Social-Aware User Cooperation in Full-Duplex and Half-Duplex Multi-Antenna Systems

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Abstract—Social and communications networks interact with each other in multifaceted ways, yet these interactions are often considered to be secondary in throughput, privacy and security analysis for communications networks. In this paper, full-duplex (FD) and half-duplex (HD) multi-antenna cooperative communication systems are studied by taking both physical links and social connections into account. An optimal beamformer for maximizing communication rate in the proposed socio-technological setting aims to balance between the direct link and the cooperating link as well as respecting the trust degree between the users. The resulting optimization problems are nontrivial to solve, even numerically, as they are not convex. The complexity of the problems is significantly reduced by showing that a linear combination of the direct and cooperating links’ channel vectors maximizes the achievable rate. Then, a computationally efficient numerical solution is used to maximize the rates both in the FD and HD modes. Numerical results demonstrate that significant gains in communication rates can be obtained with the proposed optimal beamforming design.

Index Terms—Mobile/social networks, MIMO beamforming, cooperative communication, full-duplex, half-duplex.

I. INTRODUCTION

A salient characteristic of mobile data networks is the dominant operating mode of one person behind each device, which differentiates them from other technological networks such as sensor networks. A direct consequence of this feature of mobile data networks is the existence of both physical coupling between mobile devices through shared communication resources such as wireless spectrum, and virtual coupling among the users behind these devices in the social domain. These virtual ties, in many ways, shape the data traffic flows and quality-of-service (QoS) requirements in the physical domain. Social interactions could be positive (representing friendship) or negative (representing antagonism), which yield non-trivial impact on users’ decision-making processes (e.g., in terms of cooperation) for data transmission and reception. We refer to this two-layered network structure as a mobile/social network.

A growing number of recent works [2]–[9], have studied the integration of mobile data networks with social networks in a variety of settings. In [2], a game theoretic framework called social group utility maximization is developed to maximize a weighted sum of the individual utilities rather than a totally selfish utility. In [3]–[6], social community-aware resource allocation is studied for device-to-device (D2D) networks. A social-aware peer discovery scheme for D2D systems is proposed in [7]. It exploits social network characteristics to assist ad hoc peer discovery and enhance data transmission performance. In [8], throughput of an ad hoc networks is maximized by adaptively utilizing different transmission schemes via a social-aware mechanism. Social-aware transmission strategies can be also employed to enhance transmission security [9].

Most of the above papers develop communication strategies based on single-antenna nodes. In a recent study, Ryu et al. [10] proposed the use of social connections for beamforming in a cooperative communication system with a single-antenna relay. This scheme uses trustworthiness between nodes to design beamforming for efficient data relaying. It is proved that a linear combination of channel vectors makes an optimal beamformer to maximize expected achievable rate. To find a closed-form solution for the optimal weights of the linear combination, a high signal-to-noise ratio (SNR) approximation is used which explicitly shows the effect of trust degrees on the beamforming design.

We exploit the concept of trust degree for user cooperation in multi-antenna cooperative systems, both in the full-duplex (FD) and half-duplex (HD) settings, in this paper. We first improve the achievable rate of [10] and extend it to the case where the relay has multiple antennas. The rate improvement is based on two observations. First, we show that maximum trust degree does not always result in the best rate performance. Thus, we consider optimization over trust degree as well as the beamforming vectors at the source and relay nodes. Second, unlike [10], our design is for the exact achievable rate, not just in the high SNR regime. We propose a computationally practical numerical solution that achieves optimal beamformer performance. The resulting beamformer improves the rate performance when compared with the closed-form solution of the approximated rate given in [10]. Another important contribution of the current paper is the extension of the trust degree based beamforming design to the case in which the relay has an arbitrary number of antennas. In fact, the proposed beamforming is developed for multi-antenna relays which can be applied to single-antenna relays as well. Lastly, while existing social-aware strategies are commonly developed based on the FD relays, we introduce the use of relays in the more practical HD mode. We consider the HD relay both with equal and unequal receive/transmission periods. Optimizing the cooperative achievable rate via HD relays is in general
more demanding than that of full duplex relays since new parameters, such as relay-receive and relay-transmit time periods, need to be optimized. Remarkably, once the transmission strategy is fixed, the same beamforming algorithm developed for the FD setting can be applied in the HD setting.

It should be mentioned that, concurrent to the conference version of this work [1], another group has independently developed a trust degree based beamforming for multi-antenna cooperative systems in [11]. The system model studied in [11] is however completely different from what we consider in this paper. Moreover, they study only time-division mod with equal time slots. Our paper studies both HD and FD cases, and in the case of HD we consider optimization over time-slots and transmission power in each time-slot. In addition, our beamforming algorithm is different from that in [11].

The paper is organized as follows. We describe mobile/social system model in Section II and formulate an optimization problem to maximize the achievable rate for an FD social-aware cooperative system in Section III. In Section IV we prove that optimal beamforming is achieved through a linear combination of the channel vectors for direct and cooperating links, respecting the trust degree. Section V is dedicated to the HD social-aware cooperative systems. Numerical results are presented in Section VI and the paper is concluded in Section VIII.

A. Notation

We use \((\cdot)^t\), \((\cdot)^*\), \((\cdot)^\dagger\), and \(\text{tr}(\cdot)\) to denote the transpose, conjugate, conjugate transpose, and trace of a matrix, respectively. Matrices are written in bold capital letters and vectors are written in bold small letters. \(I\) represents an identity matrix. For scalar \(x\), \(0 \leq x \leq 1\), we define \(\bar{x} = 1 - x\). \(\|x\|\) is the Euclidean norm of \(x\).

II. SYSTEM MODEL

In this paper, we consider a two-layered network structure, which we call a mobile/social network. As shown in Fig. 1, the network has physical connections (Fig. 1(a)) as well as social connections represented by trust degrees (Fig. 1(b)). Relays brings extra communications diversity in designing transmission strategies, which has in turn great potential to increase data rates [12], [13]. To this end, social connections quantify to what extent transmitters can depend on other devices in their vicinity (i.e., named as relay nodes) when transmitting their messages. We describe the details of this two-layered network model below, starting with the physical system model.

A. Physical System Model

Consider a multiple-input–multiple-output (MIMO) two-hop relay network as shown in Fig. 1(a). We assume that the source (S) and relay (R) have \(N_s\) and \(N_r\) antennas, respectively, and the destination (D) has one antenna only. Let \(h \in \mathbb{C}^{N_s \times 1}\), \(H \in \mathbb{C}^{N_s \times N_r}\), and \(g \in \mathbb{C}^{N_r \times 1}\) represent the channel coefficients for S-D, S-R, and R-D links, respectively. We assume that channel state information (CSI) is known at the transmitters, e.g., through training/feedback and channel reciprocity, so that they can steer their beamforming vectors to maximize their transmission performance.

Several cooperative strategies, such as amplify-and-forward, decode-and-forward and compress-and-forward can be applied in the relay node [14]. Further, the relay can operate in different modes, including the HD and FD modes. In this paper, we focus on the decode-and-forward relaying although the beamforming algorithm we develop can be applied to any cooperation strategy. The relay can, however, operate on the HD and FD modes.

B. Social System Model

We model the social system by trust degrees, similar to that in [10]. Defined for each link between every two nodes, trust degree is a level of belief that one node can help the other node for a specific action such as relaying [10], [15]. Trust degree can be quantified based on the previous interactions between the nodes or can be estimated from the history of past interactions in a social platform such as in Facebook or Twitter [10], [15]–[17]. Specifically, in our cooperative communication system (see Fig. 1), the relay would be willing to participate in cooperation with probability \(\alpha_1\) (trust degree) and would use a fraction \(\beta_1\) of its power for this purpose, with \(\alpha_1, \beta_1 \in [0, 1]\).

\(^1\) In general, for link \(i\) we can define the pair \((\alpha_i, \beta_i)\), \(\alpha_i, \beta_i \in [0, 1]\), as a virtual (social) tie in which \(\alpha_i\) denotes the probability of cooperation and \(\beta_i\) denotes the fraction of the power that the node uses for collaboration [10]. Here, trust degrees are assumed to be bidirectional.
The prime motivation for us to consider trust degrees in this cooperative communication setup is the observation that deploying fixed relay nodes to help mobile users in relaying data to their intended receivers is usually not a cost-effective solution. Such a fixed relay deployment scenario puts an upward pressure on the operating expenses (OPEX) and capital expenses (CAPEX) of network operators. Hence, opportunistic utilization of other devices, operated by human beings and physically close to the transmitter, emerges as an affordable and practical approach to realize the realm of cooperative relaying in future communications system architectures. The main problem with this cooperative approach is, however, the incentivization human-operated-devices to participate in the cooperative communication phase. To this end, social reciprocity and social trust are the two key mechanisms providing these incentives, alongside satisfactory performance, privacy and security protection, for cooperative relaying in machine-type communications and other edge-networking technologies [9, 10, 17]. To keep the analysis simpler and obtain clean insights into relay operation, we will focus only on trust degrees in the current paper and relegate the joint investigation of social reciprocity and trust to a future work.

Understanding the trust establishment process and trust variations over time is also important in this setup. For trust establishment, we assume that an application layer trust establishment algorithm runs at each device, which crawls the registered mobile social platforms of the device operators, collects social data traces from these social platforms and processes the collected social data to produce a trust degree database for each device. It is not within the scope of this paper to design such a trust establishment algorithm and some approaches can be found in [13, 18–20]. Rather, given any trust degree levels, we consider that these trust values can be pushed down from application layer to the physical layer in order to determine the cooperative communication strategy, at the protocol level and oblivious to the device operator, maximizing the data rates. The developed solutions in this paper will hold correct for any given trust degree levels. Once a trust degree level is established for using a particular device as a relay, this value will change much more slowly than the time-scale of change of channel states. However, due to transmitter mobility, the relay devices in the vicinity of the transmitter can vary, which can, in turn, trigger a relay selection and switching problem based on the trust degree levels of available relay devices at a given time instant. For the simplicity of analysis below, we will assume that transmitter mobility is not fast enough to initiate switching the relay node during a communication session. However, it should be noted that our optimum transmission strategy below will allow the flexibility of switching between cooperative and noncooperative communication modes at the time-scale of change of channel states based on the observed channel conditions and given trust degree levels. We will show that this extra design degree-of-freedom will help to improve achievable physical layer data rates.

Without loss of generality, and for simplicity of presentation, we absorb $\beta_i$ in the relay power. That is, we assume $P_i$ is the fraction of the relay’s power used for cooperation. Further, assume that the cooperative link is used with a probability $\alpha$, and $0 \leq \alpha \leq \alpha_1$.

Then, the expected trust degree based rate is given by

$$R_T = \alpha R_{DF} + \tilde{\alpha} C_{SD},$$

in which $R_{DF}$ represents the achievable rate of the cooperative link, $C_{SD}$ is the capacity of the direct transmission link, and $\tilde{\alpha} = 1 - \alpha$.

The goal of this work is to find beamforming algorithms, both at the source and the relay nodes, as well as the optimal value of $\alpha$ in order to maximize the expected trust degree based rate ($R_T$). It is worth noting that in the case of $\alpha_1 = 1$ we get $R_T = R_{DF}$ and our system simplifies to a decode-and-forward MIMO relay beamforming problem which is a nonconvex problem and has been extensively studied, e.g., see [21–23] and the reference therein. On the other hand, for $\alpha_1 = 0$ we get $R_T = R_{SD}$ and the system simplifies to multiple-input-single-output (MISO) channel which is a well-understood problem.

III. PROBLEM FORMULATION AND TRANSFORMATION FOR FD RELAY

In this section, we formulate a beamforming design problem to maximize the objective function in (1) when the relay works in the FD mode. We then transform this problem into some equivalent problems that can be tackled through easier analytical approaches.

A. Problem Formulation

Consider the physical system model described in Section II-A where the relay works in the FD mode and applies the decode-and-forward cooperation strategy. In this setting, the information symbol $x_s$ is first multiplied by a beamforming vector $w_s$ before being transmitted at S. The complex baseband signal received at R can be represented as

$$y_R = H^t w_s x_s + n_R,$$

in which $n_R \sim CN(0, I)$ represent the complex additive Gaussian noise at R. In an FD mode, the relay can receive and transmit at the same time. It decodes the information, re-encodes it to $x_t$, and transmits it to D. The complex baseband signal received at D, which is a combination of the information sent by S and R, can be represented as

$$y_D = h^t w_s x_s + g^t w_t x_t + n_D,$$

in which $n_D \sim CN(0, 1)$.

Next, let us define

$$\gamma_{SD} = \|h^t w_s\|^2 P_s,$$

$$\gamma_{SR} = \|H^t w_t\|^2 P_s,$$

$$\gamma_{RD} = \|g^t w_t\|^2 P_R,$$

2Although, the relay is willing to participate in cooperation with probability $\alpha_1$, we may use it with a probability $\alpha$ and $\alpha < \alpha_1$. This is because, depending on the channel conditions, such an $\alpha$ may result in a higher achievable rate when compared to $\alpha = \alpha_1$. 

$$\gamma_{SD} = \|h^t w_s\|^2 P_s,$$

$$\gamma_{SR} = \|H^t w_t\|^2 P_s,$$

$$\gamma_{RD} = \|g^t w_t\|^2 P_R,$$
in which $P_s$ is the average transmission power at S and $P_t$ is the fraction of the relay’s power used for the cooperation. An achievable rate for the decode-and-forward system can be written as

$$R_{DF}^{FD} = \min \left\{ \log_2(1 + \gamma_{SR}), \log_2(1 + \gamma_{SD} + \gamma_{RD}) \right\}. \quad (5)$$

This can be obtained using (12) equation (15)] or by letting $\beta = 0$ in equation (7) of (24). We next look at the direct transmission case, i.e., the case where the relay is not involved in the communication between S and D. It is easy to see that the maximum achievable rate of communication over the direct transmission link from S to D is given by

$$C_{SD} = \log_2(1 + \gamma_{SD}). \quad (6)$$

Finally, using (1), the expected achievable rate for the FD system is obtained as

$$R_T^{FD} = \alpha R_{DF}^{FD} + \bar{\gamma} C_{SD}$$

$$= \alpha \min \left\{ \log_2(1 + \gamma_{SR}), \log_2(1 + \gamma_{SD} + \gamma_{RD}) \right\} + \bar{\gamma} \log_2(1 + \gamma_{SD}). \quad (7)$$

Our goal is to jointly optimize the beamforming vectors $w_s$ and $w_r$ as well as $\alpha$, to maximize (7). Mathematically, this can be expressed as

$$\max_{w_s, w_r, \alpha} R_T^{FD}$$

s.t. $\|w_s\|^2 = 1$, $\|w_r\|^2 = 1$, $0 \leq \alpha \leq \alpha_1$, \quad (8)

where $R_T^{FD}$ is defined in (7).

This is a nonconvex optimization problem because $R_T$ is in the form of functional compositions of logarithmic and quadratic functions of $w_s$; thus, it is a combination of concave and convex functions, which is not necessarily convex. Therefore, standard convex optimization techniques cannot be applied directly. As a first step to simplifying (8), we observe that $\gamma_{SD}$ and $\gamma_{SR}$ are independent of $w_r$, the beamforming vector at the relay. In other words, $w_r$ merely affects $\gamma_{RD}$. Hence, the maximal ratio transmission (MRT) beamformer $22, 23, 25$ is optimal, i.e.,

$$w_r = \frac{g}{\|g\|}. \quad (9)$$

With this, the problem reduces to

$$\max_{w_s, \alpha} \left[ \alpha \min \{ \log_2(1 + \gamma_{SR}), \log_2(1 + \gamma_{SD} + \gamma_{RD}) \} + \bar{\gamma} \log_2(1 + \gamma_{SD}) \right]$$

s.t. $\|w_s\|^2 = 1$, $0 \leq \alpha \leq \alpha_1$, \quad (10)

in which $\gamma_{RD} = \|g\|^2 P_t$ is obtained from (9) and (4c).

B. An Analytical Solution

Before proceeding to further simplify (10), in this subsection we show that under certain channel conditions this optimization problem lends itself to the analytical solution. To show this, we first introduce an upper bound in the following.

**Lemma 1.** An upper bound on the achievable rate in (1) is given by

$$R_T \leq \alpha_1 \log_2(1 + \gamma_{SD}^* + \gamma_{RD}^*) + \bar{\alpha}_1 \log_2(1 + \gamma_{SD}) \quad (11)$$

$$= \alpha_1 \log_2(1 + \|h\|^2 P_s + \|g\|^2 P_t) + \bar{\alpha}_1 \log_2(1 + \|h\|^2 P_s)$$

in which $\gamma_{SD}^*$ is obtained from (4a) for $w_s = \frac{h}{\|h\|}$.\quad \Box

**Proof.** Using (10), we can write

$$R_T = \alpha \min \{ \log_2(1 + \gamma_{SR}), \log_2(1 + \gamma_{SD} + \gamma_{RD}) \} + \bar{\gamma} \log_2(1 + \gamma_{SD})$$

$$\leq \alpha \log_2(1 + \gamma_{SD}^* + \gamma_{RD}^*) + \bar{\alpha}_1 \log_2(1 + \gamma_{SD})$$

$$\leq \alpha_1 \log_2(1 + \gamma_{SD}^* + \gamma_{RD}^*) + \bar{\alpha}_1 \log_2(1 + \gamma_{SD})$$

$$\leq \alpha_1 \log_2(1 + \gamma_{SD} + \gamma_{RD}) + \bar{\alpha}_1 \log_2(1 + \gamma_{SD}) \quad (12)$$

in which the first inequality is due to the fact that for any $a, b \in \mathbb{R}$ we have $\min\{a, b\} \leq b$, the second inequality follows from $1 + \gamma_{SD}^* + \gamma_{RD}^* \geq 1 + \gamma_{SD}$ and the fact that $\alpha \leq \alpha_1$, and the third inequality is due to a simple optimization over $\gamma_{SD}$ in which $\gamma_{SD}^*$ is obtained from (4a) for $w_s = \frac{h}{\|h\|}$.

It is worth noting that the above upper bound is achievable if the link between S and R is strong enough so that it is not the bottleneck of the system. This is expected if the relay has multiple antennas and/or is close enough to the transmitter. More accurately, we have the following result.

**Theorem 1.** If $\|H w_s\|^2 P_s \geq \|h\|^2 P_s + \|g\|^2 P_t$ for $w_s = \frac{h}{\|h\|}$, then $w_s = \frac{h}{\|h\|}$ is an optimal beamformer of (10), and

$$R_T = \alpha_1 \log_2(1 + \|h\|^2 P_s + \|g\|^2 P_t) + \bar{\alpha}_1 \log_2(1 + \|h\|^2 P_s)$$

is achievable for the system depicted in Fig. 7 with an FD decode-and-forward relaying strategy.\quad \Box

**Proof.** The proof follows from Lemma 1 because the upper bound above is achievable for $w_s = \frac{h}{\|h\|}$ if $\|H w_s\|^2 P_s \geq \|h\|^2 P_s + \|g\|^2 P_t$.

Although the condition leading to the optimal beamformer in Theorem 1 can happen in practical networks, it captures a very special case. We are interested in obtaining the optimal beamformer under any channel conditions. To this end, in the next subsection, we further simplify (10) in order to pave the road to introduce a general solution in Section V.

C. Transforming the Optimization Problem

The optimization problem (8) is a nonconvex problem, even for a fixed $\alpha$. In the following, we aim at simplifying it by converting it into more tractable problems. This is based on the fact that for any fixed $w_s$ (including the optimal $w_s$), we will have one of the following three cases: $\gamma_{SR} \leq \gamma_{SD}, \gamma_{SR} \geq \gamma_{SD} + \gamma_{RD}^*$, or $\gamma_{SD} < \gamma_{SR} < \gamma_{SD} + \gamma_{RD}^*$. Based on this, we can break (8) into three optimization problems, as described below.
1) $\gamma_{SR} \leq \gamma_{SD}$: In such a case, the relay link is not strong enough to be helpful. Then, from (10), it is straightforward to check that the optimal $\alpha$ is zero ($\alpha^* = 0$) and the problem simplifies to

$$\max_{w_s} \log_2(1 + \gamma_{SD})$$
$$\text{s.t. } \|w_s\|^2 = 1,$$  \quad (13)

Then, without the last constraint, $w_s$ can be easily designed as an MRT beamformer; i.e., $w_s = \frac{h^\dagger}{\|h\|}$. Thus, an optimal beamformer is given by $w_s = \frac{h^\dagger}{\|h\|}$ provided that the inequality $\gamma_{SR} \leq \gamma_{SD}$ holds for this $w_s$. The optimal $w_s$ is not trivial to obtain, in general.

2) $\gamma_{SR} \geq \gamma_{SD} + \gamma_{RD}^2$. This happens, for example, when the relay is very close to the transmitter. In this case, $R_T = \alpha \log_2(1 + \gamma_{SD} + \gamma_{RD}) + \bar{\alpha} \log_2(1 + \gamma_{SD})$. Now, since the argument of the first logarithm is always greater than or equal to that of the second one, we have $\alpha^* = \alpha_1$, and (10) reduces to

$$\max_{w_s} \alpha_1 \log_2(1 + \gamma_{SD} + \gamma_{RD}^2) + \bar{\alpha}_1 \log_2(1 + \gamma_{SD})$$
$$\text{s.t. } \|w_s\|^2 = 1,$$  \quad (14)

Then again, $w_s = \frac{h^\dagger}{\|h\|}$ is optimal if the inequality $\gamma_{SR} \geq \gamma_{SD} + \gamma_{RD}^2$ holds for $w_s = \frac{h^\dagger}{\|h\|}$. In such a case, we get $R_T = \alpha_1 \log_2(1 + \|h\|^2P_s + \|g\|^2P_l) + \bar{\alpha}_1 \log_2(1 + \|h\|^2P_s)$. Note that this is the best achievable rate we can expect from the optimization problem in (10).

3) $\gamma_{SD} < \gamma_{SR} < \gamma_{SD} + \gamma_{RD}$. In this case, since $\gamma_{SR} < \gamma_{SD}$, it can be checked from (10) that $\alpha^* = \alpha_1$ is optimal. Physically, this means that the relay is strong enough and the social connection is fully exploited. Further, in view of $\gamma_{SR} < \gamma_{SD} + \gamma_{RD}$, (10) simplifies to

$$\max_{w_s} \alpha_1 \log_2(1 + \gamma_{SR}) + \bar{\alpha}_1 \log_2(1 + \gamma_{SD})$$
$$\text{s.t. } \|w_s\|^2 = 1,$$  \quad (15)

Then, the new optimization problems defined by (13), (14), and (15) are simpler than (10) in that the optimization over the variable $\alpha$ is already carried out. With this divide and conquer approach, we readily obtain the following useful lemmas.

**Lemma 2.** The optimal value of $\alpha$ in (10) is either zero or $\alpha_1$.

**Lemma 3.** Let $R_{T1}$, $R_{T2}$ and $R_{T3}$ be the solutions to the optimization problems in (13), (14) and (15), respectively. Then, the optimum rate achieved by the solution of (10) is equal to $\max \{R_{T1}, R_{T2}, R_{T3}\}$. Further, the optimum beamforming vector at the source $w_s$ is the one corresponding to the maximum $R_{T1}$ for $i = 1, 2, 3$.

By converting the original optimization problem into three subproblems corresponding to different scenarios, we fixed the value of $\alpha$ in the previous subsection. However, the new optimization problems are still nonconvex and difficult to solve. A computationally efficient way to tackle the optimization problem in (13) is to use semidefinite relaxation (SDR) [26], as described in [1]. However, unfortunately, SDR is not applicable to the optimization problems in (14) and (15), as their objective functions are neither quadratic nor convex. This motivates as to find another solution for these problems, and (10) in general, in the following section.

IV. AN EFFICIENT HEURISTIC APPROACH FOR BEAMFORMING

In the previous section, we explained that the optimization problem (10) is hard to solve even when the trust degree is fixed. The main problem lies in the fact that the resulting optimization formulation is not convex, which does not lend itself to an analytically or numerically efficient solution. Likewise, the SDR method is not applicable to this problem in general. In order to alleviate this problem, we propose an efficient heuristic methodology to construct the beamforming vector and reduce the dimension of the optimum beamforming problem in this section. This approach will lead to a numerically efficient solution for trust degree based beamforming in cooperative mobile data and social networks.

The optimization problem formulated in (10) clearly indicates that the beamforming vector should alter the values of $\gamma_{SD}$ and $\gamma_{SR}$ in a way to maximize the transmission rates. On the other hand, from (13) and (14), we observe that $\gamma_{SD}$ and $\gamma_{SR}$ are functions of $h$ and $H$, respectively. Thus, it is intuitive to expect an efficient construction of the beamforming vector based on the knowledge of $h$ and $H$. More concretely, the optimal $w_s$ should be a vector in the span of the space generated by the columns of $h$ and $H$.

One immediate solution to this end would be the normalization of $ch + \sum_{i=1}^{N_r} c_i h_i$, where $h_i$ is the $i$th column of $H$. However, such a $w_s$ leads to an intractable form when substituted back into (10), as $h$ and the $h_i$’s are not necessarily orthogonal. A more amenable form would be obtained if an orthogonal basis of the subspace defined by these vectors is used. This is the approach we follow below.

The orthogonal projection of $H$ onto the column space of $h$ is given by $\Pi_h H$, where $\Pi_h \triangleq h h^\dagger h^\dagger h^\dagger$ is the standard orthogonal projection matrix [27]. Since $\Pi_h$ is a projection onto $h$, it is easy to check that each column of $\Pi_h H$ is equal to $h_i h_i^\dagger$. Also, the orthogonal projection of $H$ onto the orthogonal complement column space of $h$ is given by $\Pi_{h_i} H$, where $\Pi_{h_i} \triangleq I - \Pi_h$.

Next, let $\Pi_h H \triangleq \hat{H}_1 [\hat{H}_2]$ where $\hat{H}_1$ is the first column of $\Pi_h H$ and $\hat{H}_1$ contains the remaining $N_r - 1$ columns of $\Pi_h H$. We can find the orthogonal projection of $\hat{H}_1$ onto the column space of $\hat{H}_1$ by $\Pi_{\hat{H}_1} \hat{H}_1$, where $\Pi_{\hat{H}_1} \triangleq \Pi_{\hat{H}_1} \hat{H}_1 \hat{H}_1^\dagger - \hat{H}_1 \hat{H}_1^\dagger$. Likewise, the orthogonal projection of $\hat{H}_1$ onto the orthogonal complement column space of $\hat{H}_1$ is given by $\Pi_{\hat{H}_2} \hat{H}_1$. We now denote $\Pi_{h_i} \hat{H}_1 \triangleq [\hat{H}_2]$. Find 3 Let $\Xi \triangleq X (X^\dagger X)^{-1} X^\dagger$ be the orthogonal projection onto the column space of $X$. Then, $\Xi \triangleq I - \Pi_X$ represents the orthogonal projection onto the orthogonal complement of the column space of $X$ [27].
orthogonal projections of $\bar{H}_2$ onto the column space and orthogonal complement column space of $\bar{H}_2$. We repeat this procedure until $\bar{H}_i$ becomes empty. Note that $i \leq N_r$ as $\Pi_i^\perp H$ has $N_r$ (independent) columns at most. At this point, the orthogonalization process is completed and we can write

$$w_0 \triangleq \frac{h_0}{\|h_0\|},$$

$$w_i \triangleq \frac{h_i}{\|h_i\|}, \quad i \in \{1, \ldots, N_r\}. \quad (16)$$

By definition, we have $w_0 \perp w_i^\perp$, i.e., $w_i^\perp w_i^\perp = 0 \forall i \in \{1, \ldots, N_r\}$. The optimal structure of the beamforming vector is then given by the following lemma.

**Theorem 2.** The transmit beamformer that maximizes the achievable rate in (5) can be represented as

$$w_s = \sqrt{\gamma_0} w_0 + \sum_{i=1}^{N_r} \sqrt{\gamma_i} w_i,$$  \quad (18)

in which $w$ and the $w_i$’s are orthonormal bases spanning the column space of $h$ and $H$, respectively, as defined in (16) and (17), and $\gamma_0 + \gamma_1 + \ldots + \gamma_{N_r} = 1$.

**Proof.** We prove this lemma by contradiction, similar to [10]. To simplify the proof, let us rewrite (18) as

$$w_s = \sqrt{\gamma_0} w + \sqrt{\gamma_1} w_i,$$  \quad (19)

in which we define $w = \frac{1}{\sqrt{\gamma_0}} \sum_{i=1}^{N_r} \sqrt{\gamma_i} w_i$. Next, suppose that $w_s \in \mathbb{C}^{N_r \times 1}$ is an optimal beamformer. Obviously, $w_s$ can be represented as $w_s = \sqrt{\gamma_1} f_1 + \sqrt{\gamma_2} f_2 + \ldots + \sqrt{\gamma_{N_r}} f_{N_r}$, where the $f_i$’s form an orthonormal basis. Since $w$ and $w_i^\perp$ are orthonormal vectors, without loss of generality, let $f_1 = w$ and $f_2 = w_i^\perp$. Then, we get

$$w_s = \sqrt{\gamma_1} f_1 + \sqrt{\gamma_2} f_2 + \ldots + \sqrt{\gamma_{N_r}} f_{N_r},$$ \quad (20)

in which $f_1^\perp \cdot w = 0$ and $f_i^\perp \cdot w = 0$ for $i \geq 3$. Next, we argue that $\eta_i$ should be zero for any $i > 3$. To this end, we show that using $w_s$ as a beamformer, $\gamma_{SD}$ and $\gamma_{SR}$ do not depend on $\eta_i$, $i \geq 3$. The former is rather simple as $\gamma_{SD} = \|Hw_s\|^2 P = \eta_1 \|h\|^2 P_h$, in view of (20) and (16). It is also easy to verify that $\gamma_{SR} = \|H^\dagger w_s\|^2 P$ is independent of $\eta_i$, $i \geq 3$. To see this, recall from the orthogonalization process in (17) that the $w_i$’s form an orthonormal basis spanning the column space of $H$. Hence, each column of $H$ is a linear combination of the $w_i$’s, and thus, is orthogonal to $f_i$ for $i \geq 3$ due to the construction of the $f_i$’s in (20) and that of $w_i^\perp$ in (19). This means that $\bar{w}_i = \sqrt{\gamma_i} w + \sqrt{\gamma_i} w_i^\perp$ with $\eta_2 = \eta_3 + \eta_4 + \ldots + \eta_{N_r}$ can strictly increase the optimum value of the data rate in (10) when compared with $w_s$. But, this contradicts the assumption that $w_s$ is an optimal beamformer and not equal to $w_s$. Therefore, any optimal beamformer must be equal to $w_s$ given in (19). This completes the proof. \hfill \Box

Note that, in practical systems, $N_r \gg N_s$ and $N_r$ is usually 1, 2, or 4 as relay nodes are assumed to be network users rather than being extra middle-boxes to help for cooperative communication.

So far, we have proved that optimal beamformer is a linear combination of at most $N_r + 1$ orthonormal vectors, as given in (18). To complete the process, we need to find the coefficients of the linear combination, i.e., $\gamma_{S\bar{D}}$ in (18), $i = 0, \ldots, N_r$. For $N_r = 1$ this is very simple and a linear search over one parameter is enough. For $N_r = 2$ and $N_r = 4$, we generate the coefficients randomly. Numerical tests show that for these cases using 100 different random weights (vectors of length $N_r + 1$) is enough to achieve an optimal solution. This is because the difference between the rates obtained via using 100 and 1000 random weights is negligible and thus there is no need to use a higher number of random weights.

**Remark 1.** For the MISO case ($N_r = 1$), from (19), it is easy to see that $w_i^\perp = w_1$. Then, to determine $w_s$ in (18), we only need to find $\gamma_0$. We should highlight that this case was studied in [10], where the authors found a closed-form solution for $\gamma_0$ based on an approximated achievable rate. However, the optimization problem in [10] is solved for a fixed $\alpha$, i.e., $\alpha = \alpha_1$, whereas in Lemma 2 we proved that the optimal $\alpha$ can be 0 or $\alpha_1$. In addition, the solution obtained for $\gamma_0$ is only for an approximated achievable rate in the high SNR regime, which tends to lose its accuracy when SNR values become smaller.

V. **HALF DUPLEX RELAY**

In this section, we investigate the performance of our baseline mobile/social network model when the relay operates on a HD mode. This is a more practical constraint on the relay where it transmits and receives on different time-slots, which is also called time-division (TD) relay channel [24]. In this setting, the relay is in the receive mode for a fraction $\hat{\tau}$ of the time ($0 \leq \hat{\tau} \leq 1$), and in the transmit mode for a fraction $\hat{\tau}$ of the time. These are called the relay-receive period and the relay-transmit period, respectively. The source node can, however, transmit in both periods.

A. **Fixed Power at Source**

We explain the information transmission in each phase and clarify the achievable cooperative rate assuming that the source uses a fixed power ($P_s$) in both phases.

- **Relay-receive phase ($\hat{\tau}$):** In this phase, the information symbol $x_s$ is first multiplied by a beamforming vector $w_s$ before being transmitted at $S$. The complex baseband signal received at $D$ and $R$ can be represented as

$$y_{SD} = h^\dagger w_s x_s + n_{SD},$$ \quad (21)

$$y_{SR} = H^\dagger w_s x_s + n_{SR},$$ \quad (22)

where $n_{SD} \sim \mathcal{CN}(0,1)$ and $n_{SR} \sim \mathcal{CN}(0,1)$ represent complex additive Gaussian noise at $D$ and $R$, respectively. The maximum achievable rates of communication from $S$ to $D$ and $S$ to $R$ are obtained by

$$C_{SD} = \log_2(1 + \gamma_{SD}),$$ \quad (23)

$$C_{SR} = \log_2(1 + \gamma_{SR}),$$ \quad (24)
average power is

\[ \gamma_{SD} \] and \( \gamma_{SR} \), the received SNRs at D and R, are
given by \( (4a) \) and \( (4b) \), respectively.

- **Relay-transmit phase** (\( \bar{\tau} \)) In this phase, R forwards its
decoded symbol \( x_r \) to D with power \( P_{RD} \) through transmission
over \( N_t \) antennas. The complex baseband signal received at D
can be written as

\[ y_{RD} = g^t w_r x_r + n_{RD}, \]

where \( n_{RD} \sim CN(0, 1) \). The received SNR at D, in this phase,
given by \( |g^t w_r|^2 = \frac{x_r}{\bar{\tau}} \). Then, D can combine the signal
received from S and R to improve the SNR. Specifically, we can have
the following result.

**Proposition 1.** An achievable rate for the TD HD relay is obtained by

\[ R_{TD}^{DF} = \max_{\theta} \min\{ R_1^{TD}, R_2^{TD} \}, \]

in which

\[ R_1^{TD} = \tau \log_2(1 + \gamma_{SR}) + \bar{\tau} \log_2(1 + \theta \gamma_{SD}), \]

\[ R_2^{TD} = \tau \log_2(1 + \gamma_{SD}) + \bar{\tau} \log_2 \left( 1 + \gamma_{SD} + \frac{1}{\bar{\tau}} \gamma_{RD} + 2 \sqrt{\frac{\theta}{\bar{\tau}}} \gamma_{SD} \gamma_{RD} \right), \]

\[ 0 \leq \tau \leq 1, \quad \text{and} \quad 0 \leq \theta \leq 1. \]

**Proof.** The proof can be found in the literature of the HD relay,
see for example [24], Proposition 2, [28], and [29]. Note that to get the above rate we have assumed that the relay uses the source power \( P_s \) in both transmission phases so that the total average power is \( \bar{\tau} P_s + \tau P_s = P_s \). Contrary to the source, the relay is silent during the relay-receive phase.

The optimal value of \( \theta \) can be obtained in closed-form as
proved in the following lemma.

**Lemma 4.** The optimal value of \( \theta \) in Proposition 1 is zero
if \( \left( \frac{1 + \gamma_{SR}}{1 + \gamma_{SD}} \right) \) is less than or equal to \( \frac{1 + \gamma_{SD} + \frac{1}{\bar{\tau}} \gamma_{RD}}{1 + \gamma_{SD}} \). Otherwise, the optimal value is obtained by

\[ \theta^* = \frac{1}{c \gamma_{SD}} \left[ -\frac{1}{\bar{\tau}} \gamma_{RD} + \sqrt{\frac{1}{\bar{\tau}} \gamma_{RD} - c \gamma_{SD} (1 + \gamma_{SD})(1 - c) + \frac{1}{\bar{\tau}} \gamma_{RD}} \right]^2, \]

in which \( c = \left( \frac{1 + \gamma_{SR}}{1 + \gamma_{SD}} \right)^{\bar{\tau}} \).

**Proof.** From (27a) and (27b), it can be seen that \( R_1^{TD} \) and \( R_2^{TD} \) are decreasing and increasing in \( \theta \), respectively. Thus, if the two curves \( (R_1^{TD} \) and \( R_2^{TD} \) with respect to \( \theta \) intersect, then that point will give \( \max \min \{ R_1^{TD}, R_2^{TD} \} \) and the optimal \( \theta \). Otherwise, the optimal value of \( \theta \) is obtained based on the maximum of the lower curve, which is rather simple. We first consider the first case. For notational convenience, let

\[ x = \gamma_{SR}, \quad y = \gamma_{SD}, \quad \text{and} \quad z = \frac{1}{2} \gamma_{RD}. \] Then, it is straightforward to check that \( R_1^{TD} = R_2^{TD} \) is equivalent to

\[ \frac{1 + x^{\bar{\tau}}}{1 + y^{\bar{\tau}}} = 1 + y + z + 2 \sqrt{\theta y z} \]

(29)

This is a quadratic equation with respect to \( \sqrt{\theta} \) which results in \( \sqrt{\theta} = -\sqrt{\frac{z}{c}} + \sqrt{\frac{z}{c} + 2 \gamma_{SR}(1 + \gamma_{SR})(1 - c) + \frac{1}{c} \gamma_{SD} \gamma_{RD}} \)

(30)

This completes the proof.

1) **Equal time allocation:** We first consider the special case of \( \tau = \frac{1}{2} \), i.e., equal time for the relay-receive and relay-transmit phases. This is the most common case in the literature. Obviously, a better rate can be achieved if we optimized \( \tau \) in Proposition 1. However, to avoid the complexity of practical relaying system as well as the complexity of the optimization problem, we fix \( \tau = \frac{1}{2} \) in this subsection. With this, the condition required for \( \theta = 0 \) reduces to \( \gamma_{SR} \leq \gamma_{SD} + 2 \gamma_{RD} \). Thus, we differentiate the following two sub-cases:

- **Case I.** \( \gamma_{SR} \leq \gamma_{SD} + 2 \gamma_{RD} \): In such a case, from Lemma 4 we know that \( \theta = 0 \) is optimal. Further, it can be checked that \( R_1^{TD} \leq R_2^{TD} \). Then, the TD HD rate (26) simplifies to

\[ R_{TD}^{HD} = \frac{1}{2} \left[ \log_2(1 + \gamma_{SD}) + \log_2(1 + \gamma_{SR}) \right]. \]

Next, using (I), the expected achievable rate for the HD mobile/social network system is obtained as

\[ R_T^{HD} = \alpha R_{TD}^{HD} + \alpha \gamma_{SD} \]

(31)

\[ = \frac{\alpha}{2} \left[ \log_2(1 + \gamma_{SD}) + \log_2(1 + \gamma_{SR}) \right] + \alpha \log_2(1 + \gamma_{SD}). \]

(32)

Similar to the FD case in Section III our goal is to jointly optimize the beamforming vectors \( w_s \) and \( w_r \) as well as \( \alpha \) to maximize (32), which can be cast as

\[ \max_{w_s, w_r, \alpha} R_T^{HD} \]

\[ \text{s.t.} \quad \|w_s\|^2 = 1, \]

\[ \|w_r\|^2 = 1, \]

\[ 0 \leq \alpha \leq \alpha_1, \]

where \( R_T^{HD} \) is defined in (32). We note that the objective function is independent of \( \gamma_{SR} \) and thus \( w_r \). Hence, we obtain

\[ \max_{w_s, \alpha} \log_2(1 + \gamma_{SD}) + \frac{\alpha}{2} \log_2 \left( \frac{1 + \gamma_{SR}}{1 + \gamma_{SD}} \right) \]

(33)

\[ \text{s.t.} \quad \|w_s\|^2 = 1, \]

\[ 0 \leq \alpha \leq \alpha_1. \]

---

The reason behind using \( P_r \), rather than \( P_s \), as transmit power of R in this phase is to keep the total average transmit power of the relay equal to \( P_s \). Recall that the relay was silent in the relay-receive phase. This is to make the comparison between the FD and HD schemes fair. Recall that the total average power in the FD case was \( P_s \).
With this representation of the objective function, it is clear that the optimal value of $\alpha$ is restricted to either zero or $\alpha_1$. Specifically, if $\gamma_{SR} \leq \gamma_{SD}$, then $\alpha = 0$ is optimal, i.e., the direct transmission outperforms the relaying. Further, the optimal $w_s$ is given as an MRT beamformer; i.e., $w_s = h_s^\dagger$. Otherwise ($\gamma_{SR} > \gamma_{SD}$), the relaying is better than the direct transmission, i.e., $\alpha = \alpha_1$ is optimal. In this case, finding $w_s$ is not trivial and we use the same hubristic solution developed in Section IV.

**Case II.** $\gamma_{SR} > \gamma_{SD} + 2\gamma_{RD}$: In such a case, the optimal $\theta$ is obtained from (28). We know that such a point is obtained by equating $R_{HD}^T$ and $R_{2}^T$. Then, the TD HD rate in (26) simplifies to $R_{HD}^T = R_{1}^T = R_{2}^T$. Finally, to get the expected achievable rate for the HD mobile/social network system, we replace $R_{HD}^T = R_{2}^T$ in (1) which results in

$$R_T^1 = \alpha R_{DF}^T + \alpha C_{SD}$$

$$= \frac{\alpha}{2} \log \left(1 + \gamma_{SD}\right) + \frac{\alpha}{2} \log \left(1 + \gamma_{SD} + 2\gamma_{RD} + 2\sqrt{2\theta \gamma_{SD} \gamma_{RD}}\right) + \alpha \log \left(1 + \gamma_{SD}\right).$$

(35)

Note that we have used $\tau = \frac{1}{2}$ in (27b).

Then again we need to solve the optimization problem (33), this time for $R_{HD}^T$ given by (35). Seeing that $\gamma_{SD}$ is independent of $w_r$, we conclude that $w_r = h_r^\dagger$ is optimal, similar to what we had in (9). Next, it is obvious that $w_s = h_s^\dagger$, is the optimal beamformer at the source. Finally, it is easy to see that the objective function is a linear function of $\alpha$. Thus, its maximum is achieved in one of the two ends; i.e., $\alpha = 0$ or then $\alpha = \alpha_1$.

In the above analysis, we have assumed an equal time slots allocation to the relay-receive and relay-transmit phases. Because of the latter, the pre-log factor $\frac{1}{2}$ appears in (31). A more general setting would be obtained if time is arbitrarily divided between the two phases, as can be seen in the next subsection.

2) Unequal time allocation: With the insight, we got from equal time relaying, we now consider the unequal case here. Again we need to solve the optimization problem (33), but this time for the more general $R_T^T$ obtained by using (26) for unspecified $\tau$. To this end, very similar to what we showed in the equal time allocation case, for a given $\tau$ we can treat the following to cases separately.

**Case I.** ($1 + \gamma_{SR})^{\tau} \leq \frac{1}{1 + \gamma_{SD}} + \frac{1}{1 + \gamma_{RD}}$: In this case, from Lemma 4, we know that $\theta = 0$ is optimal. We can also check that $R_{1}^T \leq R_{TD}^T$, in such a case. Thus, from (26), we see that the TD HD rate simplifies to $R_{HD}^T = R_{1}^T$. In this case, $R_T^1 = \alpha R_{DF}^T + \alpha C_{SD}$ is independent of $\gamma_{SR}$ and thus $w_r$. Further, $\alpha = 0$ and $\alpha = \alpha_1$ are optimal when $R_{HD}^T \leq C_{SD}$ and $R_{HD}^T > C_{SD}$, respectively. In the former case, the problem is rather simple and $w_s = h_s^\dagger$ is the optimal beamformer at the source. To obtain the optimal $w_r$ in the latter case, we use the algorithm developed in Section IV.

**Case II.** ($1 + \gamma_{SR})^{\tau} > \frac{1}{1 + \gamma_{SD}} + \frac{1}{1 + \gamma_{RD}}$: In this case, the optimal $\theta$ is obtained from (28) and we have $R_{HD}^T = R_{1}^T = R_{2}^T$. Then, in (1), we let $R_{DF}^T = R_2^T$ for which it is straightforward to check that $w_r = \frac{g_r}{||g_r||}$ and $w_s = \frac{h_s^\dagger}{||h_s^\dagger||}$ are the optimal beamforming vectors at the relay and source, respectively.

Very similar to case I, we see that $\alpha = 0$ and $\alpha = \alpha_1$ are optimal if $R_{DF}^T \leq C_{SD}$ and $R_{DF}^T > C_{SD}$, respectively.

Eventually, to find the optimal $\tau$ of the above cases we use a linear search over that.

**B. Varying Power at Source**

In the previous subsection, we have assumed that the source uses a fixed power in both phases of transmission. A more general setting would be obtained if the source power can vary in different phases as long as it satisfies the total average power $P_s$. Then, the achievable rate for the decode-and-forward system is modified as in the following proposition [24], [28], [29].

**Proposition 2.** A more general achievable rate for the TD HD relaying is given by

$$R_{DF}^T = \max_{\theta} \min \left\{R_{1}^T, R_{2}^T \right\},$$

(36)

in which

$$R_{1}^T = \tau \log_2 \left(1 + \frac{\gamma_{SR}}{\gamma_{SD}}\right) + \tau \log_2 \left(1 + \frac{\gamma_{SD}}{\gamma_{RD}}\right),$$

(37a)

$$R_{2}^T = \tau \log_2 \left(1 + \frac{\gamma_{SD}}{\gamma_{RD}}\right) + \tau \gamma_{SD} \gamma_{RD},$$

(37b)

where

$$\gamma_{SR} = |h_s^\dagger w_s|^2 P_s^{(1)},$$

(38a)

$$\gamma_{SD} = |h_s^\dagger w_s|^2 P_s^{(2)},$$

(38b)

$$\gamma_{SR} = ||H^\dagger w_s||^2 P_s^{(1)}.$$
1) Step 1: To simplify the problem, we first find the optimal $\theta$ (and $R_{TD}^{*}$) of (36) for a given $\tau$, $P_{s}^{(1)}$, and $P_{s}^{(2)}$. We then optimize over $\tau$ and $P_{s}^{(1)}$. The optimal value of $\theta$ can be obtained in closed-form as proved in the following lemma.

**Lemma 5.** The optimal value of $\theta$ in Proposition 7 is zero if $(1+\gamma_{SR}) \frac{1}{1+\gamma_{SR}} \geq \frac{1+\gamma_{SD}^{(2)}}{1+\gamma_{SD}}$. Otherwise, the optimal value is obtained by
\[
\theta^* = \frac{1}{c \gamma_{SD}^{(2)}} \left[ -\sqrt{\frac{1}{\tau} \gamma_{SD}^{(2)}} + \sqrt{\frac{1}{\tau} \gamma_{RD} - c \gamma_{SD}^{(2)} - \gamma_{SD}^{(2)}} \right],
\]
in which $c = (1+\gamma_{SR}) \frac{1}{1+\gamma_{SD}}$.

**Proof.** The proof is very similar to that of Lemma 4 and is omitted.

2) Step 2: To find the optimal beamforming vectors for this scenario, very similar to what we saw in Section V-A2 for the latter case, we use the algorithm developed in Section IV to find the optimal $w_{r}$. In this case, the optimal $\theta$ is obtained from (41) and we have $R_{TD}^{1} = R_{TD}^{2}$. Then, in (40), we let $R_{TD}^{*} = R_{TD}^{2}$ for which $w_{r} = \frac{\mathbf{g}_{n}^{H}}{\| \mathbf{g}_{n} \|$ and $w_{s} = \frac{\mathbf{h}_{m}^{H}}{\| \mathbf{h}_{m} \|$ are the optimal beamforming vectors at the relay and source, respectively. Also, similar to case I, we see that $\alpha = 0$ and $\alpha = \alpha_{1}$ are optimal when $R_{TD}^{1} \leq C_{SD}$ and $R_{TD}^{2} > C_{SD}$, respectively.

3) Step 3: The last step is to find the optimal value of $\tau$ as well as the corresponding power allocation, i.e., $P_{s}^{(1)}$. To this end, we use a linear search over $[0,1]$ for $\tau$ and over $[0,\frac{P_{s}^{(1)}}{P_{s}^{(1)}}]$ for $P_{s}^{(1)}$.

Numerical results in Section VI confirm that the rate region can be improved to a good extent by optimizing over the time slot length and source power. Such an improvement, however, comes at the expense of complexity in the transmission scheme.

VI. NUMERICAL RESULTS

We evaluate the performance of the proposed approach for generating beamforming vectors and compare it with the existing schemes in this section. For the purpose of simulation, we will assume that the channels $H$ and $g$ are complex Gaussian vectors whose entries are independent and identically distributed (i.i.d.) Gaussian random variables with zero means and variances $\sigma_{h}^{2}$, $\sigma_{H}^{2}$, and $\sigma_{g}^{2}$, respectively. Unlike [10], we optimize the exact achievable rate given by (10), not just a high SNR approximation of that. All graphs are based on averaging over 10,000 channel realizations.

We first focus on the MISO case and compare our results with those of [10] for $N_{s} = 4$ and $(\sigma_{h}^{2}, \sigma_{H}^{2}, \sigma_{g}^{2}) = (-5,-4,10)$ dB. In Fig. 2, we show the effect of optimizing over $\alpha$ rather than fixing it at $\alpha = \alpha_{1}$, which is the case in [10]. Recall from Lemma 2 that $\alpha^* \in \{0,\alpha, \alpha_{1}\}$. Hence, we choose between $\alpha = 0$ or $\alpha = \alpha_{1}$, whichever choice gives a better rate. In Fig. 4 we take $\alpha_{1} = 0.7$ and direct transmission refers to the case with $\alpha = 0$. This figure clearly shows that using the maximum trust degree ($\alpha_{1}$) is not always the best approach to optimize data rates, and, depending on the channel conditions, the transmission rate may increase by using a fine-tuned $\alpha$ smaller than $\alpha_{1}$. As can be seen in Fig. 4, there is a visible gap between our scheme and the one proposed in [10] for different values of $N_{s}$ at all SNRs, although this gap reduces as $N_{s}$ increases or the relay link becomes stronger.

In Fig. 4 we consider the MIMO case and compare the achievable rates for different values of $N_{t}$. Expectedly, as the number of antennas increases, the achievable rate goes up. In Fig. 5 we compare the achievable rate for the HD and FD cases with a different number of antennas at the relay. Note that in this figure $\alpha_{h}^{2}$ has increased from $-5$ dB to $-2$ dB compared to that of Fig. 4. Due to this increase in the strength of the direct link, all curves including the direct transmission curve has slightly shifted up compared to their corresponding curves in Fig. 4. It is also seen that the achievable rates for the HD relaying with $N_{t} = 4$ and $N_{r} = 2$ are almost as high as the achievable rates of the FD relaying with $N_{t} = 2$ and $N_{r} = 1$, respectively. Therefore, the HD relaying loss, compared to the FD relaying, can be compensated by increasing the number of antennas at the relay. In Fig. 6 we study the effect of the HD relaying with unequal time allocation to the relay-transmit and relay-receive phases. This figure shows that optimizing $\tau$ can noticeably increase the trust degree based rate when expected compared to the case with $\tau = \frac{1}{2}$. This relative gain is higher.
Interestingly, when the relay has a single antenna.

In Fig. 7 we investigate the effect of the trust degree on the achievable rate of HD schemes with fixed and varying power. As can be seen from both Fig. 7(a) and Fig. 7(b) as the level of trust increases, the achievable rates of both FD and HD social/mobile systems significantly increase. From these two figures, we can also see the difference between the achievable rates for the HD social/mobile systems where the relay power is fixed or can take different values in different transmission phases. The achievable rate (for the HD case) is visibly higher for Fig. 7(b) where the relay power can take different values in different transmission phases compared to Fig. 7(a) where the relay uses a fixed power in both transmission phases.

Interestingly, when the number of antennas at the relay is high, this increase is almost linear in $\alpha_1$. This is because in such cases $\gamma_{SR} > \gamma_{SD}$ with high probability which implies that $\alpha = \alpha_1$ is optimal in (1), and from which it is clear that $R_T$ is linear in $\alpha_1$. Note that the graphs in this figure are for $(\sigma_h^2, \sigma_H^2, \sigma_g^2) = (-5, -4, 10)$ dB. If $\sigma_H^2 \gg \sigma_h^2$, which is mostly the case in practice because the relay is usually much closer than the destination to the source, then even with a single antenna at relay $R_T$ will be linear in $\alpha_1$. On the contrary, if $\sigma_H^2 \leq \sigma_h^2$ and $N_r$ is small, then the cooperative link may not be superior to the direct link. Such a case implies $\alpha = 0$ is optimal and then $R_T$ will not be linear in $\alpha_1$.

VII. SCOPE OF THE PAPER AND EXTENSIONS

In this section, we revisit the scope of the paper and describe some possible extensions to the proposed social-aware cooperative communications approach.

A. Social-Aware FD Communications with Self-Interference

When considering social-aware FD cooperative communications in Section III we used the FD achievable rate in which practical issues such as self-interference (SI) is not considered. However, high-powered SI is the main practical challenge in FD radio which swamps the receiver. Over the past few years, several research groups have proposed various designs to
build FD radios, e.g., see [30]–[33]. An important engineering insight arising from these studies is that FD communication is feasible and practical as the SI can be reduced below the noise floor [33] through a series of analog and digital cancellation techniques. In fact, it has been demonstrated that a point-to-point FD communication link can achieve close to the theoretical doubling of throughput. Considering this, and the fact that we emphasize on the social-awareness of the communication, in this work, we have investigated the trust degree based beamforming design in multi-antenna cooperative systems in the absence of SI. This helps to focus on understanding the benefits of social-aware user cooperation. Nonetheless, to illustrate how the residual SI affects achievable rates in practice, we have also assessed the performance of the proposed FD method with SI, and compared it with the SI cancellation capability in Section VI. As can be seen in Fig. 8(a), SI can largely affect the achievable rate in the single-antenna relay. However, when the number of antennas increases the rate loss becomes much less, specifically if the SI is small (e.g., SI = 3 dB in Fig. 8(a)).

### B. Cooperative Strategies Beyond the Decode-And-Forward Relaying

Relay-assisted multi-hop wireless networks have been extensively studied in the literature of cooperative communications. In particular, in addition to the decode-and-forward relaying scheme considered in this paper, compress-and-forward and amplify-and-forward are the two other major relaying schemes extensively analyzed in the literature. We know that the decode-and-forward approach does not work when $\gamma_{SR} < \gamma_{SD}$, since it is unable to reach the data rates achievable through direct transmission. To see this more clearly in Proposition 1, we can write

$$R_{DD}^{TD} = \max_{\theta} \min \{ R_1^{TD}, R_2^{TD} \},$$

$$\leq \max_{\theta} R_1^{TD} = \tau \log_2(1+\gamma_{SR}) + \tau \log_2(1+\gamma_{SD})$$

$$< \log_2(1+\gamma_{SD}), \quad (42)$$

Fig. 7. Achievable rates versus $\alpha_1$ for the FD and HD MIMO cases with $(\sigma_H^2, \sigma_S^2, \sigma_{g_k}^2) = (0, 0, 5)$ dB, $N_s = 4$, and different $N_r$'s.

Fig. 8. Achievable rates versus $\alpha_1$ for the FD MIMO with and without self-interference with $(\sigma_H^2, \sigma_S^2, \sigma_{g_k}^2) = (-2, 2, 5)$ dB, $N_s = 4$, and different $N_r$'s.
where the second equality is straightforward from (27a) and the last step is due to the assumption $\gamma_{SR} < \gamma_{SD}$. In contrast, the compress-and-forward approach can be used for all channel conditions, which always gives a rate higher than the one achievable through direct transmission. Therefore, the overall transmission rate of the proposed cooperative and social-aware communications system, in both cases of the FD and HD modes, can be improved if the compress-and-forward scheme is used when $\gamma_{SR} < \gamma_{SD}$. However, when $\gamma_{SR}$ becomes larger than $\gamma_{SD}$, the decode-and-forward rate would be eventually larger than the compress-and-forward rate. Hence, another interesting future research direction of the authors is the social-aware design of cooperative communications systems with other cooperation strategies such as amplify-and-forward and compress-and-forward, as well as dynamic switching between these cooperation modes based on trust degrees and channel conditions.

VIII. CONCLUSION

We have developed a trust degree based beamforming design formulation for multi-antenna cooperative communication with decode-and-forward relaying both in the FD and HD cases. Observing that maximum trust degree may not result in the best rate performance, we have optimized the achievable rate over the trust degrees as well as the beamforming vectors at the source and relay nodes. We have proved that the optimal beamforming vector is a linear combination of the direct and cooperating links beamforming vectors, respecting the trust degree. Unlike existing works, the proposed scheme has been developed for the MIMO setting and is applicable to any SNR regime. For the MISO case, our scheme noticeably improves the achievable rate over those of the existing schemes. An interesting direction to extend this work is to study the effect of trustworthiness in enhancing transmission security. It would be also useful to explore the effect of untrusted relays in social-aware cooperative networks.

REFERENCES


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