On the Number of Users Served in MIMO-NOMA Cellular Networks

2016.09.21

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What is **NOMA** (Non-Orthogonal Multiple Access)?

- **Key Concept for NOMA (Two User Case)**
  - 2 users can be served by BS at the same freq., but with *different power levels*
    - Superimposed mixture containing two messages for the two users (UE1 and UE2)
    - The message to the UE2 is allocated more transmission power
      - UE2 can detect its message directly (TIN)
      - UE1 needs to first detect UE2’s information and then to subtract this information from its observation before decoding its own message (SIC)

*Source: Saito’13*

*TIN: Treating Interference as Noise
SIC: Successive Interference cancellation*
Why (Power-Domain) NOMA?

**Theoretical Promise** (spectral efficiency and user fairness)
- The BW allocated to a user with very poor channel condition may not be used efficiently
  - For low-rate users, e.g., sensors, the use of OMA may give more than what they need
- NOMA can support more users than the number of resource blocks
  - NOMA offers wider BW to both users

**Processors Evolution for Interference Cancellation**
- Moore’s law: x100 processing power every 10 year (e.g. NAICS in 3GPP Rel. 12)

**In the Literature**
- Extension to MIMO-NOMA [Ding-Ada-Poor'16], [Ding-Schober-Poor'16]
- Cooperative NOMA [Ding-Peng-Poor'16], Clustering [Ding-Fan-Poor’16], Power Allocation etc.

How Does The *Inter-cell Interference* Impact Performance?

*MIMO: Multiple-Input Multiple-Output*
Related Work

- **NOMA w/o Inter-Cell Coordination**
  - Inter-cell interference (ICI) is a big issue in multi-cell networks
  - ICI reduces the cell-edge users QoS and deteriorates users fairness

- **NOMA-Joint Transmission (JT) [Choi’15]**
  - Coordinated Superposition Coding (CSC) with Distributed Alamouti Code
  - Data Sharing through backhaul link (an excessive backhaul overhead)

NOMA-Coordinated Beamforming Has NOT Been Studied Yet!
**System Model**

- **Multi-cell MIMO Cellular System**
  - \(L\)-cell network \((L \geq 2)\) scenario
  - Each cell consists of \(K\)-cluster
  - \# of BS ant: \(M\), \# of UE ant: \(N\)

- **CSI (Channel State Information)**
  - Full CSI at BS

- **Inter-Cell Interference Pattern**
  - Center users (free of interference)

- **Our Contributions**
  - New NOMA-CB to mitigate inter-cell as well as Intra/inter-cluster interferences
  - How many users can be served simultaneously with NOMA-CB under given \# of BS and UE antennas \((M \& N)\)?
Simple Extension \((L=2, K=2)\)

- **Simple Extension of MIMO-NOMA** [Ding-Ada-Poor’16]
  - ICI Zero-forcing condition at BS 1 \((N=2)\):
    \[
    y_A^{[1]} \perp \begin{bmatrix} G_2^{[2,1]} \\ G_4^{[2,1]} \end{bmatrix}
    \]
  - By considering this,
    - **MIMO-NOMA**
      - # of Bs ant: \(M \geq 2 + 4 = 6\)
    - ICI zero-forcing
  - Tx ant. should be greater than \(K + (L - 1)K^2\)

Can We Reduce # of Tx Ant. to Support 2K User Per Cell in L-cell Network?
Example of NOMA-CB \((L=2, K=2, M=3, N=2)\)

- **Phase 1**: Interfering Channel Alignment
  - At Cell-Edges UE,
    \[ \tau^{[2]} = G^{[1,2]^\dagger}_2 w_2 = G^{[1,2]^\dagger}_4 w_4 \]
    matrix form
    \[ \begin{bmatrix}
    I & -G^{[1,2]^\dagger}_2 \\
    I & 0 \\
    \end{bmatrix}
    \begin{bmatrix}
    \tau^{[2]} \\
    w_2 \\
    w_4 \\
    \end{bmatrix} = 0 \]
    \[ A: 2M \times (M + 2N) \]
  - Due to channel randomness, \( \exists \tau^{[1]} \) and \( \tau^{[2]} \)

- **Phase 2**: Zero-forcing BF Design
  - \( \nu_A \perp [\tau^{[1]} H^{[1]^\dagger}_4 w_4] \)
  - \( \nu_B \perp [\tau^{[1]} H^{[1]^\dagger}_2 w_2] \)
Example of NOMA-CB (L=2, K=2, M=3, N=2)

- **Phase 3: Inter-Cluster Interference**
  - At Cell-Center UE,
    \[ w_1 \perp H_1^{[1]} v_B, \quad w_3 \perp H_3^{[1]} v_A \]
  - \( \exists w_1, w_2 \) due to \( N \geq K \)

- **Through Phase 1-3,**
  - NOMA-CB decomposes 2-cell MIMO-NOMA into 2K pairs of SISO-NOMA

- **Phase 4: Intra-Cluster Interference**
  - At user \( k \) at cell 1,
    \[ w_k^\dagger y_k = \tilde{h}_k \left( \sqrt{\lambda_2^{[\frac{k}{2}]} s_2^{[\frac{k}{2}]} - 1} + \sqrt{\lambda_2^{[\frac{k}{2}]} s_2^{[\frac{k}{2}]} - 1} \right) \]
    where \( \tilde{h}_k = w_k^\dagger H_k^{[1]} v_A, \quad k \in \{1, 2\} \)
    \( \tilde{h}_k = w_k^\dagger H_k^{[1]} v_B, \quad k \in \{3, 4\} \)
Main Result 1 *(Feasibility conditions)*

**Lemma 1:** For an L-cell MIMO network, to simultaneously support K clusters per cell we must have

\[ M \geq K + \Delta \quad \text{and} \quad N \geq \max \left\{ \frac{(L-1)K - \Delta}{K} M + \varepsilon, K \right\} \]

where \( \Delta \) is an arbitrary between 1 and \( \min \{(L - 1)K, M - 1\} \) and \( \varepsilon \) is an arbitrary small positive number, i.e., \( 0 < \varepsilon \ll 1 \).

**Sketch of proof:**

- In order to confine all \( Q = (L - 1)K \) interfering channels of each BS within \( \Delta \)-dimentional signal space, we must have

\[
\text{span} \left[ \begin{bmatrix} \tau_1^{[\ell]} & \tau_2^{[\ell]} & \cdots & \tau_{\Delta}^{[\ell]} \end{bmatrix} \right] \\
= \text{span} \left\{ \begin{bmatrix} G_2^{[1,\ell]} & G_2^{[2,\ell]} & \cdots & G_2^{[L,\ell]} \end{bmatrix} \right\} \\
\text{where} \quad G_2^{[\ell',\ell]} = [G_2^{[\ell',\ell]}]_{1 \times \Delta} \begin{bmatrix} w_{2,1}^{[\ell']} & \cdots & G_2^{[\ell',\ell]}_{2,K} w_{2,K}^{[\ell']} \end{bmatrix}
\]
Main Result I *(Feasibility conditions)*

- **Sketch of proof:**
  - Considering all cells in the network, we can unify a system matrix equation.
  - The size of the unified matrix in general is $L(L - 1)KM \times (LM\Delta + LKN)$.
  - Since all the channel matrices are completely random, the unified matrix has full rank a.s.
  - Thus, to guarantee the existence of null space of the matrix (inter-cell interference),
    \[
    N > \frac{(L - 1)K - \Delta}{K} M
    \]
  - On the other hand, $N \geq K$ in order to cancel inter-cluster interference at cell-center user.
  - To ensure zero inter-cell and inter-cluster interference at cell-edge users, $M \geq K + \Delta$

\[
\begin{align*}
\mathbf{v}_{1}^{[1]} & \perp \begin{bmatrix} \tau_{1}^{[1]} & \ldots & \tau_{1}^{[\Delta]} \end{bmatrix} \\
\mathbf{w}_{2}^{[1]} & \mathbf{H}_{2}^{[1]} & \mathbf{w}_{2}^{[1]} & \mathbf{H}_{2}^{[1]} & \mathbf{w}_{2}^{[1]} & \mathbf{H}_{2}^{[1]} & \cdots & \mathbf{w}_{2}^{[1]} & \mathbf{H}_{2}^{[1]}
\end{align*}
\]

*Inter-cell* \hspace{5cm} *Inter-cluster*
**Main Result II (Analysis of # of users)**

**Theorem 1:** The maximum number of users supported by the proposed NOMA-CB scheme in an $L$-cell MIMO network with $M$ transmit antennas at each BS and $N$ receive antennas at each UE is given by

$$2 \min \{ \max \{ M - \lfloor \delta^* \rfloor, \lfloor f(\lfloor \delta^* \rfloor) \rfloor - \epsilon, g(1) \}, N \}$$

where $\delta^* = \frac{(L-1)M^2-MN}{LM-N}$, $f(x) = \frac{N-(L-2)x+\sqrt{[N-(L-2)x]^2+4(L-1)x^2}}{2(L-1)}$, $g(x) = \min\{M-x, |f(x) - \epsilon|\}$

**Sketch of Proof:** From the 3 conditions,

$$\frac{NK}{(L-1)K-\Delta} > M > K + \Delta$$

which results in

$$K < \frac{N-(L-2)\Delta+\sqrt{[N-(L-2)\Delta]^2-4(L-1)\Delta^2}}{2(L-1)} \triangleq f(x)$$
Main Result II (Analysis of # of users)

Sketch of Proof:

- The number of clusters per cell is bounded as
  \[ K \leq \min\{M - \Delta, f(\Delta) - \epsilon, N\} \]

- Since \( \Delta \) and \( K \) must be integers, we formulate the following optimization problem:
  \[
  \max_{\Delta} 2K = 2\max_{\Delta} \min\{M - \Delta, |f(\Delta) - \epsilon|, N\} \\
  \text{s.t.} \quad L, M, K \geq 2 \\
  \Delta \in \{1, 2, \ldots, \min\{(L - 1)K, M - 1\}\}
  \]

- Note that \( M - \Delta \) is linearly decreasing with \( \Delta \) and \( f(\Delta) \) is a strictly convex, i.e.,
  \[
  \frac{\partial^2 f(\Delta)}{\partial \Delta^2} > 0
  \]
Main Result II (Analysis of # of users)

Sketch of Proof:

- Note that \( M - \Delta \) is linearly decreasing with \( \Delta \) and \( f(\Delta) \) is a strictly convex.

- By considering integer constraint,

\[
K^* = \min \left\{ \max \left\{ g(\lfloor \Delta' \rfloor), g(\lfloor \Delta' \rfloor), g(1) \right\}, N \right\}
\]
\[
= \min \left\{ \max \left\{ M - \lfloor \delta^* \rfloor, [f(\lfloor \delta^* \rfloor) - \epsilon], g(1) \right\}, N \right\}
\]

where \( g(x) = \min \left\{ M - x, [f(x) - \epsilon] \right\} \)
Numerical Results

- **Special Case** \((L = 2, M = K + 1, N = K)\)
  - What is maximum \# of users?
    \[
    2 \min \left\{ \max \left\{ M - \left\lfloor \delta^* \right\rfloor, \left\lfloor f(\delta^*) \right\rfloor - \epsilon, g(1) \right\}, N \right\}
    = 2 \min \left\{ \max \left\{ K + 1, \frac{K}{2} \right\}, K \right\} = 2K
    \]
  - (Per-Cell) System Throughput
    \[
    R = \sum_{k=1}^{K} [R_{2k-1}(1 - P_{2k-1}) + R_{2k}(1 - P_{2k})]
    \]
    - Users are randomly distributed
    - Compare with existing schemes
      - OMA: \((K+1)/2\) users
      - SA-NOMA [Ding-Schober-Poor'16], MIMO-NOMA [Ding-Ada-Poor'16] : \(K\) users

Proposed Scheme Achieves **Better Throughput** than Existing Works (Larger \# of Users)
Summary

- We introduced multi-cell NOMA techniques, called NOMA-CB, which does not rely on data sharing among BSs (Less Backhaul Overhead)
- We completely analyzed the number of supported users with NOMA-CB scheme, which shows that the significant gains over existing schemes

Future Work

- NOMA-CB with imperfect CSIT (Delayed/Limited Feedback)