Deep Learning based Precoding for the MIMO Gaussian Wiretap Channel

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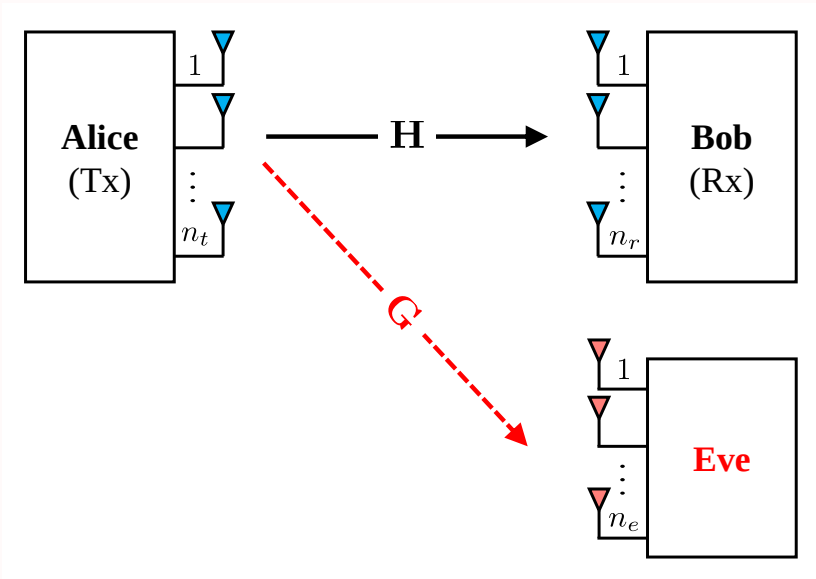
Channel Model

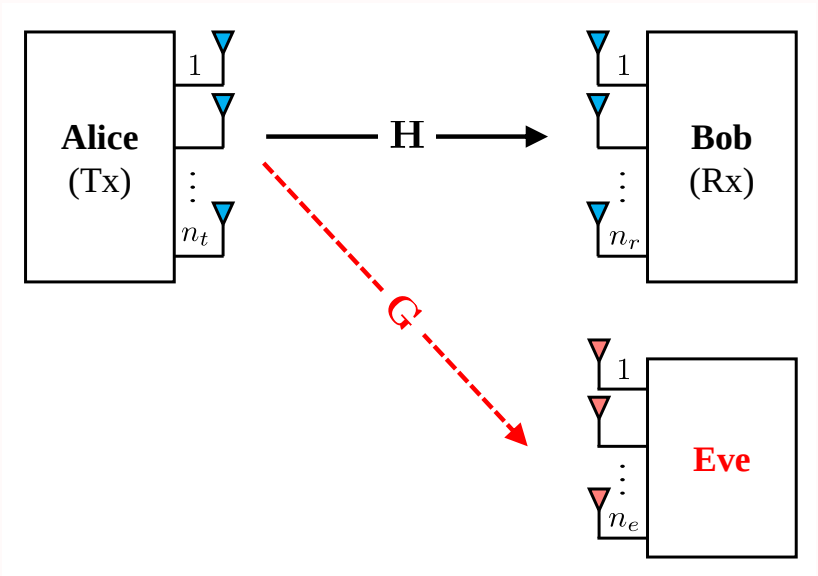


Multiple-input multiple-output (MIMO) Gaussian wiretap channel

$$\mathbf{y}_r = \mathbf{H}\,\mathbf{x} + \mathbf{w}_r$$
$$\mathbf{y}_e = \mathbf{G}\,\mathbf{x} + \mathbf{w}_e$$

- $\mathbf{w}_r$ and $\mathbf{w}_e$ are i.i.d. Gaussian noise vectors
- $\mathrm{tr}(\mathbb{E}\{\mathbf{x}\mathbf{x}^T\}) = \mathrm{tr}(\mathbf{Q}) \leq P$ (average power constraint)
- $\mathbf{H}$ and $\mathbf{G}$ are given

Channel Model



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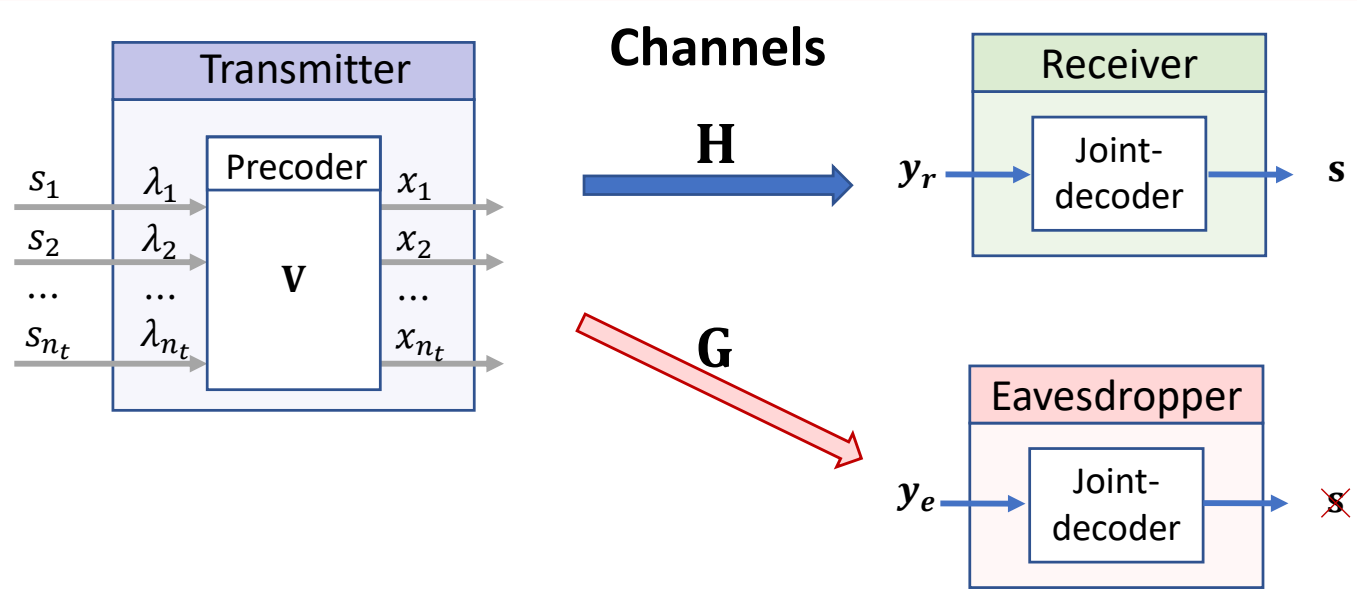
Secrecy capacity [Khisti-Wornell'08] [Oggier-Hassibi'08]

$$\max_{\mathbf{Q}} \quad \frac{1}{2}\log\frac{|\mathbf{I}_{n_r} + \mathbf{H}\mathbf{Q}\mathbf{H}^T|}{|\mathbf{I}_{n_e} + \mathbf{G}\mathbf{Q}\mathbf{G}^T|}$$
$$\text{s.t.} \quad \mathbf{Q} \succeq \mathbf{0},\ \mathbf{Q} = \mathbf{Q}^T,\ \mathrm{tr}(\mathbf{Q}) \le P$$



Precoding for the three-nodes wireless system

The Q in precoding

$$\mathbb{E}\{\mathbf{s}\mathbf{s}^T\} = \mathbf{I}, \text{ (i.i.d)}$$

$$\mathbf{x} = \mathbf{V}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{s}$$

$$\mathbf{Q} \triangleq \mathbb{E}\{\mathbf{x}\mathbf{x}^T\} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T$$

- Precoding matrix **V**
  Rotation model [Zhang-Qi-Vaezi'19]
  (arxiv.org/abs/1908.00994)
- Power allocation matrix

$$\mathbf{\Lambda} \triangleq \begin{bmatrix} \lambda_1^2 & & \\ & \ddots & \\ & & \lambda_{n_t}^2 \end{bmatrix}$$

Existing Solutions for **Q**

This problem is still difficult because it is non-convex and existing solutions are:

- not effective for all antenna settings
- iterative (time consuming)

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- iterative (time consuming)

Closed-form solution is known only for

- MISO case ($n_r = 1$) [Khisti-Wornell'10]
- $n_t = n_r = 2$ and $n_e = 1$ [Shafiee-Liu-Ulukus'09]
- Strictly degraded channel ($\mathbf{H}^H\mathbf{H} \succ \mathbf{G}^H\mathbf{G}$) only if $P > P_0$ [Loyka-Charalambous'16][Fakoorian-Swindlehurst'13]
- $n_t = 2$ [Vaezi-Shin-Poor'17]

Existing Solutions for **Q**

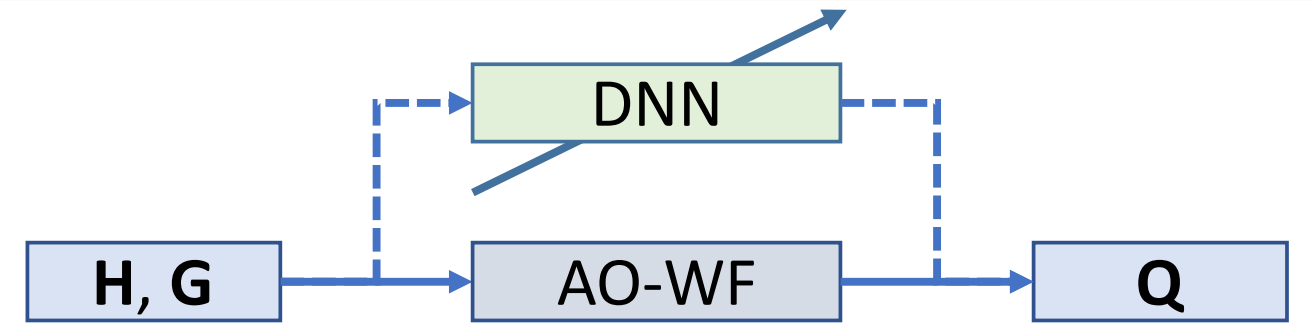
Existing numerical solutions

- Generalized singular value decomposition (GSVD) based beamforming [Fakoorian-Swindlehurst'12]
- Alternative optimization and water-filling (AO-WF) [Li-Hong-Wai-Liu-Ma-Luo'13]
- Rotation model [Zhang-Qi-Vaezi'19]

Problems

1. May provide a sub-optimal solution
2. Time cost usually high

This Talk

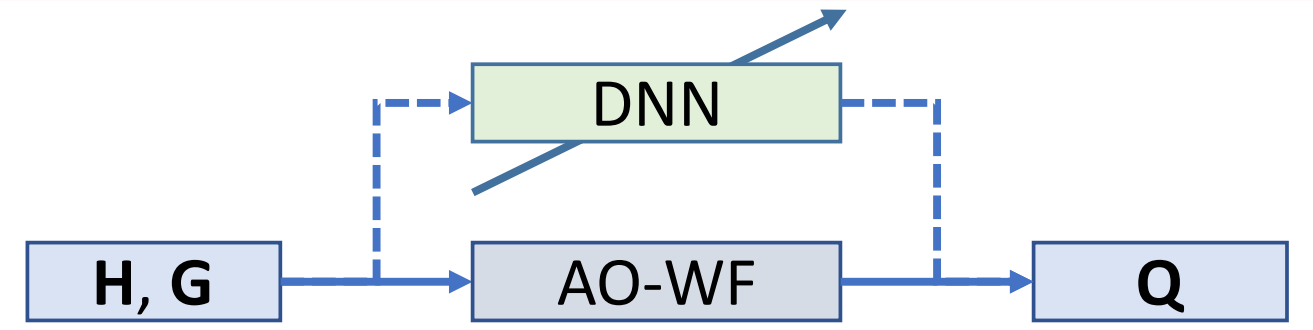


Training process

Why do we use deep neural network (DNN)?

1. **Fast**: finite matrices computation, no iterations
2. **Feasible to learn**: input/output mapping
3. **Easy data generation**: build data set including random channels and find corresponding optimal **Q**

This Talk



Training process

The proposed method

1. Build data set using numerical method (**off line**)
   − high time cost + close to the global optimal
2. Train the DNN (**off line**)
   − high time cost + accurate regression
3. Implement the well trained DNN (**on the fly**)
   + time efficient + close to the global optimal

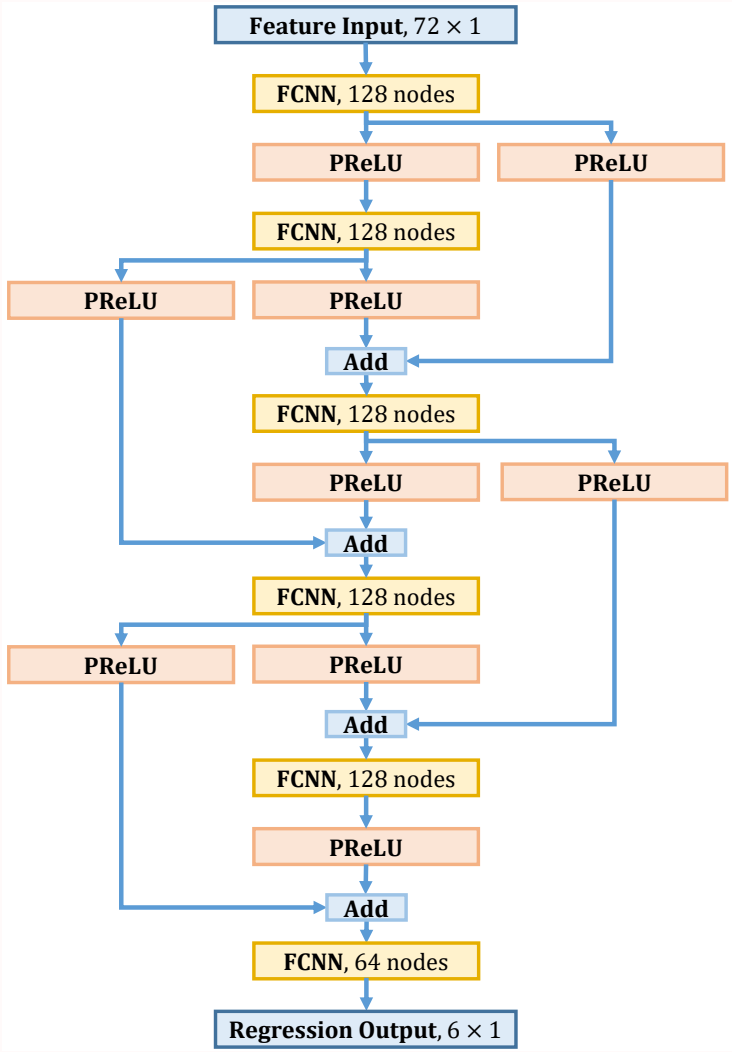
DNN design (for $n_t = 3$)

Components

- DNN architecture
  - Fully-connected neural networks (FCNN)
  - Activation functions
  - Residual connections (short cuts)

Ideas of the design

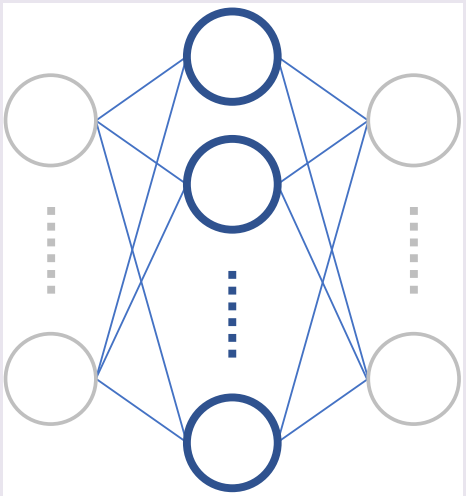
- **Non-linear**: to be able to learn the mapping
- **Simple**: small memory and computation, avoid over-fitting



Proposed DNN architecture

DNN architecture

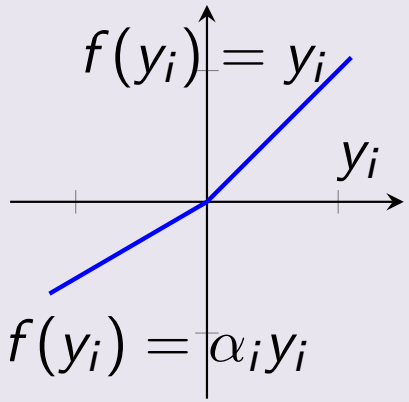
FCNN



Fully-connected neural network

Activation function

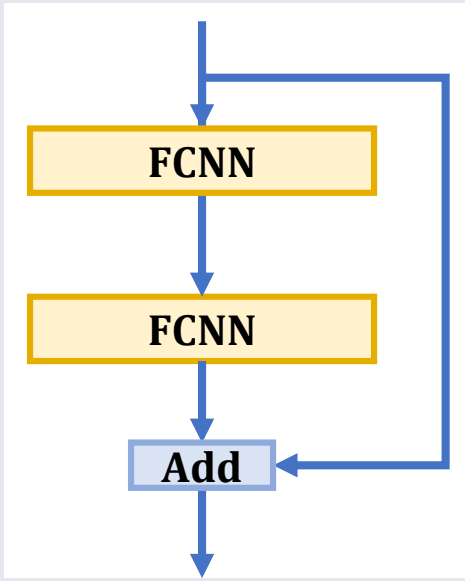
Parametric Rectified Linear Unit (PReLU)
[He-Zhang-Ren-Sun'15]



- Additional adaptive parameters
- Less computation

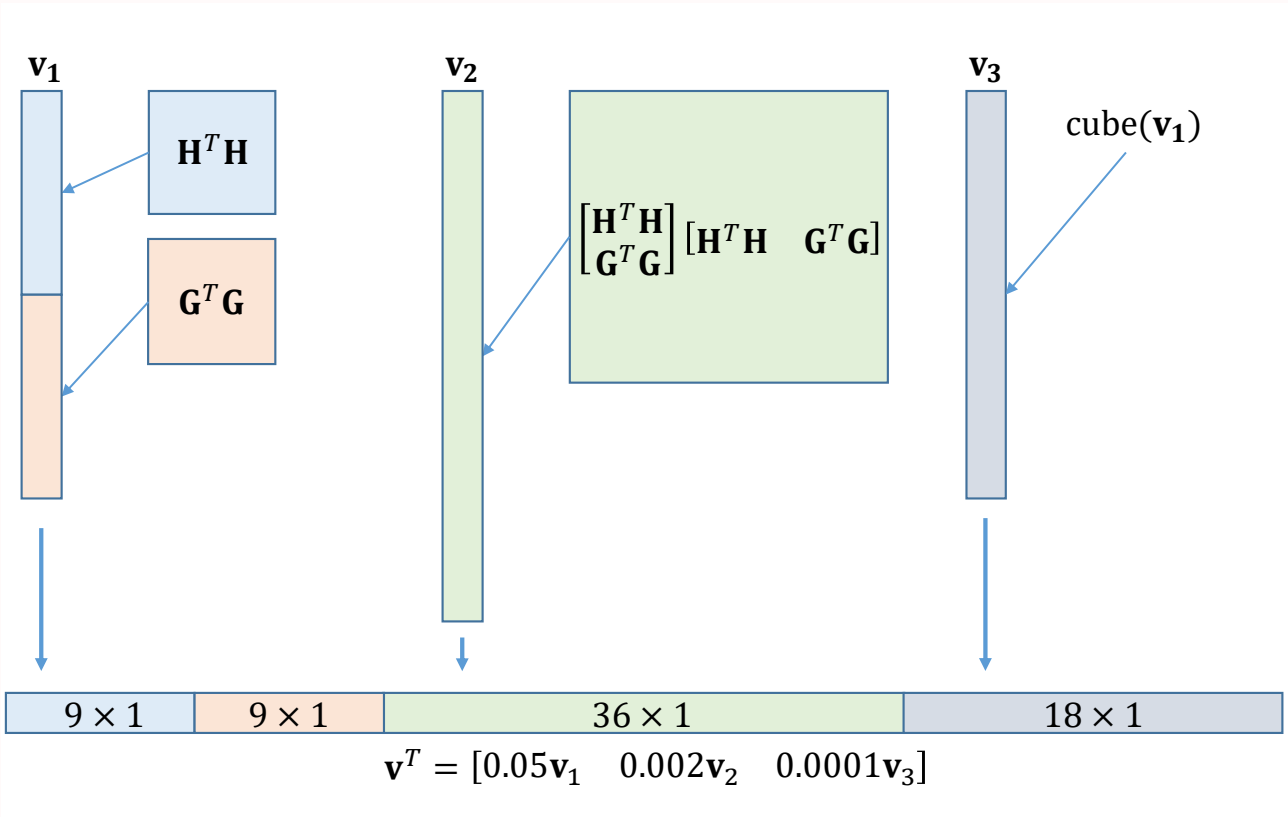
Residual connection

[He-Zhang-Ren-Sun'16]



- Increases complexity
- Converges faster

DNN input feature design

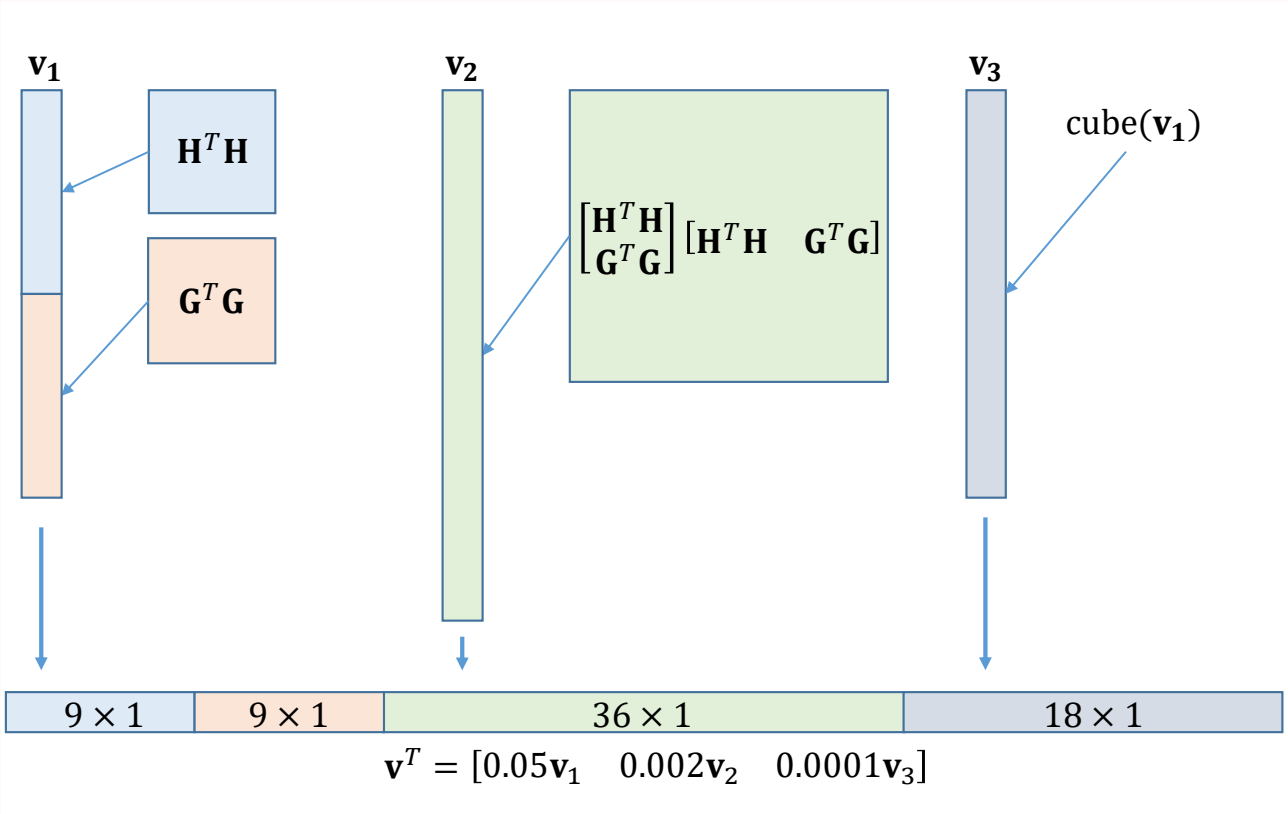


Secrecy capacity

$$\max_{\mathbf{Q}} \quad \frac{1}{2}\log\frac{|\mathbf{I}_{n_t}+\mathbf{H}^T\mathbf{H}\mathbf{Q}|}{|\mathbf{I}_{n_t}+\mathbf{G}^T\mathbf{G}\mathbf{Q}|}$$

$$\text{s.t.} \quad \mathbf{Q}\succeq\mathbf{0},\ \mathbf{Q}=\mathbf{Q}^T,\ \text{tr}(\mathbf{Q})\le P$$

DNN input feature design



Principles of input design

- The solution is only related to $\mathbf{H}^T\mathbf{H}$ and $\mathbf{G}^T\mathbf{G}$
- Nonlinear features can improve the performance of regression, since the DNN is a basicaly linear system

DNN output layer

Elements in the output vector (for $n_t = 3$)

Regression Output, 6×1

$$\mathbf{q} \triangleq [q_{11}, q_{22}, q_{33}, q_{12}, q_{23}, q_{13}]^T$$

$$\mathbf{Q}^* = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{12} & q_{22} & q_{23} \\ q_{13} & q_{23} & q_{33} \end{bmatrix}$$

Symmetric matrix

Data set

Data Set construction

For each sample of data set,

- Generate **H** and **G** randomly (Gaussian)
- Find the optimal **Q** using AO-WF or rotation method
- Record input and output vectors

Table: Details of the data sets.

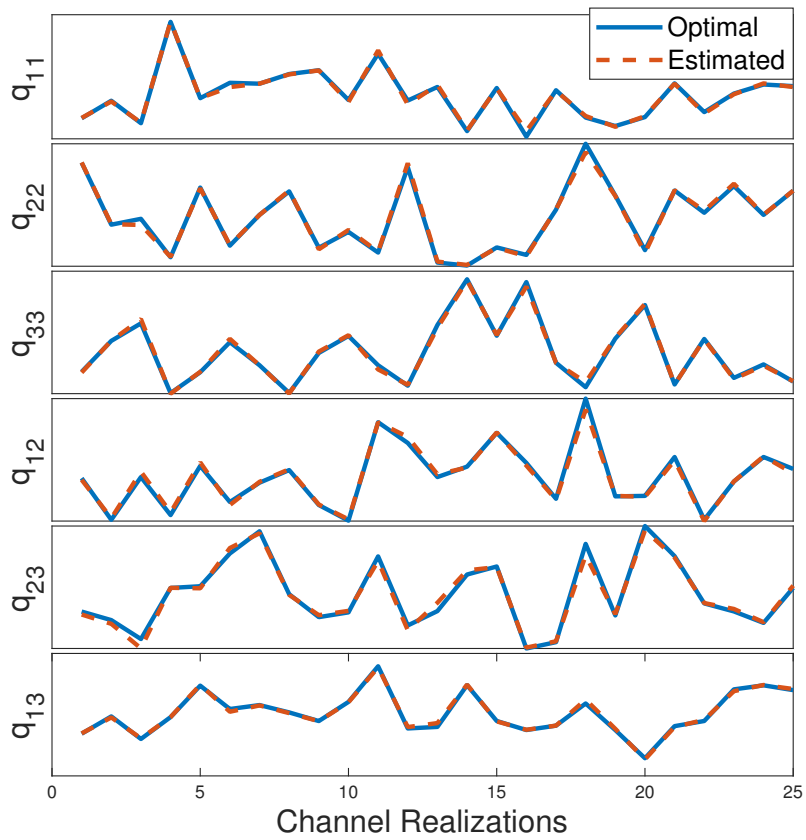
	n_t	n_r	n_e	number of samples
TrainingSet-I	3	2	1	2,000,000
TrainingSet-II	3	4	3	2,000,000
TrainingSet-III	Cascade of *TrainingSet-I* and *TrainingSet-II*			
TestSet-I	3	2	1	1000
TestSet-II	3	4	3	1000

Simulation Results

Accuracy of regression

Table: Mean square error (MSE) of the regression.

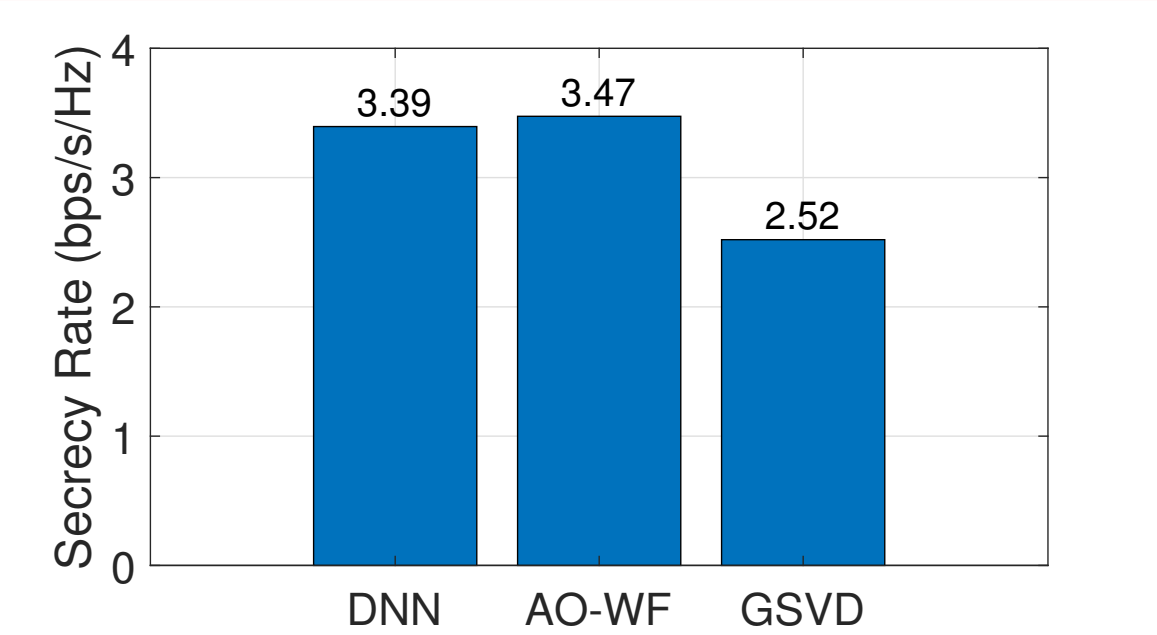
	TrainingSet-III	
	TestSet-I	*TestSet-II*
$\hat{q}_{11}$	0.1313	0.1564
$\hat{q}_{22}$	0.1234	0.1386
$\hat{q}_{33}$	0.1219	0.1364
$\hat{q}_{12}$	0.1526	0.1351
$\hat{q}_{23}$	0.1057	0.1596
$\hat{q}_{13}$	0.1384	0.1123



Comparison between the DNN output and the ideal values

Simulation Results

Achievable secrecy rate



Averaged time cost

Table: Average time for one realization.

	DNN	AO-WF	GSVD
Time Cost (ms)	**0.0255**	19.49	0.513

Simulation Results

Promising for the internet of things (IoT) devices

Limited computational ability and battery life
Require low delay and high speed

Averaged time cost

Table: Average time for one realization.

	DNN	AO-WF	GSVD
Time Cost (ms)	**0.0255**	19.49	0.513

Conclusions

Summary

- Problem: secure transmission over Gaussian wiretap channel
- Solution: DNN-based precoding
- Advantage: fast and reliable, efficient for IoT devices

Future Work

- Train for arbitrary n_r and n_e
- Generalize to complex channel
- Multi-task precoding realization

Thank you!