Lecture 5
Amplitude Modulation

- Why Modulation?
- Amplitude Modulation (AM)
- AM Modulation/Demodulation: A Deeper Look

Contents

Check Yourself

Example:

For efficient transmission and reception, the antenna length should be on the order of the wavelength.

Telephone-quality speech contains frequencies between 200 Hz and 3000 Hz.

How long should the antenna be?

1. < 1 mm
2. ~ cm
3. ~ m
4. ~ km
5. > 100 km

Check Yourself

The wavelength and frequency are related by $\lambda = c/f$, where $c = 3 \times 10^8 m/s$ is the light speed. Thus,

- The lowest frequencies (200 Hz) produce the longest wavelengths

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{200} = 1.5 \times 10^6 m = 1500 km$$

- The highest frequencies (3000 Hz) produce the shortest wavelengths

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3000} = 10^5 m = 100 km$$

On the order of hundreds of miles!

Check Yourself

Example:

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

1. $<$ 100 KHz
2. ~ 1 MHz
3. ~ 10 MHz
4. ~ 100 MHz
5. $>$ 1 GHz

- A wavelength of 10 cm corresponds to a frequency of

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{0.1m} = 3 \times 10^9 Hz = 3GHz$$

- Modern cell phones use frequencies near 2 GHz.

Check Yourself

Example:

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

1. < 100 KHz
2. ~ 1 MHz
3. ~ 10 MHz
4. ~ 100 MHz
5. > 1 GHz

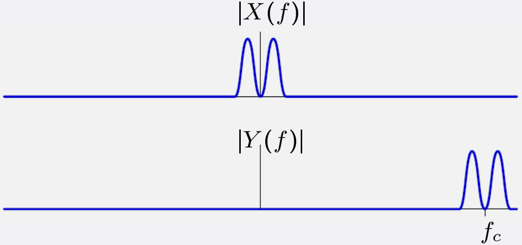
- A wavelength of 10 cm corresponds to a frequency of

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{0.1m} = 3 \times 10^9 Hz = 3GHz$$

- Modern cell phones use frequencies near 2 GHz.

Check Yourself

Example: Construct a signal Y that codes the audio frequency information in X using frequency components near $f = f_c$, e.g., 2GHz.



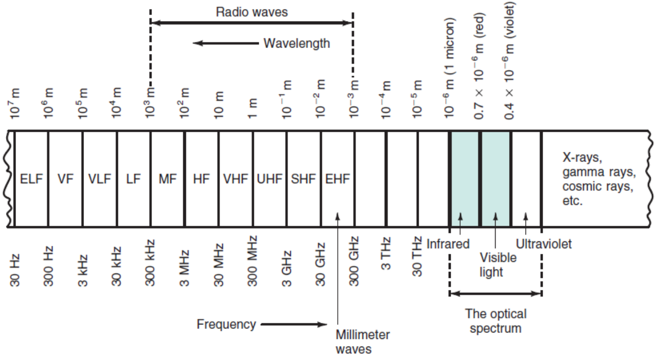
Determine the expression for Y in terms of X.

1. $y(t) = x(t)e^{j2\pi f_c t}$
2. $y(t) = x(t) \star e^{j2\pi f_c t}$
3. $y(t) = x(t)\cos(2\pi f_c t)$
4. $y(t) = x(t) \star \cos(2\pi f_c t)$

$\star$ means convolution

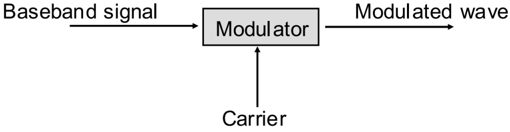
Electromagnetic Spectrum

- Speech is not well-matched to the wireless medium
- Many other applications are so (require the use of signals that are not well-matched to the required media)
- We can often **modify** the signals to obtain a better match

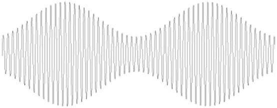


What is Modulation?

- Process of putting a baseband signal (voice or data) onto a carrier wave



- Two celebrated analog modulation techniques



Amplitude Modulation (AM)

Frequency Modulation (FM)

Why Modulation?

- **What is Modulation?** two definitions
  - The process by which some characteristic of a **carrier wave** is varied in accordance with an **information-bearing signal**
  - To transform an information signal into a signal more appropriate for the transmission on a physical medium (channel)

  In this process, a baseband signal (voice, video, or digital signal) modifies another, higher-frequency signal called the carrier, which is usually a *sinusoidal wave*.

- **Why Modulation?** There are two principal motivating reasons:
  1. To **match** the transmission characteristics of the physical medium, and considerations of power and antenna size
  2. The desire to **multiplex**, or share, a communication medium among many concurrently active users

Continuous-Wave Modulation

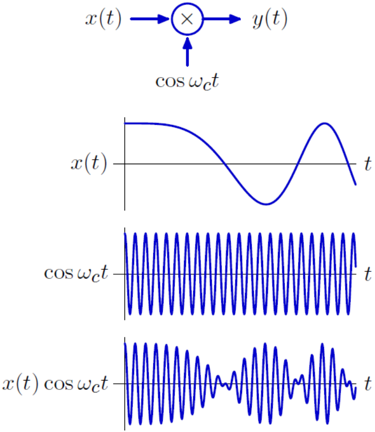
When the **information-bearing signal is of an analog** kind, we speak of continuous-wave (CW) modulation, a term that stresses continuity of the carrier wave as a function of time.

- A commonly used carrier is a sinusoidal wave
- CW modulation is made possible by varying the **amplitude** or **angle** of the sinusoidal carrier wave
- **CW Modulation:** is classified into two broadly defined families:
  - Amplitude Modulation (chapter 3)
  - Angle Modulation (chapter 4)

Contents

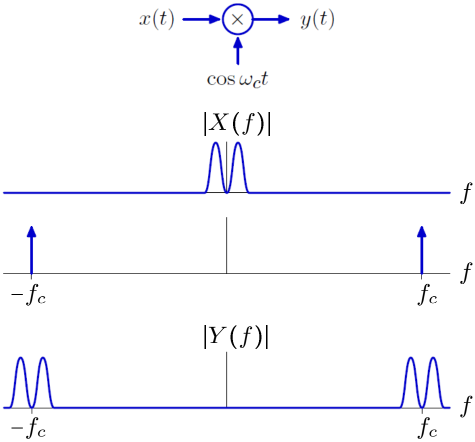
Amplitude Modulation (AM)

- **Amplitude Modulation:** Multiplying a signal by a sinusoidal carrier signal is called amplitude modulation (AM). The signal *modulates* the amplitude of the carrier.



Amplitude Modulation (AM)

- AM shifts the frequency components of X by $+f_c$ and $-f_c$

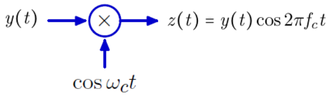


Demodulation

How can we recover $x(t)$ from $y(t)$?

Such a process is called **demodulation**.
A circuit that accepts the modulated signal and recovers the original modulating information is demodulator.

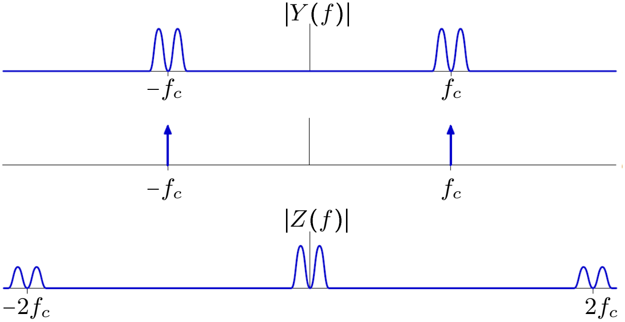
- X can be recovered by multiplying by the carrier wave and then low-pass filtering. This process is called synchronous demodulation.



$$
\begin{aligned}
z(t) &\triangleq y(t)\cos 2\pi f_c t \\
&= x(t)\cos 2\pi f_c t \times \cos 2\pi f_c t \\
&= x(t)\left(\frac{1}{2} + \frac{1}{2}\cos 4\pi f_c t\right)
\end{aligned}
$$

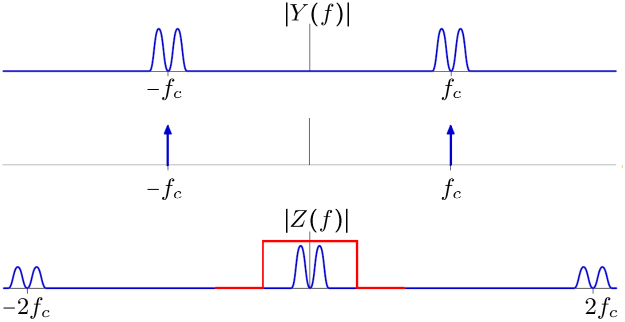
Synchronous Demodulation

- Synchronous demodulation shifts the frequency components of Y by $+f_c$ and $-f_c$



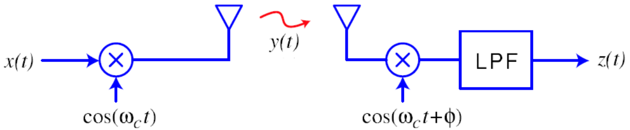
Synchronous Demodulation

- Now, all we need to do is a low-pass filtering to throw away the components at $\pm 2f_c$



Modulator/Demodulator

The demodulator is the key circuit in any radio receiver. In fact, demodulator circuits can be used alone as simple radio receivers.

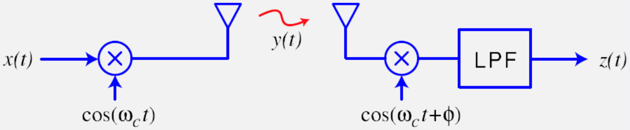


The problem with making an inexpensive radio receiver is that you must know the carrier signal exactly!

Check Yourself

Example:

What happens if there is a phase shift ϕ between the signal used to modulate and that used to demodulate?



$$y(t)\cos(\omega_c t + \phi) = x(t)\cos\omega_c t \;\times\; \cos(\omega_c t + \phi)$$
$$= \frac{1}{2}x(t)\left[\cos\phi + \cos(2\omega_c t + \phi)\right]$$

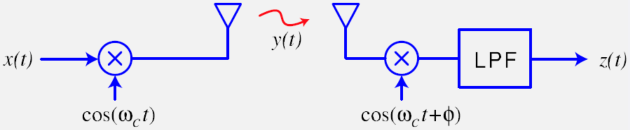
Passing y(t) through a low pass filter yields $z(t) = \frac{1}{2}x(t)\cos\phi$.

- If ϕ changes with time, then the signal *fades*.
- Specifically, when $\phi = \frac{\pi}{2}$ the output is zero!

Check Yourself

Example:

What happens if there is a phase shift ϕ between the signal used to modulate and that used to demodulate?



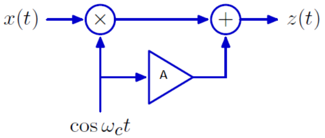
$$y(t)\cos(\omega_c t+\phi) = x(t)\cos\omega_c t \;\times\; \cos(\omega_c t+\phi)$$
$$= \frac{1}{2}x(t)\left[\cos\phi + \cos(2\omega_c t+\phi)\right]$$

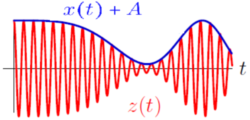
Passing y(t) through a low pass filter yields $z(t) = \frac{1}{2}x(t)\cos\phi$.

- If ϕ changes with time, then the signal *fades*.
- Specifically, when $\phi = \frac{\pi}{2}$ the output is zero!

AM with Carrier

One way to synchronize the sender and receiver is to send the carrier along with the message.

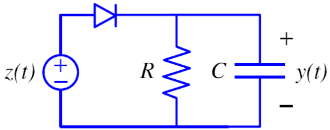


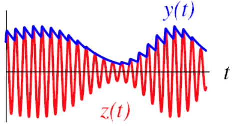


- $z(t) = x(t)\cos\omega_c t + A\cos\omega_c t = (x(t) + A)\cos\omega_c t$
- Adding carrier is equivalent to shifting the DC value of $x(t)$
- If we shift the DC value sufficiently, the message is easy to decode: it is just the envelope (minus the DC shift).

Inexpensive Radio Receiver

If the carrier frequency f_c is much greater than the highest frequency in the message, AM with carrier can be demodulated using an **envelope detector**.

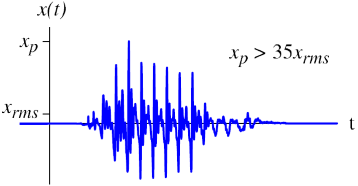




In AM radio, the highest frequency in the message is 5 kHz and the carrier frequency is between 500 kHz and 1500 kHz. This circuit is simple and inexpensive. *But there is a problem!*

Inexpensive Radio Receiver

AM with carrier requires much more power to transmit the carrier than to transmit the message!



- Speech sounds have high *crest factors* (peak value divided by rms value).
- For envelope detection to work, the DC offset A must be larger than x_p.
- If we shift the DC value sufficiently, the message is easy to decode: it is just the envelope (minus the DC shift).
- The power needed to transmit the carrier can be $35^2 \approx 1000$ time that needed to transmit the message.

- Okay for broadcast radio (WBZ: 50 kwatts). - Not for point-to-point (cell phone batteries wouldn't last long!).

Contents

CW Modulation Schemes

- **CW Modulation:** is classified into two broadly defined families:
  - Amplitude Modulation (chapter 3)
    - Amplitude modulation (AM)
    - Double sideband-suppressed carrier (DSB-SC)
    - Single sideband (SSB)
    - Vestigial sideband (VSB)
  - Angle Modulation (chapter 4)
    - Frequency modulation (FM)
    - Phase modulation (PM)
- Amplitude modulation family are also called ***linear modulation*** strategies

> Why linear? They obey superposition! (modulated signal of $x_1 + x_2$ is sum of modulated signals of x_1 and x_2)

Amplitude Modulation

- Consider a *sinusoidal carrier* wave defined by

$$c(t) = A_c \cos 2\pi f_c t$$

in which A_c is the carrier amplitude and f_c is the carrier frequency.

- Let $m(t)$ be the information-bearing signal (message signal)

Let W be the highest frequency component of the message signal $m(t)$. W is called the **message bandwidth**.

- AM wave usually is described as a function of time as follows:

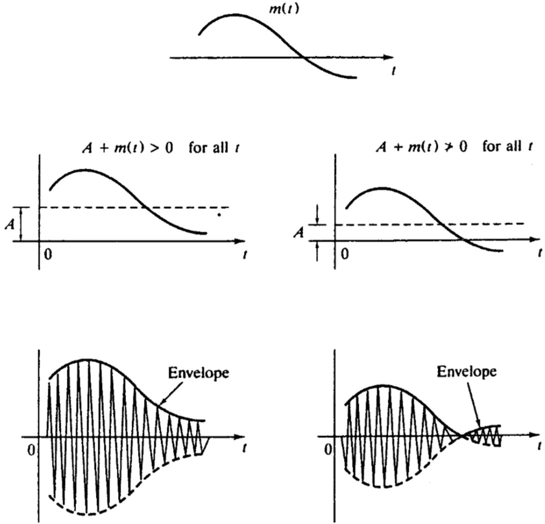
$$\boxed{s_{AM}(t) = [A_c + m(t)] \cos \omega_c t}$$

Amplitude Modulation

- The envelope of $s_{AM}(t)$ has essentially the same shape as $m(t)$ provided that
  1. $A_c + m(t) \geq 0 \quad \forall t$,
     Otherwise phase reversals occur and envelope is not going to be proportional to the message
  2. $f_c \gg W$ (the carrier frequency is much greater than the message bandwidth)

Amplitude Demodulation

- For AM signal $s_{AM}(t) = [A + m(t)] \cos \omega_c t$, envelope is similar to message $m(t)$ only if $A + m(t) \geq 0$

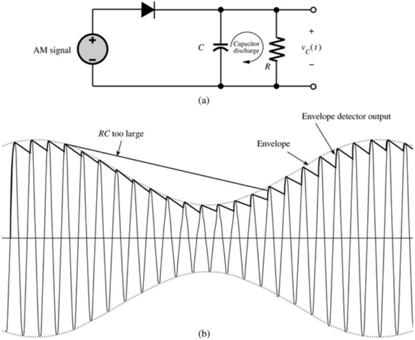


Demodulation of AM Signal

- AM signal can be demodulated using a very simple scheme know as envelope detector, if sufficient carrier power is transmitted
- Let AM signal has the form $s_{AM}(t) = [A_c + m(t)] \cos \omega_c t$.
- When A_c is large enough such that $A_c + m(t) \geq 0$ for all t, then the envelope (amplitude) of modulated signal, given by $A_c + m(t)$, will be proportional to $m(t)$
- Otherwise, if $A_c + m(t) < 0$ for some t, the envelope will not be proportional to $m(t)$ (see figures in the previous page). In such a case, phase reversal occurs.
- So to have the luxury of simple envelope detector, we need to keep A_c large, i.e., transmit high carrier power

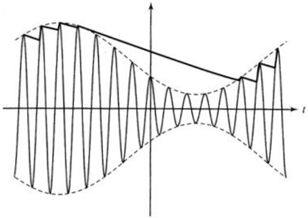
AM Demodulator

- How does this simple demodulator (envelope detector) work?
- What happens when RC is too large or too small?



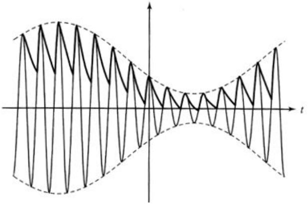
AM Demodulator

- The discharge time constant RC is critical to follow the variation in the envelope of the modulated signal
  - **If** RC **is too large** the discharge of the capacitor is too slow and the output will not follow the variation in the envelope
  - **If** RC **is too small** the discharge of the capacitor is too fast and the output will have big ripples



- For good performance of envelop detector we should have

$$\frac{1}{f_c} \ll RC \ll \frac{1}{W}$$



Example: AM Demodulator

Example: Find a good range for RC when

$W = 5$kHz and $540 \leq f_c \leq 1600$ kHz

Answer: $18.5\ \mu s \leq RC \leq 20 \mu s$

AM Signal - A Slightly Different Presentation

- An AM wave can also be described as

$$\boxed{s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t}$$

  where k_a is a constant (design parameter) called the amplitude sensitivity of the modulator.

- Then, the envelope of $s(t)$ will have the same shape as $m(t)$ provided that
  1. $1 + k_a m(t) \geq 0 \quad \forall t$, or $|k_a m(t)| \leq 1 \quad \forall t$
  2. $f_c \gg W$ (W is the message bandwidth)

- k_a is introduced (designed) to make $1 + k_a m(t)$ positive for all t.

Example: AM Demodulator

Example: If the message is a square wave with magnitude ± 5 voltes, what is the acceptable range of k_a for an AM modulator?

Answer: To have $1 + k_a m(t) \geq 0 \quad \forall t$, we need $1 - 5k_a \geq 0$, i.e. $k_a \leq 0.2$. Thus, $\boxed{0 < k_a \leq 0.2}$ gives the acceptable range.

Q1: What happens for $k_a = 0$?

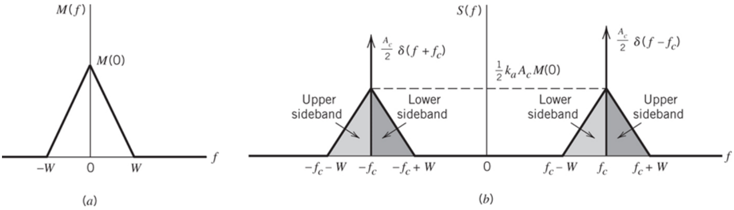
Q2: What happens for $k_a > 0.2$?

AM Spectrum

- What are the highest and lowest frequency components of the AM wave (for positive frequencies)?

  - **Transmission bandwidth**: the difference between the highest and lowest frequency components of the modulated wave ($B_T = 2W$)
  - **Message bandwidth** (W): the highest frequency component of the message signal

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$



Blank Page

Example: Single-Tone Modulation

Consider a modulating wave that consists of a single tone or frequency component; that is, $m(t) = A_m \cos 2\pi f_m t$

- The corresponding AM wave is thus given by

$$s(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t,$$

  where $\mu \triangleq k_a A_m$.

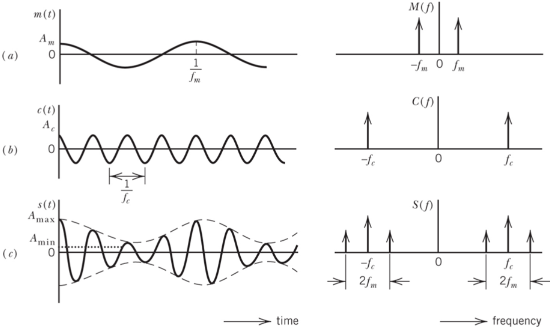
- The dimensionless constant μ is called the modulation factor
- To avoid envelope distortion, the modulation factor μ must be kept below unity. (otherwise, $1 + \mu \cos 2\pi f_m t$ could be negative and makes distortion)

Example: Single-Tone Modulation

We can write

$$s(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} [\cos 2\pi (f_c - f_m)t + \cos 2\pi (f_c + f_m)t]$$

$$\begin{aligned} S(f) = & \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] \\ & + \frac{\mu A_c}{4} [\delta(f - f_c + f_m) + \delta(f + f_c - f_m)] \\ & + \frac{\mu A_c}{4} [\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] \end{aligned}$$



Example: Single-Tone Modulation

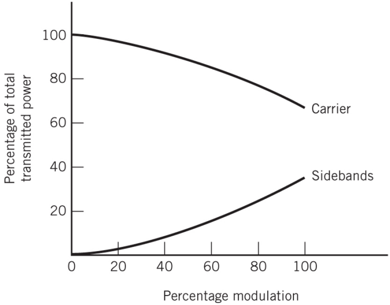
- The power of $A\cos(2\pi f_0 t + \phi)$ is $\frac{A^2}{2}$ for any ϕ and $f_0 \neq 0$.

- carrier power = $\frac{1}{T}\int_0^T A_c^2 \cos^2 2\pi f_c t \, dt = \frac{1}{2}A_c^2$
- upper-side power = $\frac{1}{T}\int_0^T (\frac{\mu A_c}{2})^2 \cos^2 2\pi(f_c + f_m)t \, dt = \frac{1}{8}\mu^2 A_c^2$
- lower-side power = $\frac{1}{T}\int_0^T (\frac{\mu A_c}{2})^2 \cos^2 2\pi(f_c - f_m)t \, dt = \frac{1}{8}\mu^2 A_c^2$
- message power = upper-side power + lowers-side power

$$= \frac{1}{8}\mu^2 A_c^2 + \frac{1}{8}\mu^2 A_c^2 = \frac{1}{4}\mu^2 A_c^2$$

- total power = carrier power + message power = $\frac{1}{2}A_c^2 + \frac{1}{4}\mu^2 A_c^2$
- power efficiency = $\frac{\text{message power}}{\text{total power}} = \frac{\frac{1}{4}\mu^2 A_c^2}{\frac{1}{2}A_c^2 + \frac{1}{4}\mu^2 A_c^2} = \frac{\mu^2}{2+\mu^2}$
- Note that $\frac{A_{\max}}{A_{\min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)} \Longrightarrow \mu = \frac{A_{\max}-A_{\min}}{A_{\max}+A_{\min}}$

Example: Single-Tone Modulation



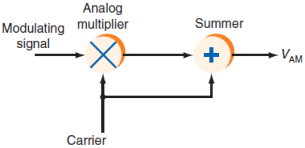
- power efficiency = $\frac{\text{message power}}{\text{total power}} = \frac{\mu^2}{2+\mu^2}$
- 100% modulation $(\mu = 1) \Rightarrow$ power efficiency is $\frac{1}{3} = 33.3\%$
- less than 20% modulation $(\mu \leq 0.20) \Rightarrow$ power efficiency $< 1\%$

- AM is not power efficient.
- AM is not bandwidth efficient.

Blank Page

AM Modulator: Circuitry

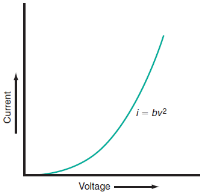
- Examining the basic equation for an AM signal gives us clues as to how AM can be generated
- Clearly, that we need a circuit that can **multiply** the carrier by the modulating signal and then **add** the carrier



- One way to do this is to develop a circuit whose gain is a function of (e.g., square) the input signals

AM Modulator: Circuitry

- A *square-law* function is one that varies in proportion to the square of the input signals
- A square-law circuit for producing AM

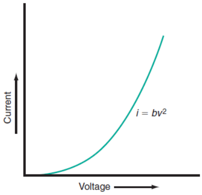


What element gives a good approximation of a square-law response?

- The current variation in a typical **diode** can be approximated as $i = av + bv^2$

AM Modulator: Circuitry

- A *square-law* function is one that varies in proportion to the square of the input signals
- A square-law circuit for producing AM



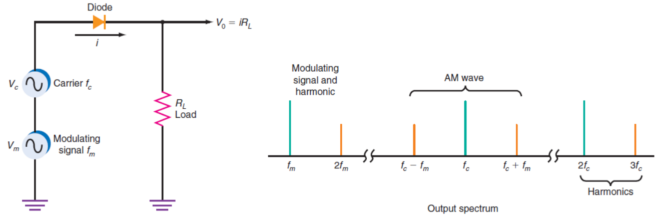
What element gives a good approximation of a square-law response?

- The current variation in a typical **diode** can be approximated as $i = av + bv^2$

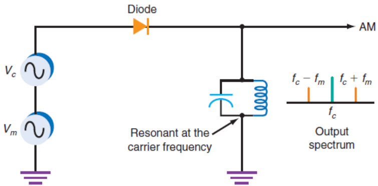
Appendix

AM Modulator: Circuitry

- A square-law circuit for producing AM

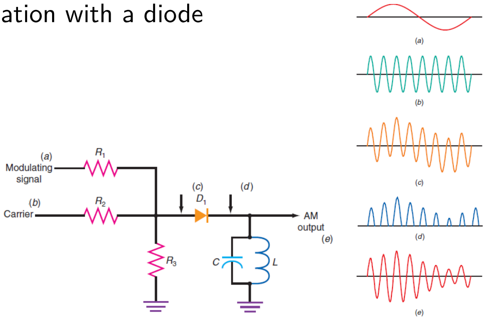


- The tuned circuit filters out the modulating signal and carrier harmonics, leaving only the carrier and sidebands

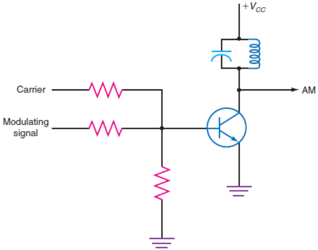


AM Modulator: Circuitry

- Amplitude modulation with a diode



- Simple transistor modulator



AM Modulator: Circuitry

- Differential amplifier modulator