# Lecture 4
# Signal Classification and Filters

- Classification of Signals
- Signal Transmission via Linear Systems
- Filters

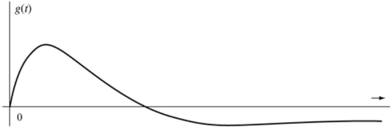
# Contents

# Classification of Signals

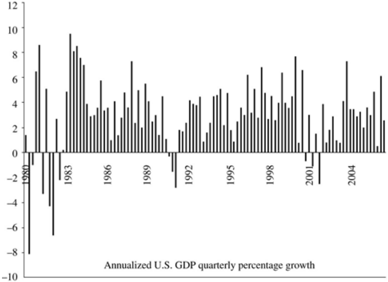
- Continuous time and discrete time signals
- Analog and digital signals
- Periodic and aperiodic signals
- Energy and power signals
- Deterministic and probabilistic signals

# Continuous vs. Discrete Time

- **Continuous-Time Signal:** is specified for every value of time $t$ (e.g., audio and video recording)
- **Discrete-Time Signal:** is specified only at discrete points of $t = nT$ (e.g., daily stocks value, quarterly GDP)



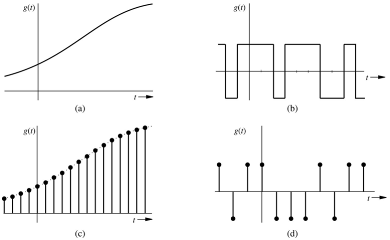
(a)



(b)

# Analog vs. Digital

- **Analog Signal:** A signals whose amplitude can have any value in a continuous range
- **Digital Signal:** A signals whose amplitude can take only a finite number of values
- (a) analog/continuous time (mathematics) (b) digital/continuous time (c) analog/discrete time (digital signal processing) (d) digital/discrete time (digital switching)




Don't mix up analog signals with continuous time signals; the two concepts are not the same.

# Energy vs. Power Signals

Measures of size of a signal

- **Signal Energy:** the energy of a signal $g(t)$ is the area under $|g(t)|^2$

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

- **Signal Power:** mean squared value of $|g(t)|$

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |g(t)|^2 dt$$

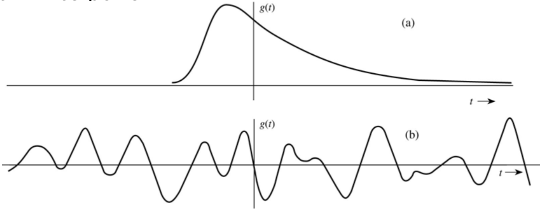
  - $\sqrt{P_g}$ is the familiar **rms** (root mean square) value
- **Energy vs. Power Signals:**
  - A signal with finite energy ($E_g < \infty$) is an *energy signal*
  - A signal with finite non-zero power ($0 < P_g < \infty$) is a *power signal*

- A signal cannot be both an energy signal and a power signal
- Certain signals are neither energy nor power signals ($P_g = \infty$).

# Example

A necessary condition for the energy to be finite is that the signal amplitude must $\to 0$ when $|t| \to \infty$
(otherwise the energy integral will not converge)

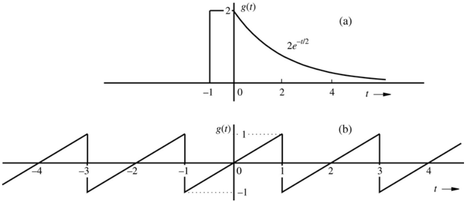
- (a) signal with finite energy
  (b) signal with finite power




- Can you come up with a neither energy and nor power signal?
  Examples: $g(t) = e^t$ and $g(t) = t$

# Example

- Determine a suitable measure (energy or power) for the below signals




- (a) $E_g = \int_{-\infty}^{\infty} g(t)^2 dt = \int_{-1}^{0} 2^2 dt + \int_{0}^{\infty} 4e^{-t} dt = 4 + 4 = 8 \Rightarrow$ energy signal
- (b) obviously not an energy signal (why?)

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |g(t)|^2 dt$$

$$= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |g(t)|^2 dt$$ because $g(t)$ is periodic

$$= \frac{1}{2} \int_{-1}^{1} t^2 dt = \frac{1}{3}$$ $\Rightarrow$ power signal

# Units of Power

1. **Watts**
   - power level may vary from megawatts (TV transmitter) to $10^{-12}$ watts (satellite receiver)
2. **dBW and dBm**
   - powers ratio can be expressed in decibels (dB). dB is logarithmic and relative to some standard power[1]

$$\boxed{\left(\frac{P}{P_0}\right)_{dB} = 10\log_{10}\left(\frac{P}{P_0}\right)}$$

   - if P is measured in watts, then
     - power in **dBW** = $10\log_{10}\left(\frac{P}{1W}\right)$ (power relative to $P_0 = 1W$)
     - power in **dBm** (or dBmW) = $10\log_{10}\left(\frac{P}{10^{-3}W}\right) = 30 + 10\log_{10} P$;
     - thus

$$\boxed{\textbf{dBm} = \textbf{dBW} + 30}$$

[1] Equivalently, number of decibels (dB) in terms of voltage is equal to $20\log_{10}(\frac{V}{V_0})$.

# Units of Power

- dBW and dBm are power units
- dB is a relative, unit-less number; it represents gain/loss in a logarithmic scale

**Examples:**

- 3dB loss means the signal strength has halved
- 10dB gain means the signal strength has increased 10 times
- **Q:** typical cell-phone sensitivity is $-107$ dBm. What is the sensitivity in watts?

$$10\log_{10}\left(\frac{P}{10^{-3}W}\right) = -107 \Rightarrow P = 1.995 \times 10^{-14}W$$

Find your-cell phone's received signal strength

- typical macro base station (cell tower) transmission power is 20W-69W. Express this in dBW and dBm. (Answer: 13-18 dBW or 43-48 dBm)

# Examples of Communication Channels

- optical fiber (attenuation 4dB/km)
- broadcast TV (50 kW transmit)
- voice telephone line (under -9 dBm or 110 $\mu$W)
- low-power not-for-profit FM (10W or 100W transmitter) (range of approximately 3.5 miles)
- walkie-talkie: 500 mW, 467 MHz
- Bluetooth: 20 dBm, 4 dBm, 0 dBm
- minimal received signal of wireless network (802.11 variants): 0.1 pW (-100 dBm)
- typical received signal power from a GPS satellite: -127.5 dBm

# Gain and Power

| Gain as a ratio | Gain in dB (= 10 $\log_{10}$[gain as ratio]) |
|---|---|
| 2 | 3 dB |
| 4 | 6 dB |
| 10 | 10 dB |
| 50 | 17 dB |
| 100 | 20 dB |
| 1000 | 30 dB |
| 0.5 (a loss of a factor of 2) | -3 dB |
| 0.1 (a loss of a factor of 10) | -10 dB |

| Power in milliwatts | Power in dBm (= 10 $\log_{10}$[power in mW]) |
|---|---|
| 2 | 3 dBm |
| 10 mW | 10 dBm |
| 100 mW | 20 dBm |
| 1000 mW | 30 dBm |
| 20 W = 20000 mW | 43 dBm |
| 1 μW | -30 dBm |

# Energy Conservation

**Parseval's Theorem:** $\int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$

- Parseval's theorem says that energy in time domain is equal to energy in the frequency domain
- **Example:**
  Find the energy of the $\mathrm{sinc}$ pulse $g(t) = 3\,\mathrm{sinc}(2t)$.

$$E = \int_{-\infty}^{\infty} 3^2 \mathrm{sinc}^2(2t)\, dt$$

$$= \left(\frac{3}{2}\right)^2 \int_{-\infty}^{\infty} \mathrm{rect}^2\left(\frac{f}{2}\right) df = \left(\frac{3}{2}\right)^2 \int_{-1}^{1} df = \frac{9}{2}$$

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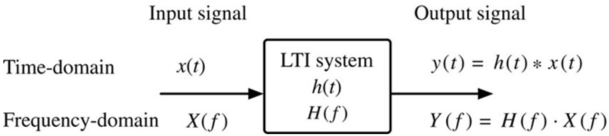
# Linear Systems

- A *linear time-invariant* (LTI) can often be used to characterize communication channels.




- A stable LTI is characterize in time domain by its *impulse response* $h(t)$
- Impulse response is the system response to a *unit impulse* input $\delta(t)$ that is $y(t) = h(t)$ when $x(t) = \delta(t)$

The system response to a bounded input $x(t)$ is the following convolution relationship: $y(t) = h(t) \star x(t)$

# Transfer Function

- $H(f)$, the FT of the impulse response $h(t)$, is generally referred to as **transfer function** or **frequency response**
- $H(f)$ is generally a complex number that can be written as

$$H(f) = |H(f)|e^{j\theta_h(f)}$$

- Signal distortion via transmission

$$Y(f) = H(f)X(f)$$
$$\Rightarrow |Y(f)|e^{j\theta_y(f)} = |H(f)|e^{j\theta_h(f)}|X(f)|e^{j\theta_x(f)}$$

or, equivalently,

$$\begin{cases} |Y(f)| & = |H(f)|\,|X(f)| \\ \theta_y(f) & = \theta_h(f) + \theta_x(f) \end{cases}$$

During transmission

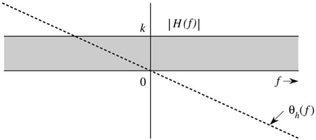
- the input amplitude spectrum $|X(f)|$ has changed to $|H(f)|\,|X(f)|$
- the input phase spectrum $\theta_x(f)$ has changed to $\theta_h(f) + \theta_x(f)$

# Distortionless Transmission

- In signal transmission over a communication channel, we require the output waveform is required to be a *replica* of the input waveform
- Transmission is said to be **distortionless** if the output has the same waveform as input except for a multiplicative constant and/or a delay

$$\boxed{y(t) = k \cdot x(t - t_d)}$$

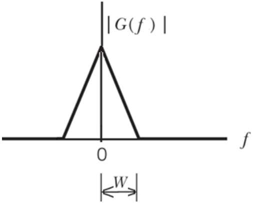
$$\Rightarrow Y(f) = kX(f)e^{-j2\pi f t_d} \quad \text{or, equivalently,} \quad \begin{cases} |H(f)| & = k \\ \theta_h(f) & = -2\pi f t_d \end{cases}$$




- Hence, the distortionless system's magnitude is constant and its phase is linear with frequency

# Low-Pass Signals

- A signal is said to be low-pass if its non-zero frequency content is limited by $|f| \leq W$, where $W$ is the signal bandwidth and is a constant.

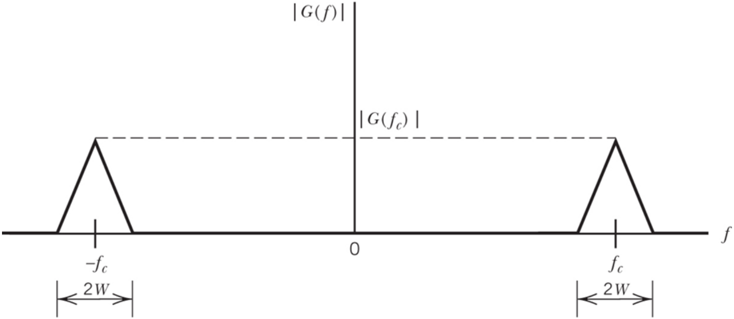



**Example**: spectrum of a low-pass signal

- A communication using low-pass signals is referred to as **baseband communication**

# Band-Pass Signals

- **Band-Pass Signal:**
  - A bandpass signal is a signal containing a band of frequencies not adjacent to zero frequency ($|f - f_c| \leq W$)
  - $f_c$ is referred to as carrier frequency
  - if the bandwidth ($2W$) is small compared to $f_c$, the signal is called *narrow-band*



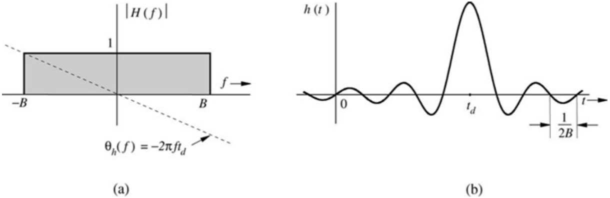

**Example**: spectrum of a band-pass signal

# Contents

# Ideal Low-Pass Filters

- *Ideal* filters allow distortionless transmission of a certain frequency band and suppress all other frequencies

  **Low-Pass Filter:**

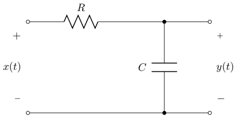



$$H(f) = |H(f)|e^{j\theta_h(f)} = \Pi\left(\frac{f}{2B}\right)e^{-j2\pi f t_d}$$

$$\Rightarrow h(t) = F^{-1}[H(f)] = 2B\mathrm{sinc}[2B(t - t_d)]$$

- Fig. (b) shows that this "ideal" filter is practically unrealizable. (why?)

# Non-Ideal Low-Pass Filters

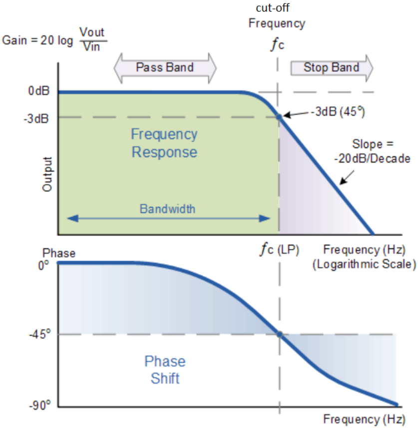



**Example:** first-order Butterworth filter

- $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1/jC\omega}{R+1/jC\omega} = \frac{1}{1+jRC\omega}$
  - magnitude of frequency response: $|H(j\omega)| = \frac{1}{\sqrt{1+(RC\omega)^2}}$
  - phase of frequency response: $\angle H(j\omega) = -\tan^{-1} RC\omega$
- for $RC\omega \ll 1$
  - $|H(j\omega)| \approx 1$
  - $\angle H(j\omega) \approx -RC\omega$
- therefore, for $RC\omega \ll 1$ (equivalently $f \ll \frac{1}{2\pi RC}$) the magnitude is constant and the phase is linear with frequency (as required in a distortionless transmission)
- for $RC\omega \ll 1$, the RC circuit approximates the ideal low-pass filter
- cut-off frequency (bandwidth) is where $|H(j\omega)| = \frac{1}{\sqrt{2}}$ (power halves)

# Frequency Response of First Order Low-Pass Filter

- Frequency response curve or Bode Plot
- $V_{in}$ and $V_{out}$ are the input (x(t)) and output (y(t))



# Filters Example

**Example:** A Low-pass filter circuit consisting of a resistor of $4.7k\Omega$ in series with a capacitor of $47nF$ is connected across a $10v$ sinusoidal supply. Calculate the output voltage at a frequency of $100Hz$ and again at frequency of $10kHz$. Which one passes the filter? What is the cut-off frequency?

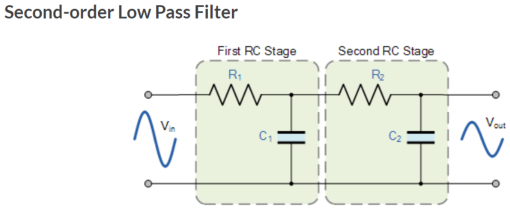
- $f = 100Hz \Rightarrow \omega RC = 2\pi fRC = 0.1388 \Rightarrow$
  $|H(j\omega)| = \frac{1}{\sqrt{1+(RC\omega)^2}} = \frac{1}{\sqrt{1+0.0193}} = 0.9905 \Rightarrow V_{out} = 0.99 * V_{in} = 9.9V$

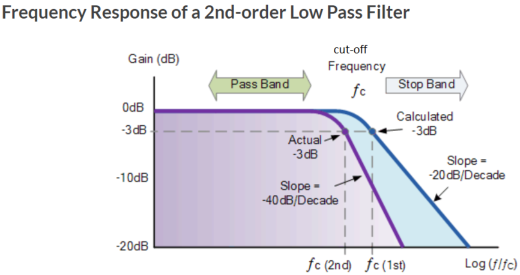
- $f = 10,000Hz \Rightarrow \omega RC = 2\pi fRC = 13.88 \Rightarrow$
  $|H(j\omega)| = \frac{1}{\sqrt{1+(RC\omega)^2}} = \frac{1}{\sqrt{1+192.64}} = 0.0719 \Rightarrow V_{out} = 0.0719 * 10 = 0.72V$

- cut-off frequency: $2\pi fRC = 1 \Rightarrow f = 720.5Hz$
- We say $100Hz$ passes the filter but $10kHz$ does not.

# Second Order Low-Pass Filter- An Example

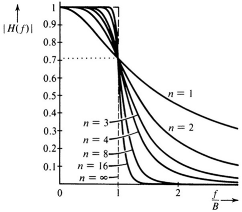
- cascaded two first-order low pass filters



# Filters

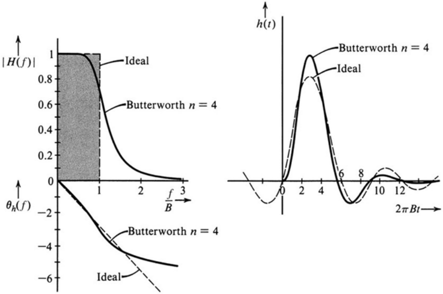
- $n$th order Butterworth filter




- Magnitude of frequency response:

$$|H(f)| = \frac{1}{\sqrt{1+(\frac{f}{B})^{2n}}}$$ where $B$ in the bandwidth

- $n = 4$ (notice the causal impulse response)

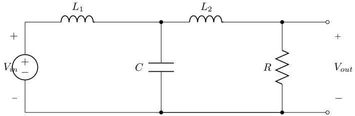


# Filters Example

**Example:** Find the transfer function of the Butterworth filter of the figure below.

- What is the order of the filter? Is it a low-pass or a high-path?
- Show that with $C = \frac{4}{3}F, R = 1\Omega, L_1 = \frac{3}{2}H$, and $L_2 = \frac{1}{2}H$ the magnitude of the frequency response is given by $|H(j\omega)| = \frac{1}{\sqrt{1+\omega^6}}$
- For what frequencies the filter is nearly ideal?

$$\frac{V_o(s)}{V_i(s)} = \frac{R}{s^3(L_1CL_2)+s^2(L_1CR)+s(L_1+L_2)+R}$$




For the given values, the transfer function simplifies to

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Then, $|H(j\omega)| = \frac{1}{\sqrt{1+\omega^6}}$ (how?)

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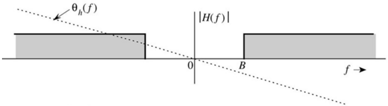
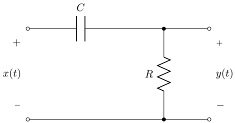
See the filter notes in blackboard.

# High-Pass Filters

- **High-Pass Filter:**
  ideal frequency response and a simple circuit



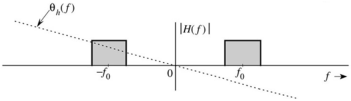

- If the phase is zero

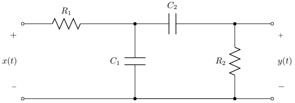
$$H(f) = 1 - \Pi\left(\frac{f}{2B}\right) \Rightarrow$$

$$h(t) = F^{-1}[H(f)] = \delta(t) - 2B\,\mathrm{sinc}[2Bt]$$

# Band-Pass Filters

- **Band-Pass Filter:**
  ideal frequency response and a simple circuit



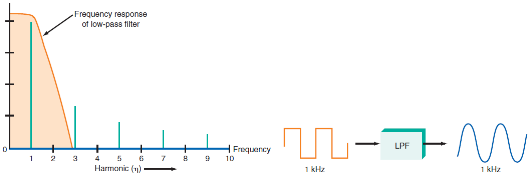

- If the phase is zero

$$h_{BP}(t) = 2\cos(2\pi f_0 t)\text{sinc}(2Bt)$$

# Filtering Examples

Converting a square wave to sine wave

- **Low-Pass Filter:** filtering out all the harmonics with a low-pass filter




- **Band-Pass Filter:** selecting the third harmonic with a bandpass filter