(2016 IEEE ISWCS, Poznan, Poland)

On the Number of Users Served in MIMO-NOMA Cellular Networks

2016. 09. 21

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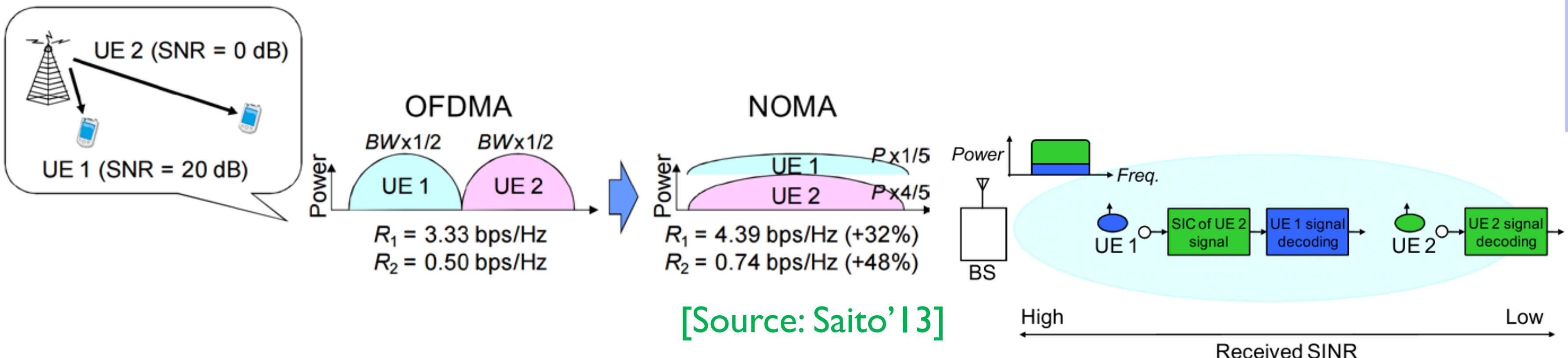
What is NOMA (Non-Orthogonal Multiple Access)?

❑ Key Concept for NOMA (Two User Case)

- 2 users can be served by BS at the same freq., but with *different power levels*
  - Superimposed mixture containing two messages for the two users (UE1 and UE2)
  - The message to the UE2 is allocated more transmission power
    - UE2 can detect its message directly (TIN)
    - UE1 needs to first detect UE2's information and then to subtract this information from its observation before decoding its own message (SIC)



[Source: Saito'13]

*TIN: Treating Interference as Noise
SIC: Successive Interference cancellation

Why (Power-Domain) NOMA?

- **Theoretical Promise** (spectral efficiency and user fairness)
  - The BW allocated to a user with very poor channel condition may not be used efficiently
    - For low-rate users, e.g., sensors, the use of OMA may give more than what they need
  - NOMA can support more users than the number of resource blocks
    - NOMA offers wider BW to both users
- **Processors Evolution for Interference Cancellation**
  - Moore's law: x100 processing power every 10 year (e.g. NAICS in 3GPP Rel. 12)
- **In the Literature**
  - Extension to MIMO-NOMA [Ding-Ada-Poor'16], [Ding-Schober-Poor'16]
  - Cooperative NOMA [Ding-Peng-Poor'16], Clustering [Ding-Fan-Poor'16], Power Allocation etc.

How Does The *Inter-cell Interference* Impact Performance?

*MIMO: Multiple-Input Multiple-Output

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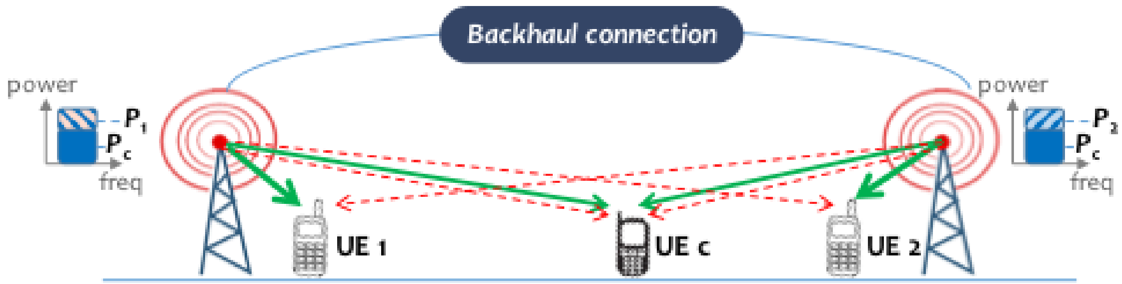
Related Work

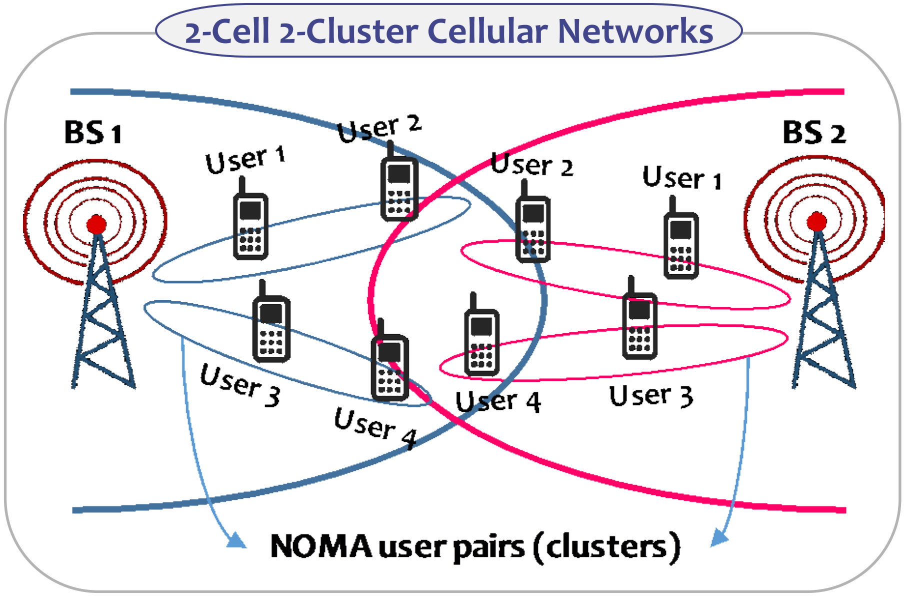
- **NOMA w/o Inter-Cell Coordination**
  - Inter-cell interference (ICI) is a big issue in multi-cell networks
  - ICI reduces the cell-edge users QoS and deteriorates users fairness
- **NOMA-Joint Transmission (JT)** [Choi'15]
  - Coordinated Superposition Coding (CSC) with Distributed Alamouti Code
  - Data Sharing through backhaul link (an excessive backhaul overhead)



NOMA-Coordinated Beamforming Has NOT Been Studied Yet!

System Model

- **Multi-cell MIMO Cellular System**
  - L-cell network ($L \geq 2$) secenario
  - Each cell consists of K-cluster
  - # of BS ant: M, # of UE ant: N
- **CSI (Channel State Information)**
  - Full CSI at BS
- **Inter-Cell Interference Pattern**
  - Center users (free of interference)



- **Our Contributions**
  - New NOMA-CB to mitigate inter-cell as well as Intra/inter-cluster interferences
  - How many users can be served simultaneously with NOMA-CB under given # of BS and UE antennas (M & N)?

Simple Extension (L=2, K=2)

❑ Simple Extension of MIMO-NOMA [Ding-Ada-Poor'16]

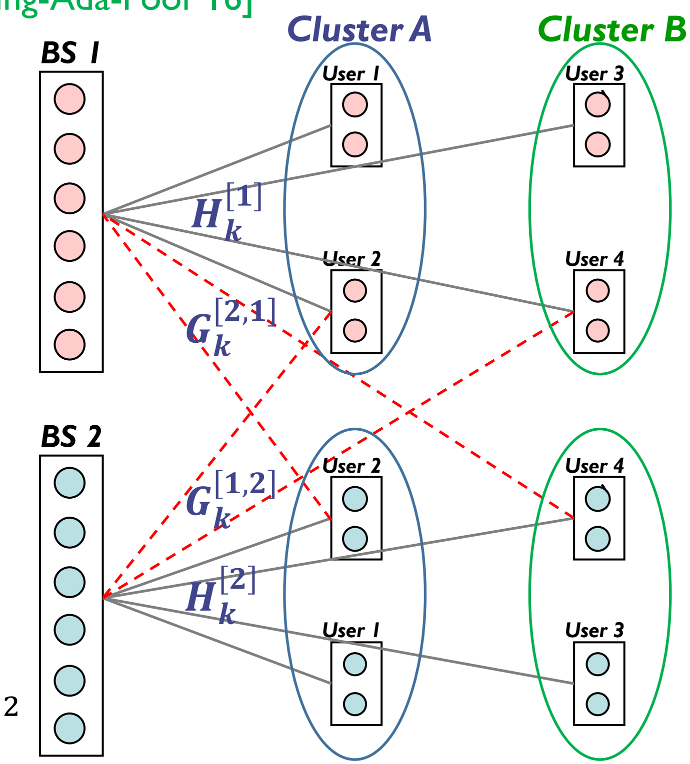
- ICI Zero-forcing condition at BS 1 (N=2):

$$v_A^{[1]} \perp \begin{bmatrix} G_2^{[2,1]} \\ G_4^{[2,1]} \end{bmatrix}$$

- By considering this,

$$\text{\# of Bs ant:}\quad M \geq \underbrace{2}_{\text{MIMO-NOMA}} + \underbrace{4}_{\text{ICI zero-forcing}} = 6$$

- Tx ant. should be greater than $K + (L-1)K^2$



Can We Reduce # of Tx Ant. to Support 2K User Per Cell in L-cell Network?

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Example of NOMA-CB (L=2, K=2, M=3, N=2)

❑ Phase 1: Interfering Channel Alignment

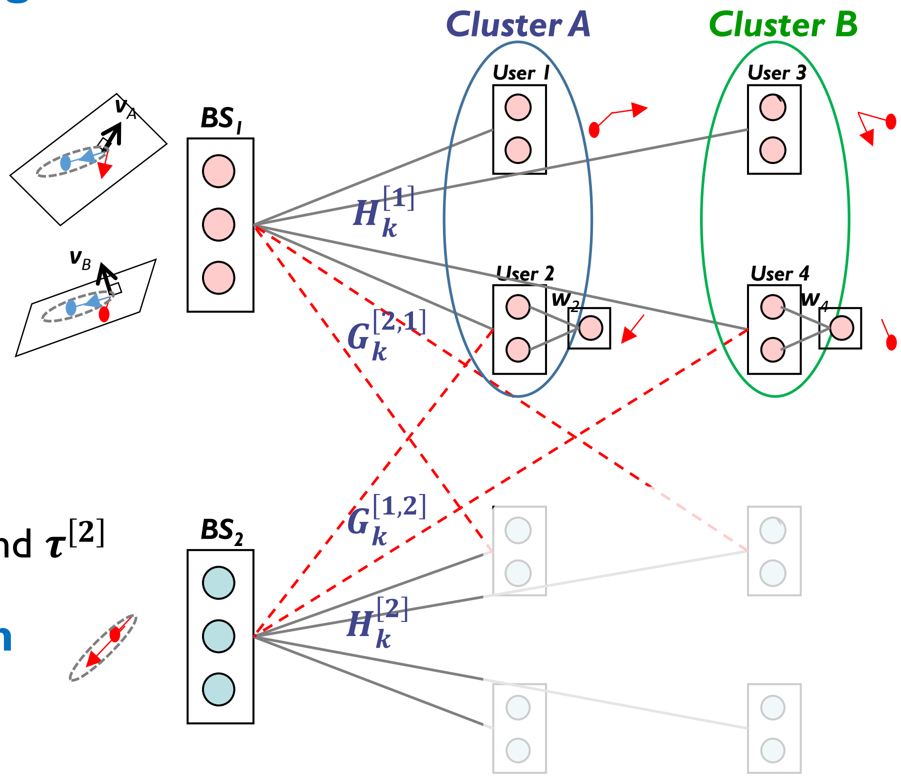
- At Cell-Edges UE,

$$\boldsymbol{\tau}^{[2]} = \boldsymbol{G}_2^{[1,2]\dagger}\boldsymbol{w}_2 = \boldsymbol{G}_4^{[1,2]\dagger}\boldsymbol{w}_4$$

⬇ *matrix form*

$$\underbrace{\begin{bmatrix} \mathbf{I} & -\boldsymbol{G}_2^{[1,2]\dagger} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & -\boldsymbol{G}_4^{[1,2]\dagger} \end{bmatrix}}_{\boldsymbol{A}:\ 2M \times (M+2N)} \begin{bmatrix} \boldsymbol{\tau}^{[2]} \\ \boldsymbol{w}_2 \\ \boldsymbol{w}_4 \end{bmatrix} = \mathbf{0}$$

- Due to channel randomness, $\exists \boldsymbol{\tau}^{[1]}$ and $\boldsymbol{\tau}^{[2]}$



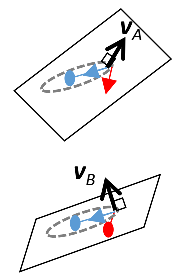
❑ Phase 2: Zero-forcing BF Design

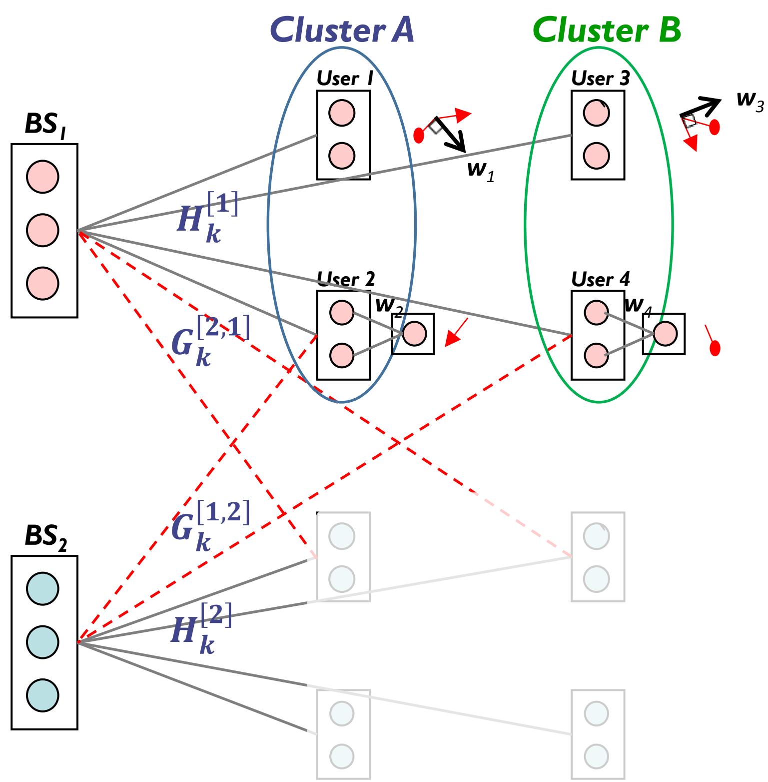
$$\boldsymbol{v}_A \perp \begin{bmatrix} \boldsymbol{\tau}^{[1]} & \boldsymbol{H}_4^{[1]\dagger}\boldsymbol{w}_4 \end{bmatrix}$$

$$\boldsymbol{v}_B \perp \begin{bmatrix} \boldsymbol{\tau}^{[1]} & \boldsymbol{H}_2^{[1]\dagger}\boldsymbol{w}_2 \end{bmatrix}$$

Example of NOMA-CB (L=2, K=2, M=3, N=2)

- **Phase 3**: Inter-Cluster Interference
  - At Cell-Center UE,
    $\boldsymbol{w_1} \perp \boldsymbol{H_1^{[1]} v_B}, \;\; \boldsymbol{w_3} \perp \boldsymbol{H_3^{[1]} v_A}$
  - $\exists \boldsymbol{w_1}, \boldsymbol{w_2}$ due to $N \geq K$





- **Through Phase 1-3,**
  - NOMA-CB decomposes 2-cell MIMO-NOMA into 2K pairs of SISO-NOMA
- **Phase 4: Intra-Cluster Interference**
  - At user k at cell 1,

$$\mathbf{w}_k^\dagger \mathbf{y}_k = \tilde{h}_k \left(\sqrt{\lambda_{2\lceil \frac{k}{2} \rceil - 1}} s_{2\lceil \frac{k}{2} \rceil - 1} + \sqrt{\lambda_{2\lceil \frac{k}{2} \rceil}} s_{2\lceil \frac{k}{2} \rceil} \right)$$

where $\tilde{h}_k = \mathbf{w}_k^\dagger \mathbf{H}_k^{[1]} \mathbf{v}_A, k \in \{1, 2\}$

$$\tilde{h}_k = \mathbf{w}_k^\dagger \mathbf{H}_k^{[1]} \mathbf{v}_B, k \in \{3, 4\}$$

Main Result I *(Feasibility conditions)*

***Lemma 1*:** For an *L*-cell MIMO network, to simultaneously support *K* clusters per cell we must have

$$M \geq K + \Delta \ \text{ and } N \geq \max\left\{\frac{(L-1)K-\Delta}{K} M + \varepsilon, K\right\}$$

where Δ is an arbitrary between 1 and $\min\{(L-1)K, M-1\}$ and ε is an arbitrary small positive number, i.e., $0 < \epsilon \ll 1$.

Sketch of proof:

- In order to confine all $Q = (L-1)K$ interfering channels of each BS within Δ-dimentional signal space, we must have

$$\text{span}\begin{bmatrix} \boldsymbol{\tau}_1^{[\ell]} & \boldsymbol{\tau}_2^{[\ell]} & \cdots & \boldsymbol{\tau}_\Delta^{[\ell]} \end{bmatrix}$$
$$= \text{span}\left\{ [\, \mathbf{G}_2^{[1,\ell]} \ \mathbf{G}_2^{[2,\ell]} \ \cdots \ \mathbf{G}_2^{[L,\ell]} \,] \backslash \mathbf{G}_2^{[\ell,\ell]} \right\}$$

where $\mathbf{G}_2^{[\ell',\ell]} = \left[\mathbf{G}_{2,1}^{[\ell',\ell]\dagger} \mathbf{w}_{2,1}^{[\ell']} \cdots \mathbf{G}_{2,K}^{[\ell',\ell]\dagger} \mathbf{w}_{2,K}^{[\ell']} \right]$

Main Result I *(Feasibility conditions)*

Sketch of proof:

- Considering all cells in the network, we can unify a system matrix equation.
- The size of the unified matrix in general is $L(L-1)KM \times (LM\Delta + LKN)$.
- Since all the channel matrices are completely random, the unified matrix has full rank a.s.
- Thus, to guarantee the existence of null space of the matrix (inter-cell interference),

$$N > \frac{(L-1)K - \Delta}{K} M$$

- On the other hand, $N \geq K$ in order to cancel inter-cluster interference at cell-center user
- To ensure zero inter-cell and inter-cluster interference at cell-edge users, $M \geq K + \Delta$

$$v_1^{[1]} \perp \Big[\underbrace{\tau_1^{[1]} \quad \cdots \quad \tau_1^{[\Delta]}}_{\text{Inter-cell}} \quad \underbrace{w_2^{[1]\dagger} H_2^{[1]} \quad \cdots \quad w_{2(k-1)}^{[1]\dagger} H_{2(k-1)}^{[1]} \quad w_{2(k+1)}^{[1]\dagger} H_{2(k+1)}^{[1]} \quad \cdots \quad w_{2K}^{[1]\dagger} H_2^{[1]}}_{\text{Inter-cluster}}\Big]$$

Main Result II *(Analysis of # of users)*

Theorem 1: The *maximum* number of users supported by the proposed NOMA-CB scheme in an *L*-cell MIMO network with *M* transmit antennas at each BS and *N* receive antennas at each UE is given by

$$2\min\left\{\max\left\{M-\lceil\delta^*\rceil, \lfloor f(\lfloor\delta^*\rfloor)-\epsilon\rfloor, g(1)\right\}, N\right\}$$

where $\delta^* = \frac{(L-1)M^2-MN}{LM-N}$, $f(x) = \frac{N-(L-2)x+\sqrt{[N-(L-2)x]^2+4(L-1)x^2}}{2(L-1)}$, $g(x) = \min\{M-x, \lfloor f(x)-\epsilon\rfloor\}$

- **Sketch of Proof:** From the 3 conditions,

$$\frac{NK}{(L-1)K-\Delta} > M > K+\Delta$$

which results in

$$K < \frac{N-(L-2)\Delta+\sqrt{[N-(L-2)\Delta]^2-4(L-1)\Delta^2}}{2(L-1)} \triangleq f(x)$$

Main Result II *(Analysis of # of users)*

Sketch of Proof:

- The number of clusters per cell is bounded as

$$K \leq \min\{M - \Delta, f(\Delta) - \epsilon, N\}$$

- Since Δ and K must be integers, we formulate the following optimization problem:

$$\max_{\Delta} 2K = 2\max_{\Delta} \min\{M - \Delta, \lfloor f(\Delta) - \epsilon \rfloor, N\}$$

$$\text{s.t.} \quad L, M, K \geq 2$$

$$\Delta \in \{1,2, \ldots, \min\{(L-1)K, M-1\}$$

- Note that $M - \Delta$ is linearly decreasing with Δ and $f(\Delta)$ is a strictly convex, i.e.,

$$\frac{\partial^2 f(\Delta)}{\partial \Delta^2} > 0$$

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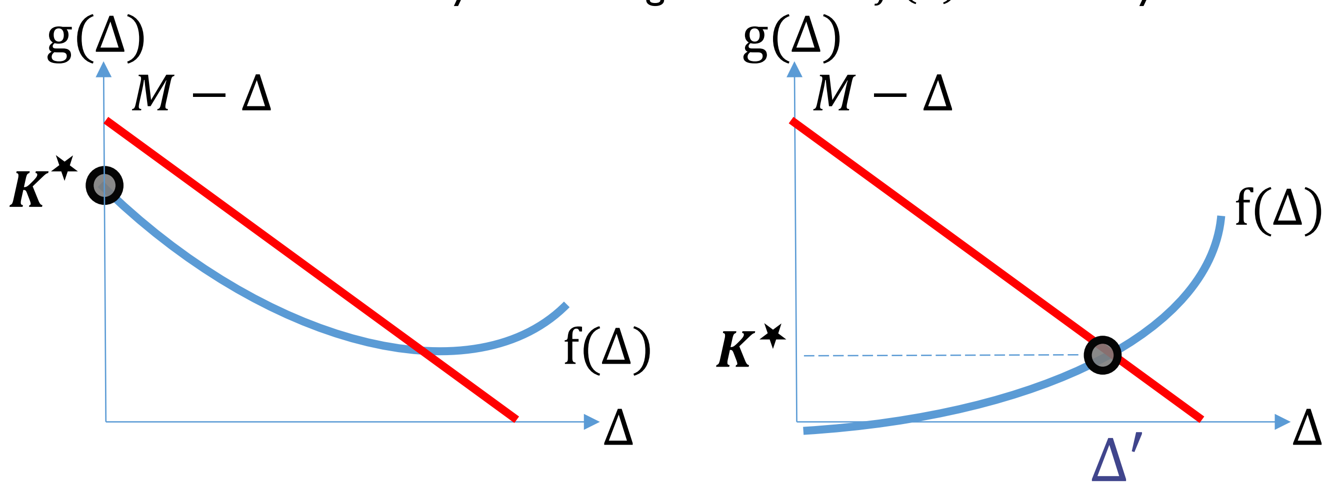
Main Result II *(Analysis of # of users)*

Sketch of Proof:

- Note that $M - \Delta$ is linearly decreasing with Δ and $f(\Delta)$ is a strictly convex.



- By considering integer constraint,

$$K^* = \min\left\{\max\left\{g(\lceil \Delta' \rceil), g(\lfloor \Delta' \rfloor), g(1)\right\}, N\right\}$$
$$= \min\left\{\max\left\{M - \lceil \delta^* \rceil, \lfloor f(\lfloor \delta^* \rfloor) - \epsilon \rfloor, g(1)\right\}, N\right\}$$
where $g(x) = \min\left\{M - x, \lfloor f(x) - \epsilon \rfloor\right\}$

Numerical Results

❑ Special Case ($L = 2, M = K + 1, N = K$)

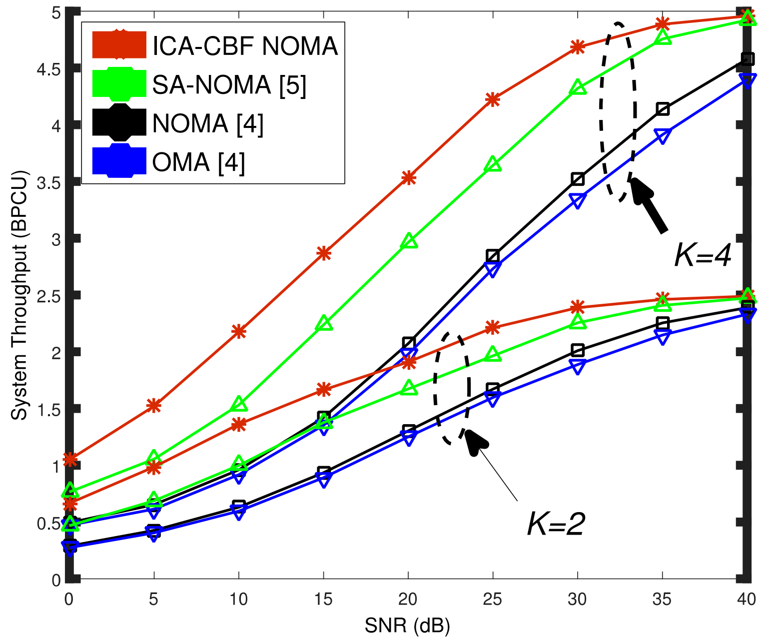
- What is maximum # of users?

$$2\min\{\max\{M - \lceil\delta^*\rceil, \lfloor f(\lfloor\delta^*\rfloor) - \epsilon\rfloor, g(1)\}, N\}$$
$$= 2\min\left\{\max\left\{K+1, \frac{K}{2}\right\}, K\right\} = 2K$$

- (Per-Cell) System Throughput

$$R = \sum_{k=1}^{K}[R_{2k-1}(1 - P_{2k-1}) + R_{2k}(1 - P_{2k})]$$

  - Users are randomly distributed
  - Compare with existing schemes
    - OMA: $(K+1)/2$ users
    - SA-NOMA [Ding-Schober-Poor'16], MIMO-NOMA[Ding-Ada-Poor'16] : K users



Proposed Scheme Achieves Better Throughput than Existing Works (Larger # of Users)

Summary

Summary

- We introduced multi-cell NOMA techniques, called NOMA-CB, which does not rely on data sharing among BSs (Less Backhaul Overhead)
- We completely analyzed the number of supported users with NOMA-CB scheme, which shows that the significant gains over existing schemes

Future Work

- NOMA-CB with imperfect CSIT (Delayed/Limited Feedback)