# The Capacity of Less Noisy Cognitive Interference Channels

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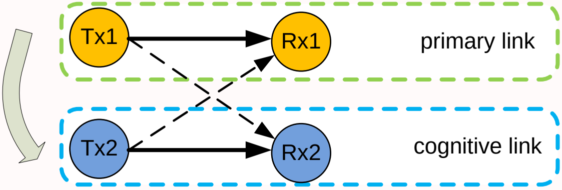
50th Annual Allerton Conference
Monticello, Illinois

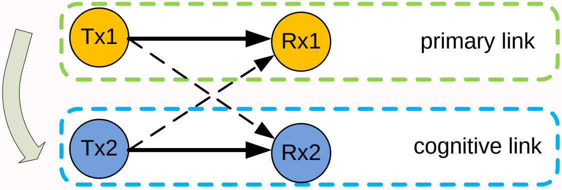
October 4, 2012

# Introduction




Cognitive Interference Channel

# Introduction




Cognitive Interference Channel

Motivation for studying the cognitive channel

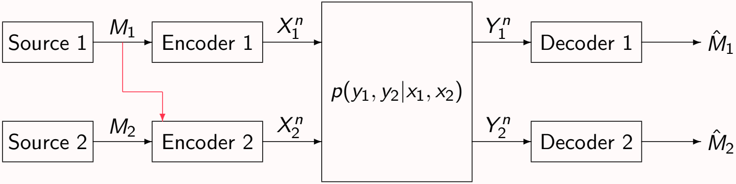
- Models an ideal cognitive radio
- Compatible for efficient spectrum utilization
- Helps understand and analyze cognitive relay networks
- Fundamental limits by themselves

# This Talk

1. Introduction
2. Two inner bounds
   - Superposition coding
3. Outer bound
   - More capable idea
4. Capacity for the less noisy CIC
   - *cognitive-less-noisy* CIC
5. Extension to more capable CIC
   - *cognitive-more-capable* CIC

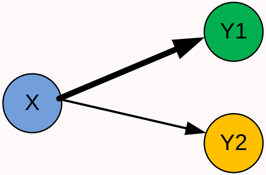
# Channel model

The DMS cognitive interference channel (DMS-CIC)




- Encoder 2 non-causally knows $X_1^n$ and $M_1$

# Broadcast Channels (BC)




if $I(U;Y_1) \geq I(U;Y_2) \quad \forall\, p(u,x) \Rightarrow Y_1$ is less noisy
if $I(X;Y_1) \geq I(X;Y_2) \quad \forall\, p(x) \Rightarrow Y_1$ is more capable

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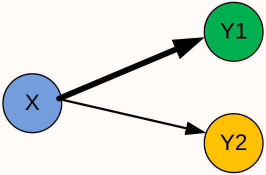


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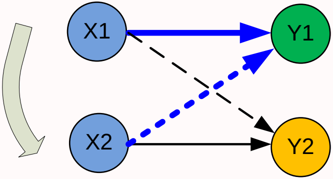
if $I(X;Y_1) \geq I(X;Y_2) \quad \forall\, p(x) \Rightarrow Y_1$ is more capable

- Capacity is know for both classes
  - less noisy [Korner-Marton 77]
  - more capable [El Gamal 79]
- Superposition coding is optimal

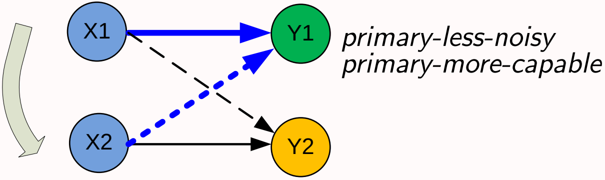
# Cognitive Interference Channels (CIC)



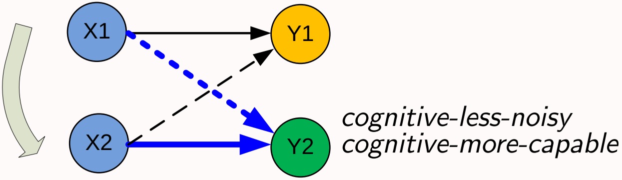

*primary-less-noisy*
*primary-more-capable*

$Y_1$ is in a better condition than $Y_2$ [Vaezi-Vu 2011]

# Cognitive Interference Channels (CIC)



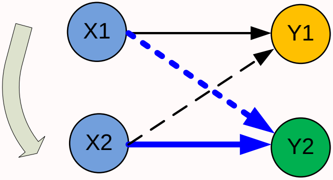

$Y_1$ is in a better condition than $Y_2$ [Vaezi-Vu 2011]




$Y_2$ is in a better condition than $Y_1$

# Cognitive-less-noisy CIC

This talk




- Cognitive receiver is less noisy than primary if $I(U; {\color{red}Y_2}) \geq I(U; Y_1)$ for all $p(u, x_1, x_2)$
- Cognitive receiver is more capable than primary if $I(X_1, X_2; {\color{red}Y_2}) \geq I(X_1, X_2; Y_1)$ for all $p(x_1, x_2)$

# Existing capacity results for DM-CIC

- Strong interference [Maric-Yates-Kramer, IT 2007]
- Weak interference [Wu-Vishwanath-Arapostathis, IT 2007]
- Cognitive-better-decoding [Rini-Tuninetti-Devroye, IT 2011]

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**Remark**

Rini et al.'s region is equal to Wu et al.'s region
-To appear in IT

# Superposition coding-based inner bound

## Theorem 1

For the DM-CIC, any rate pair $(R_1, R_2)$ that satisfies

$$\begin{aligned} R_1 &\leq I(W, X_1; Y_1), \\ R_2 &\leq I(X_2; Y_2 | W, X_1), \\ R_1 + R_2 &\leq I(X_1, X_2; Y_2), \end{aligned} \tag{1}$$

is achievable for all probability distributions $p(w, x_1, x_2)$.

## Achievability

- Superposition coding at the cognitive transmitter
- Joint typicality decoding
- $Y_1$ can only decode $M_1$ (the cloud center) while $Y_2$ can decode the satellite codewords as well

# More-capable BC capacity inspired outer bound

**Theorem 2**

The union of all rate pairs $(R_1, R_2)$ such that

$$\begin{aligned} R_1 &\leq I(U; Y_1), \\ R_2 &\leq I(V; Y_2), \\ R_1 + R_2 &\leq I(X_2; Y_2|U) + I(U; Y_1), \\ R_1 + R_2 &\leq I(X_1; Y_1|V) + I(V; Y_2), \end{aligned}$$

for some joint distribution $p(u, v, x_1, x_2)p(y_1, y_2|x_1, x_2)$ provides an outer bound on the capacity region of the DM-CIC.

**Proof.**

- Similar to the converse of the more capable BC
- Observe the symmetry of outer bound

□

# Simplified outer bound

Consider the first and third inequalities from the outer bound

$$
\begin{aligned}
R_1 &\leq I(U;Y_1),\\
R_1 + R_2 &\leq I(X_2;Y_2|U) + I(U;Y_1),
\end{aligned}
$$

This region is equal to

# Simplified outer bound

Consider the first and third inequalities from the outer bound

$$R_1 \leq I(U;Y_1),$$
$$R_1 + R_2 \leq I(X_2;Y_2|U) + I(U;Y_1),$$

This region is equal to

$$R_1 \leq I(U;Y_1),$$
$$R_2 \leq I(X_2;Y_2|U),$$

# Simplified inner bound

Consider the inner bound in Theorem 1 and let $U = W, X_1$
Then it is easy to see that

- Sum rate (the third inequality) is redundant
- The achievable region reduces to

$$R_1 \leq I(U; Y_1),$$
$$R_2 \leq I(X_2; Y_2|U),$$

# Capacity region for the cognitive-less-noisy CIC

## Theorem

*The union of all rate pairs* $(R_1, R_2)$ *such that*

$$R_1 \leq I(U; Y_1),$$
$$R_2 \leq I(X_2; Y_2|U)$$

*for some input distribution* $p(u, x_1, x_2)$ *gives the capacity region for the cognitive-less-noisy DM-CIC.*

## Remark

- In this capacity region $U$ can be replaced by $W, X_1$

# Extension to the cognitive-more-capable CIC

- This work is extended to the cognitive-more-capable CIC, and

- Theorem 1 gives the capacity of the cognitive-more-capable CIC
  -available on arXiv
- It includes all existing capacity result for the CIC
- It is the largest capacity result for the CIC

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## Concluding remark

- Superposition coding, even without requiring rate splitting, provides the largest capacity result for the DM-CIC, to date.

# Thank you!