On the Capacity of the Cognitive Z-Interference Channel

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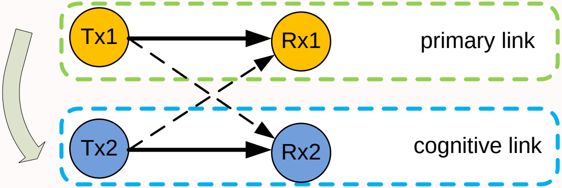
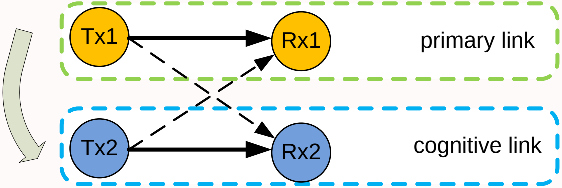
Introduction

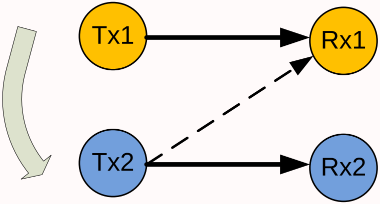


Cognitive Interference Channel

Introduction



Cognitive Interference Channel



Cognitive Z-Interference Channel

Motivation

Motivation for studying the cognitive channel

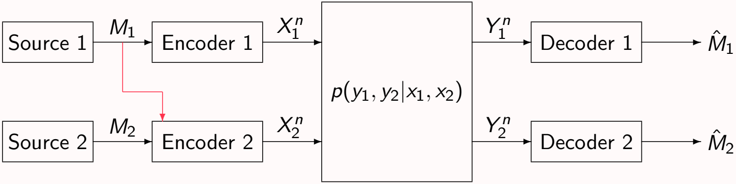
- Models an ideal cognitive radio
- Compatible for efficient spectrum utilization
- Helps understand and analyze cognitive relay networks
- Fundamental limits by themselves

This Talk

1. Introduction
2. New inner bound
   - Superposition coding
3. New outer bound
   - More capable idea
4. Capacity for the Gaussian CZIC
   - Very strong interference

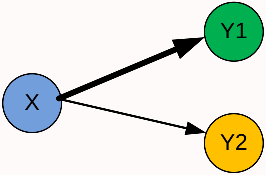
Channel model

The DMS cognitive interference channel (DMS-CIC)



- Encoder 2 non-causally knows X_1^n and M_1

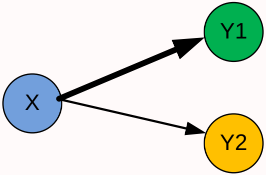
More capable channels



More capable BC

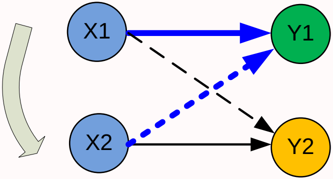
$$I(X; Y_1) \geq I(X; Y_2) \quad \text{for all } p(x)$$

More capable channels



More capable BC

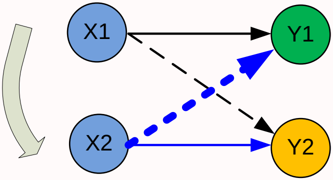
$$I(X; {\color{red}Y_1}) \geq I(X; Y_2) \quad \text{for all } p(x)$$



More capable CIC

$$I(X_1, X_2; {\color{red}Y_1}) \geq I(X_1, X_2; Y_2) \quad \text{for all } p(x_1, x_2)$$

Strong cognitive interference



Cognitive user creates strong interference

$$I(X_2; Y_1|X_1) \geq I(X_2; Y_2|X_1) \quad \text{for all } p(x_1, x_2)$$

Superposition coding-based inner bound

Theorem

For the DM-CIC, any rate pair (R_1, R_2) *that satisfies*

$$\begin{aligned} R_1 &\leq I(X_1; Y_1|U) \\ R_2 &\leq I(U; Y_2) \\ R_1 + R_2 &\leq I(X_1, X_2; Y_1) \end{aligned} \tag{1}$$

for some joint distribution that factors as $p(u)p(x_1)$ $p(x_2|x_1, u)p(y_1, y_2|x_1, x_2)$ *is achievable.*

Achievability

- Superposition coding at the cognitive transmitter
- Joint typicality decoding
- Y_2 can only decode M_2 (the cloud center) while Y_1 can decode the satellite codewords as well

More-capable BC capacity inspired outer bound

Theorem

The union of all rate pairs (R_1, R_2) *such that*

$$\begin{aligned} R_2 &\leq I(U; Y_2) \\ R_1 + R_2 &\leq I(X_1; Y_1|U) + I(U; Y_2) \\ R_1 + R_2 &\leq I(X_1, X_2; Y_1) \end{aligned} \tag{2}$$

for some joint distribution $p(u, x_1, x_2)p(y_1, y_2|x_1, x_2)$ *provides an outer bound on the capacity region of the more capable DM-CIC.*

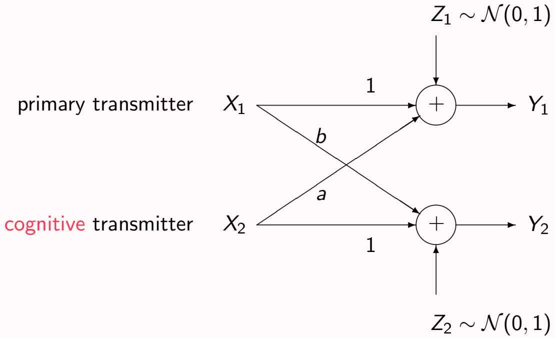
Proof.

- Similar to the converse of the more capable BC
- This theorem also applies for the DM-CIC with strong cognitive interference

□

Channel model

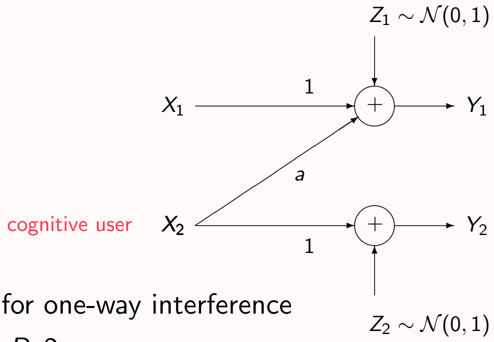
Additive white Gaussian noise (AWGN) cognitive channel



$$Y_1 = X_1(m_1) + aX_2(m_1, m_2) + Z_1$$
$$Y_2 = bX_1(m_1) + X_2(m_1, m_2) + Z_2$$

Channel model

The Gaussian cognitive Z-channel (GCZIC)



Practical situations for one-way interference

- *Tx*2 is close to *Rx*2
- Unbalanced power
- Blockage (close to primary user)

Inner bound for the GCZIC

Lemma

Any rate pair (R_1, R_2) *satisfying*

$$R_1 \leq \mathcal{C}\left((\sqrt{P_1} + a\sqrt{\alpha P_2})^2\right)$$

$$R_2 \leq \mathcal{C}\left(\frac{\bar{\alpha} P_2}{1 + \alpha P_2}\right)$$

$$R_1 + R_2 \leq \mathcal{C}\Big(P_1 + a^2 P_2 + 2a\sqrt{\alpha P_1 P_2}\Big) \tag{3}$$

with $\alpha \in [0, 1]$ *is achievable for the GCZIC.*

Achievability

- $X_1 = \sqrt{P_1} V(m_1)$, $X_2 = \sqrt{\alpha P_2} V(m_1) + \sqrt{\bar{\alpha} P_2} U(m_2)$
- The cognitive receiver: treat other codeword as interference
- The primary receiver: uses successive cancelation

Thank you!

An outer bound for the GCZIC with $a^2 \geq 1$

Denoting $\alpha \triangleq 1-\rho_2^2$, $\beta \triangleq 1-\rho_1^2$, and $\bar{x} \triangleq 1-x$ for any $x \in [0,1]$, we obtain

Corollary

An outer bound on the capacity region of the GCZIC with $|a| \geq 1$ is the set of all rate pairs (R_1, R_2) satisfying

$$
\begin{aligned}
R_2 &\leq \mathcal{C}\left(\frac{\bar{\alpha}P_2}{1+\alpha P_2}\right) \qquad (4)\\
R_1+R_2 &\leq \mathcal{C}\left((\sqrt{\beta P_1}+|a|\sqrt{\alpha P_2})^2\right)+\mathcal{C}\left(\frac{\bar{\alpha}P_2}{1+\alpha P_2}\right)\\
R_1+R_2 &\leq \mathcal{C}\left(P_1+a^2P_2+2|a|(\sqrt{\alpha\beta}+\sqrt{\bar{\alpha}\bar{\beta}})\sqrt{P_1P_2}\right)
\end{aligned}
$$

for $\alpha, \beta \in [0,1]$.

Capacity of the GCZIC at very strong interference

Converse: Corollary 1

Proof.

- If the second bound is not redundant, on the boundary of this outer bound, we must have $R_2 \leq \frac{1}{2}\log\left(1 + (\sqrt{\beta P_1} + |a|\sqrt{\alpha P_2})^2\right)$
- Comparing this inequality with the first inequality of Lemma 2, $\Longrightarrow \beta = 1$ (since otherwise the outer bound becomes less that the inner bound!)
- For $|a| \geq \sqrt{1 + P_1}$ the second inequality cannot be redundant
- Thus, for $|a| \geq \sqrt{1 + P_1}$, $\beta = 1$ is optimum and capacity region is established

□