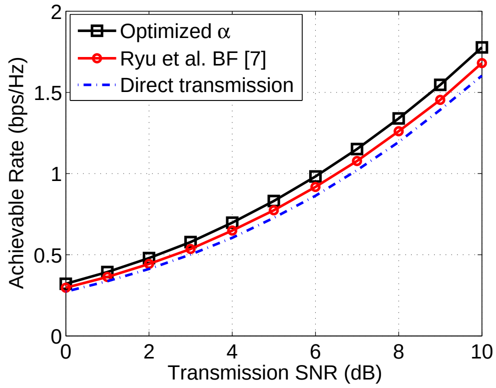

Fig. 2. Achievable rates for the MISO case with $N_{\rm s}=4$, $N_{\rm r}=1$, $\alpha_1=0.7$, and $(\sigma^2_{\mathbf{h}}, \sigma^2_{\mathbf{H}}, \sigma^2_{\mathbf{g}}) = (-5, -4, 10)$dB.

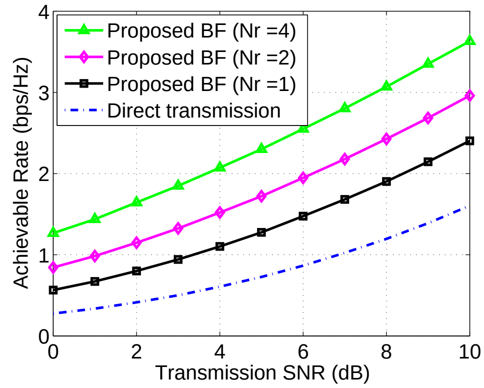

Fig. 4. Achievable rates for the MIMO case with $\alpha_1=0.7$, $(\sigma^2_{\mathbf{h}}, \sigma^2_{\mathbf{H}}, \sigma^2_{\mathbf{g}}) = (-5, 0, 10)$dB, $N_{\rm s}=4$, and different $N_{\rm r}$'s.

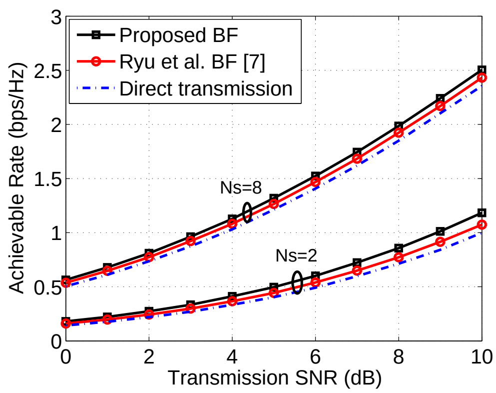

Fig. 3. Achievable rates for the MISO case ($N_{\rm r}=1$) with $\alpha_1=0.7$ and $(\sigma^2_{\mathbf{h}}, \sigma^2_{\mathbf{H}}, \sigma^2_{\mathbf{g}}) = (-5, -4, 10)$dB.

number of antennas increases, the achievable rate goes up.

VI. Conclusion

We have developed a trust degree based beamforming design formulation for multi-antenna cooperative communication. Observing that maximum trust degree may not result in the best rate performance, we have optimized the achievable rate over the trust degrees as well as the beamforming vectors at the source and relay nodes. We have proved that the optimal beamforming vector is a linear combination of the direct and cooperating links beamforming vectors, respecting the trust degree. Unlike existing works, the proposed scheme has been developed for the MIMO setting and is applicable to any SNR regime. For the MISO case, our scheme noticeably improves the achievable rate over those of the existing schemes.

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