On the Capacity of the Cognitive Z-Interference Channel

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May 18, 2011
Introduction

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Cognitive Z-Interference Channel

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Motivation

Motivation for studying the cognitive channel

- Models an ideal **cognitive radio**
- Compatible for efficient spectrum utilization
- Helps understand and analyze cognitive relay networks
- Fundamental limits by themselves
1 Introduction

2 New inner bound
   - Superposition coding

3 New outer bound
   - More capable idea

4 Capacity for the Gaussian CZIC
   - Very strong interference
The DMS cognitive interference channel (DMS-CIC)

- Encoder 2 non-causally knows $X_1^n$ and $M_1$
More capable channels

\[ I(X; Y_1) \geq I(X; Y_2) \quad \text{for all } p(x) \]
More capable channels

\[ I(X; Y_1) \geq I(X; Y_2) \quad \text{for all } p(x) \]

\[ I(X_1, X_2; Y_1) \geq I(X_1, X_2; Y_2) \quad \text{for all } p(x_1, x_2) \]
Strong cognitive interference

\[ I(X_2; Y_1 | X_1) \geq I(X_2; Y_2 | X_1) \quad \text{for all } p(x_1, x_2) \]
Superposition coding-based inner bound

Theorem

For the DM-CIC, any rate pair \((R_1, R_2)\) that satisfies

\[
R_1 \leq I(X_1; Y_1|U) \\
R_2 \leq I(U; Y_2) \\
R_1 + R_2 \leq I(X_1, X_2; Y_1)
\] (1)

for some joint distribution that factors as

\[
p(u)p(x_1) \\
p(x_2|x_1, u)p(y_1, y_2|x_1, x_2)
\]
is achievable.

Achievability

- **Superposition coding** at the cognitive transmitter
- Joint typicality decoding
- \(Y_2\) can only decode \(M_2\) (the cloud center) while \(Y_1\) can decode the satellite codewords as well
Theorem

The union of all rate pairs \((R_1, R_2)\) such that

\[
R_2 \leq I(U; Y_2) \\
R_1 + R_2 \leq I(X_1; Y_1|U) + I(U; Y_2) \\
R_1 + R_2 \leq I(X_1, X_2; Y_1)
\]

for some joint distribution \(p(u, x_1, x_2)p(y_1, y_2|x_1, x_2)\) provides an outer bound on the capacity region of the more capable DM-CIC.

Proof.

- Similar to the converse of the more capable BC
- This theorem also applies for the DM-CIC with strong cognitive interference
Additive white Gaussian noise (AWGN) cognitive channel

\[ Y_1 = X_1(m_1) + aX_2(m_1, m_2) + Z_1 \]
\[ Y_2 = bX_1(m_1) + X_2(m_1, m_2) + Z_2 \]
The Gaussian cognitive Z-channel (GCZIC)

Practical situations for one-way interference
- $Tx_2$ is close to $Rx_2$
- Unbalanced power
- Blockage (close to primary user)
Inner bound for the GCZIC

Lemma

Any rate pair \((R_1, R_2)\) satisfying

\[
R_1 \leq C \left( \left( \sqrt{P_1} + a\sqrt{\alpha P_2} \right)^2 \right)
\]

\[
R_2 \leq C \left( \frac{\bar{\alpha}P_2}{1 + \alpha P_2} \right)
\]

\[
R_1 + R_2 \leq C \left( P_1 + a^2 P_2 + 2a\sqrt{\alpha P_1 P_2} \right)
\]

with \(\alpha \in [0, 1]\) is achievable for the GCZIC.

Achievability

- \(X_1 = \sqrt{P_1} V(m_1), X_2 = \sqrt{\alpha P_2} V(m_1) + \sqrt{\bar{\alpha} P_2} U(m_2)\)
- The cognitive receiver: treat other codeword as interference
- The primary receiver: uses successive cancelation
Thank you!
An outer bound for the GCZIC with \( a^2 \geq 1 \)

Denoting \( \alpha \triangleq 1 - \rho_2^2 \), \( \beta \triangleq 1 - \rho_1^2 \), and \( \bar{x} \triangleq 1 - x \) for any \( x \in [0, 1] \), we obtain

**Corollary**

An *outer bound* on the capacity region of the GCZIC with \( |a| \geq 1 \) is the set of all rate pairs \((R_1, R_2)\) satisfying

\[
R_2 \leq C\left( \frac{\bar{\alpha} P_2}{1 + \alpha P_2} \right)
\]

\[
R_1 + R_2 \leq C\left( \left( \sqrt{\beta P_1} + |a| \sqrt{\alpha P_2} \right)^2 \right) + C\left( \frac{\bar{\alpha} P_2}{1 + \alpha P_2} \right)
\]

\[
R_1 + R_2 \leq C\left( P_1 + a^2 P_2 + 2|a|(\sqrt{\alpha \beta} + \sqrt{\bar{\alpha} \bar{\beta}}) \sqrt{P_1 P_2} \right)
\]

for \( \alpha, \beta \in [0, 1] \).
Converse: Corollary 1

Proof.

- If the second bound is not redundant, on the boundary of this outer bound, we must have
  \[ R_2 \leq \frac{1}{2} \log \left( 1 + \left( \sqrt{\beta P_1} + |a| \sqrt{\alpha P_2} \right)^2 \right) \]

- Comparing this inequality with the first inequality of Lemma 2, \( \iff \beta = 1 \) (since otherwise the outer bound becomes less than the inner bound!)

- For \( |a| \geq \sqrt{1 + P_1} \) the second inequality cannot be redundant

- Thus, for \( |a| \geq \sqrt{1 + P_1} \), \( \beta = 1 \) is optimum and capacity region is established