Simplified Han-Kobayashi Region for One-Sided and Mixed Gaussian Interference Channels

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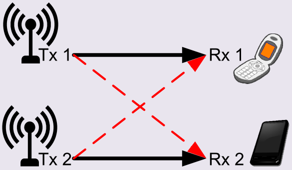
Wireless Channels

Two key features of wireless channels:

1. Fading – well-established theory, such as MIMO
2. Interference – within or across systems, e.g.
   - within adjacent cells in a cellular system
   - among multiple WiFi networks



Basic Model: Two-User Interference Channel (IC)

What do we know?

Two basic approaches to use the common spectrum:

- Orthogonalization into different bands to avoid interference
- Full sharing of the spectrum but treating interference as noise

The best known achievable region is due to Han-Kobayashi (HK), but

- general HK scheme is quite complex
- basic HK region can be enlarged by time-sharing
- cardinality of the time-sharing parameter is rather high

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Two basic approaches to use the common spectrum:

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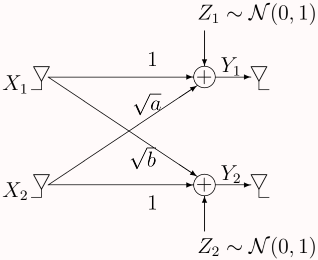
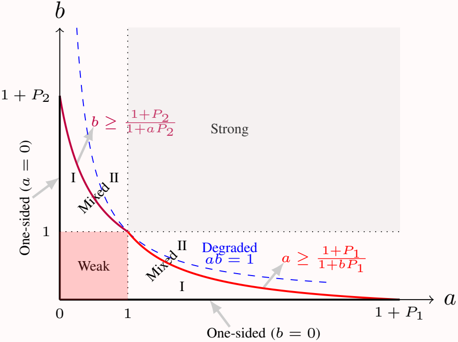
Questions

1. Is the HK scheme optimal encoding strategy?
2. Are Gaussian inputs optimal for the HK scheme?
3. What is the cardinality of the time-sharing parameter?

Outline

Channel Model





$$Y_1 = X_1 + \sqrt{a}X_2 + Z_1,$$
$$Y_2 = \sqrt{b}X_1 + X_2 + Z_2,$$

average power constraint P_i
$\mathbb{E}(\|X_i\|^2) \leq P_i, \ i = 1, 2.$

HK Scheme - Key Idea

- divide each user's message into private and common messages
- decode and cancel part of interference (other user's common message) before decoding own private message

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HK region without time-sharing ($\mathcal{R}_{\mathrm{HK}}^{\mathcal{G}_0}$)

The region $\mathcal{R}_{\mathrm{HK}}^{\mathcal{G}_0}$ for the one-sided IC with $b = 0$ is given by the union of the set of (R_1, R_2) such that

$$R_1 \leq \gamma\Big(\frac{P_1}{1 + a\beta P_2}\Big),$$

$$R_2 \leq \gamma(P_2),$$

$$R_1 + R_2 \leq \gamma\Big(\frac{P_1 + a\bar{\beta} P_2}{1 + a\beta P_2}\Big) + \gamma(\beta P_2),$$

where $\beta \in [0, 1]$, $\bar{\beta} = 1 - \beta$, and $\gamma(x) \triangleq \frac{1}{2}\log_2(1 + x)$.

Weak One-Sided IC

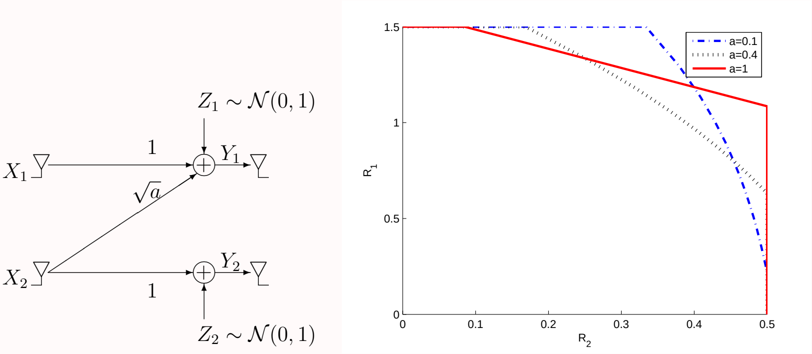
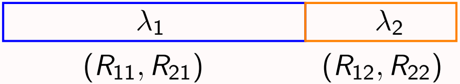


Figure: Basic HK region (without time-sharing) for $P_1 = 1$ and $P_2 = 7$.

Time-sharing

Time-sharing: (R_{11}, R_{21}) and (R_{12}, R_{22}) are achievable $\Rightarrow$ $\lambda_1(R_{11}, R_{21}) + \lambda_2(R_{12}, R_{22})$ is achievable, for any $\lambda_1 + \lambda_2 = 1$.



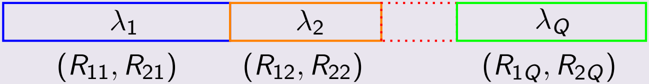
$$(R_1, R_2) = \lambda_1(R_{11}, R_{21}) + \lambda_2(R_{12}, R_{22})$$

Example

- in λ_1 interference might be treated as noise
- in λ_2 user 2 may transmits alone
- in λ_3 both users may decode both messages
- in λ_4 ...

Cardinality of Time-sharing

Let q be a time-sharing parameter with cardinality of $|q| = Q$.
Then, for $\sum_q \lambda_q = 1$ the following rate pair is achievable:



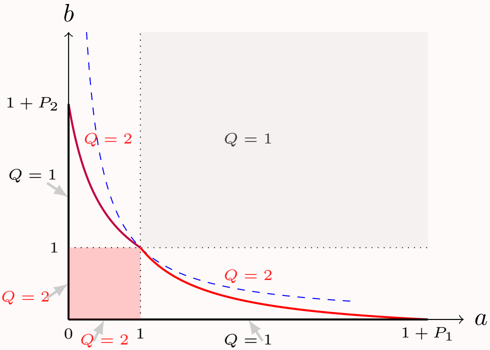
$$(R_1, R_2) = \sum_{q=1}^{Q} \lambda_q (R_{1q}, R_{2q})$$

Cardinality of time-sharing

What is the optimum value (minimum) of Q?

- $Q \leq 7$ for full IC [Chong et al.'08]
- $Q \leq 3$ for one-sided IC [Motahari-Kandani'09]

Our Contribution



$Q = 2$ is optimal for

- one-sided IC in the weak interference regime
- full IC in a large part of mixed interference regime

$\mathcal{R}_{\mathrm{HK}}^{\mathcal{G}}$ for One-Sided IC

The border of the HK region ($\mathcal{R}_{\mathrm{HK}}^{\mathcal{G}}$) is characterized by[1]

$$\text{maximize} \sum_{q=1}^{3} \lambda_q \left[\mu R_{1q} + R_{2q}\right]$$

subject to

$$\sum_{q=1}^{3} \lambda_q = 1, \quad \sum_{q=1}^{3} \lambda_q P_{1q} \leq P_1, \quad \sum_{q=1}^{3} \lambda_q P_{2q} \leq P_2,$$

$$0 \leq \beta_q \leq 1, \quad \lambda_q \geq 0,\ P_{1q} \geq 0,\ P_{2q} \geq 0,\ \forall\, q \in \{1,2,3\}.$$

One needs to optimize 11 variables at the same time.

[1] $\mu R_{1q} + R_{2q} = \mu\gamma\left(\frac{P_{1q}}{1+a\beta_q P_{2q}}\right) + \gamma\left(\frac{a\bar{\beta}_q P_{2q}}{1+P_{1q}+a\beta_q P_{2q}}\right) + \gamma(\beta_q P_{2q})$

Sketch of Proof

1. $\mu R_{1q} + R_{2q}$ is strictly increasing with P_{1q}
   $\Longrightarrow \sum_{q=1}^{3} \lambda_q P_{1q} = P_1$
2. non-interfering user need not split its power
   $\Longrightarrow P_{12} = P_{13} = 0,\ P_{11} = \frac{P_1}{\lambda_1}$
3. now, it can be check that $\beta_2 = \beta_3 = 1$ are optimal for $a < 1$
   $\Longrightarrow \lambda_2 R_{22} + \lambda_3 R_{23} = \lambda_2 \gamma(P_{22}) + \lambda_3 \gamma(P_{23})$
4. power splitting is not required in a single-user channel
   $\Longrightarrow \lambda_3 = 0$ (optimal solution needs two subbands only)
5. assuming P_{21} is used in subband λ_1, the leftover power must be used in subband λ_2 to maximize $\lambda_2 \gamma(P_{22})$.
   $\Longrightarrow \sum_{q=1}^{2} \lambda_q P_{2q} = P_2$, or $P_{22} = \frac{P_2 - \lambda_1 P_{21}}{1 - \lambda_1}$.

We need to optimize only 3 variables, namely, $\lambda_1, \beta_1, P_{21}$.

Simplified HK Inner Bound

Theorem 1

The set of non-negative (R_1, R_2) satisfying

$$R_1 \leq \lambda_1 R_{11},$$
$$R_2 \leq \lambda_1 R_{21} + \lambda_2 R_{22},$$

in which

$$R_{11} \leq \gamma\left(\frac{\frac{P_1}{\lambda_1}}{1 + a\beta_1 P_{21}}\right),$$
$$R_{21} \leq \gamma\left(\frac{a\bar{\beta}_1 P_{21}}{1 + \frac{P_1}{\lambda_1} + a\beta_1 P_{21}}\right) + \gamma(\beta_1 P_{21}),$$
$$R_{22} \leq \gamma(P_{22}),$$

is achievable for the one-sided Gaussian interference channel where $\lambda_1 + \lambda_2 = 1$, $\lambda_1 P_{21} + \lambda_2 P_{22} = P_2$, $0 \leq \beta_1 \leq 1$, and $\bar{\beta}_1 = 1 - \beta_1$.

Simplified HK - Intuitions

To achieve a rate pair (R_1, R_2) on the border of $\mathcal{R}_{\mathrm{HK}}^{\mathcal{G}}$

- Time-sharing in two dimensions
  - both users transmit for λ_1 fraction of channel uses
  - only non-interfered-with user transmits during λ_2

λ_1	λ_2
*user*1 & *user*2	*user*2
(R_{11}, R_{21})	$(0, R_{22})$

- i.i.d inputs are not optimal, in general

$$\mathrm{Cov}(X_1) = \begin{bmatrix} \frac{n}{n_1} P_1 \boldsymbol{I}_{n_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \text{ and } \mathrm{Cov}(X_2) = \begin{bmatrix} P_{21} \boldsymbol{I}_{n_1} & \mathbf{0} \\ \mathbf{0} & P_{22} \boldsymbol{I}_{n_2} \end{bmatrix}$$

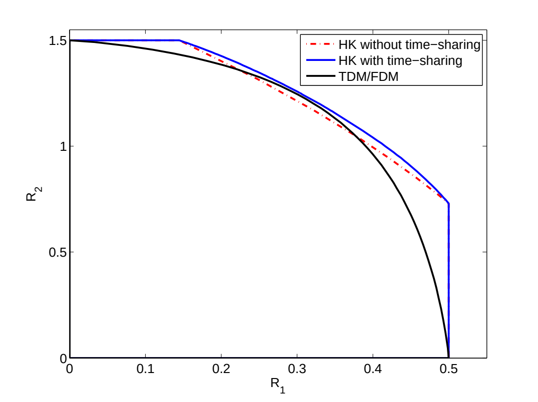
where $n_1 = \lambda_1 n$, $n_2 = \lambda_2 n$, and n is code (block) length.

Special Cases

1. $\lambda_2 = 0$ (NO time-sharing) $\Longrightarrow$ basic HK region ($\mathcal{R}_{\text{HK}}^{\mathcal{G}0}$)
2. $P_{21} = 0 \Longrightarrow$ TDM/FDM region

$$R_1 \leq \lambda_1 \gamma\left(\frac{P_1}{\lambda_1}\right),$$

$$R_2 \leq \lambda_2 \gamma\left(\frac{P_2}{\lambda_2}\right),$$



$\mathcal{R}_{\mathrm{HK}}^{\mathcal{G}}$ for Mixed IC

For full IC

- we do not have an optimization problem similar to the one-sided IC case ($\mu R_1 + R_2$ is not known)
- $\mathcal{R}_{\mathrm{HK}}^{\mathcal{G}}$ is daunting as it needs time-sharing in 7 dimensions

However, we can use the same optimization problem to achieve the HK border for a large part of mixed interference regime

- The $\mathcal{R}_{\mathrm{HK}}^{\mathcal{G}_0}$ is the same for $a < 1,\ b \geq \frac{1+P_2}{1+aP_2}$ and $a < 1,\ b = 0$
- This immediately reduces Q to 2 (formerly it was 7)
- The complexity of the HK region thus reduces significantly, in the above case

Summary

- With Gaussian inputs time-sharing in two dimensions is enough to achieve the border of HK region $a < 1$ and $b = 0$ or $b \geq \frac{1+P_2}{1+aP_2}$
  - use basic HK for λ_1 fraction of channel uses
  - single-user transmission for λ_2 fraction of channel uses
- This is the best known inner bound with Gaussian inputs; it is not however known whether
  - Gaussian inputs are optimal for the HK region
  - the HK region is optimal for the Gaussian IC
- The cardinality of time-sharing parameter is still 7 for the weak and part of mixed interference regimes

Thank you!