The Capacity of More Capable Cognitive Interference Channels

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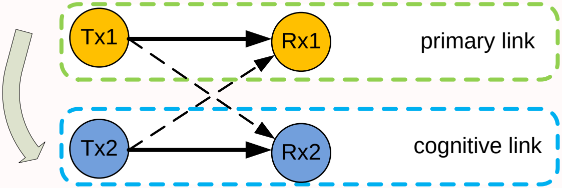
Introduction



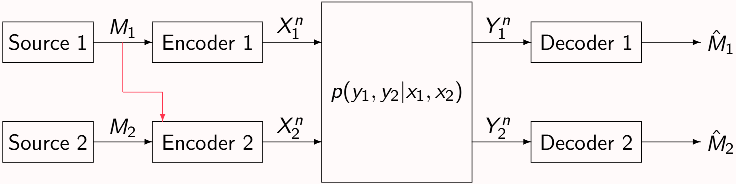
Cognitive Interference Channel

Motivation for studying the cognitive channel

- Models an ideal cognitive radio
- Compatible for efficient spectrum utilization
- Helps understand and analyze cognitive relay networks
- Fundamental limits by themselves

Channel model

The discrete memoryless cognitive interference channel (DM-CIC)



- Encoder 2 non-causally knows M_1

Existing capacity results for DM-CIC

- Strong interference [Maric-Yates-Kramer, IT 2007]
- Weak interference [Wu-Vishwanath-Arapostathis, IT 2007]
- Cognitive-better-decoding [Rini-Tuninetti-Devroye, IT 2011]

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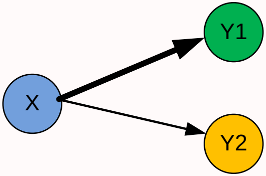
Remark

Rini et al.'s region is equal to Wu et al.'s region
-[M. Vaezi , IT 2013]

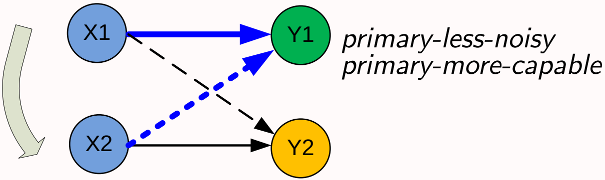
Broadcast Channels (BC)



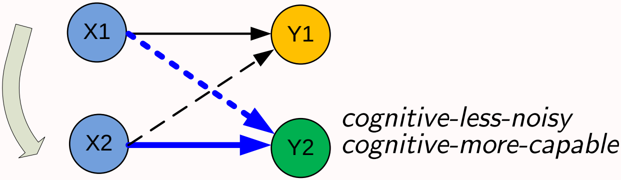
$$\text{if } I(U;Y_1) \geq I(U;Y_2) \quad \forall\, p(u,x) \Rightarrow Y_1 \text{ is less noisy}$$
$$\text{if } I(X;Y_1) \geq I(X;Y_2) \quad \forall\, p(x) \Rightarrow Y_1 \text{ is more capable}$$

- Capacity is known for both classes
  - less noisy [Korner-Marton 77]
  - more capable [El Gamal 79]
- Superposition coding is optimal

Cognitive Interference Channels (CIC)



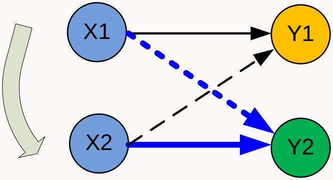
Y_1 is in a better condition than Y_2 [Vaezi-Vu 2011]



Y_2 is in a better condition than Y_1

Cognitive-more-capable CIC

This talk



- Cognitive receiver is less noisy than primary if
  $I(U; Y_2) \geq I(U; Y_1)$ for all $p(u, x_1, x_2)$
- Cognitive receiver is more capable than primary if
  $I(X_1, X_2; Y_2) \geq I(X_1, X_2; Y_1)$ for all $p(x_1, x_2)$

Superposition coding-based inner bound

Theorem 1

For the DM-CIC, any rate pair (R_1, R_2) that satisfies

$$R_1 \leq I(W, X_1; Y_1),$$
$$R_2 \leq I(X_2; Y_2 | W, X_1),$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_2),$$

is achievable for all probability distributions $p(w, x_1, x_2)$.

Achievability

- Superposition coding at the cognitive transmitter
- Joint typicality decoding
- Y_1 can only decode M_1 (the cloud center) while Y_2 can decode the satellite codewords as well

More-capable BC capacity inspired outer bound

Theorem 2

The union of all rate pairs (R_1, R_2) such that

$$\begin{aligned} R_1 &\leq I(U, X_1; Y_1), \\ R_1 + R_2 &\leq I(U, X_1; Y_1) + I(X_2; Y_2|U, X_1), \\ R_1 + R_2 &\leq I(X_1, X_2; Y_2), \end{aligned}$$

for some joint distribution $p(u, v, x_1, x_2)p(y_1, y_2|x_1, x_2)$ provides an outer bound on the capacity region of the DM-CIC.

Proof.

- The first two inequalities are from Wu et al.'s outer bound
- Similar to the converse of the more capable BC

□

Simplified outer bound

Consider the first and second inequalities from the outer bound

$$R_1 \leq I(U, X_1; Y_1),$$
$$R_1 + R_2 \leq I(U, X_1; Y_1) + I(X_2; Y_2|U, X_1).$$

But, the **convex hull** of the above region is equal to the convex hull of the below region

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$$R_1 \leq I(U, X_1; Y_1),$$
$$R_2 \leq I(X_2; Y_2|U, X_1).$$

Alternative representation of the outer bound

Theorem 3

The union of all rate pairs (R_1, R_2) such that

$$R_1 \leq I(U, X_1; Y_1),$$
$$R_2 \leq I(X_2; Y_2|U, X_1),$$
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for some joint distribution $p(u, v, x_1, x_2)p(y_1, y_2|x_1, x_2)$ provides an outer bound on the capacity region of the DM-CIC.

Proof.

- Use the simplified version of the outer bound in Theorem 2

□

Capacity region for the cognitive-more-capable CIC

Theorem 4

The capacity region of the cognitive-less-noisy DM-CIC is given by the set of all rate pairs (R_1, R_2) such that

$$\begin{aligned} R_1 &\leq I(W, X_1; Y_1), \\ R_2 &\leq I(X_2; Y_2 | W, X_1), \\ R_1 + R_2 &\leq I(X_1, X_2; Y_2), \end{aligned}$$

for some $p(w, x_1, x_2)$.

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for some $p(w, x_1, x_2)$.

Remark

- Superposition coding, even without requiring rate splitting, provides the largest capacity result for the DM-CIC, to date.

Comparison and Classification

- The capacity region of the cognitive-more-capable CIC explicitly includes all existing capacity result for the CIC

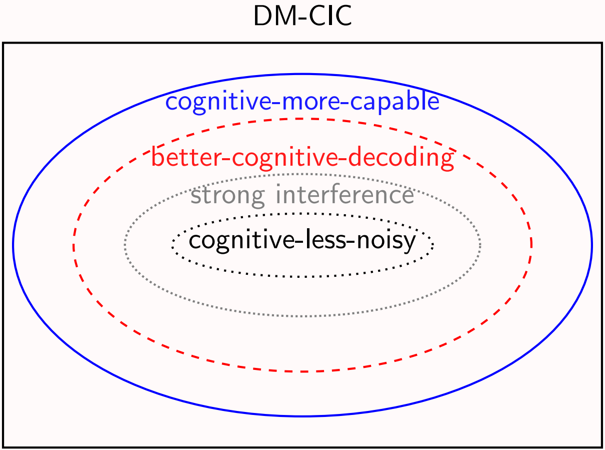

Figure: The class of the discrete memoryless cognitive interference

Comparison and Classification

Table: Summary of existing and new capacity results for the DM-CIC. The subscripts 1 and 2, respectively, denote the primary and secondary (cognitive) users.*

Label	DM-CIC class	Condition	Capacity region
$\mathcal{C}_I$	cognitive-less-noisy	$I(U;Y_1) \leq I(U;Y_2)$	$R_1 \leq I(U;Y_1)$ $R_2 \leq I(X_2;Y_2\|U)$
$\mathcal{C}_{II}$	strong interference	$I(X_1,X_2;Y_1) \leq I(X_1,X_2;Y_2)$ $I(X_2;Y_2\|X_1) \leq I(X_2;Y_1\|X_1)$	$R_1 \leq I(X_1;Y_1)$ $R_2 \leq I(X_2;Y_2\|X_1)$
$\mathcal{C}_{III}$	weak interference	$I(X_1;Y_1) \leq I(X_1;Y_2)$ $I(U;Y_1\|X_1) \leq I(U;Y_2\|X_1)$	$R_1 \leq I(U,X_1;Y_1)$ $R_2 \leq I(X_2;Y_2\|U,X_1)$
$\mathcal{C}'_{III}$	better-cognitive-decoding	$I(U,X_1;Y_1) \leq I(U,X_1;Y_2)$	$R_1 \leq I(U,X_1;Y_1)$ $R_2 \leq I(X_2;Y_2\|X_1)$ $R_1 + R_2 \leq I(U,X_1;Y_1) + I(X_2;Y_2\|U,X_1)$
$\mathcal{C}_{IV}$	cognitive-more-capable	$I(X_1,X_2;Y_1) \leq I(X_1,X_2;Y_2)$	$R_1 \leq I(U,X_1;Y_1)$ $R_2 \leq I(X_2;Y_2\|U,X_1)$ $R_1 + R_2 \leq I(X_1,X_2;Y_2)$

* It should be emphasized that $\mathcal{C}'_{III} \equiv \mathcal{C}_{III}$ [Vaezi 2013], $\mathcal{C}_I \subseteq \mathcal{C}_{II} \subseteq \mathcal{C}_{III} \subseteq \mathcal{C}_{IV}$.

Thank you!