Lecture 6
Amplitude Modulation II

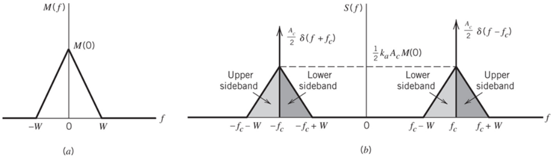
- AM Limitations
- DSB-SC Modulation
- SSB Modulation
- VSB Modulation
- Multiplexing
- Summary

Contents

AM Limitations

Amplitude modulation is the oldest method of performing modulation.

- **Advantages**:
  - AM's biggest advantage is the ease of generation and demodulation
  - AM system is relatively inexpensive to build
- **Disadvantages**:
  - AM is wasteful of power
    - The carrier wave $c(t)$ is completely independent of the message signal; its transmission therefore is a waste of power
  - AM is wasteful of bandwidth
    - Due to symmetry, the upper and lower sidebands of an AM wave are uniquely related, and the amplitude and phase of either sideband can be uniquely determined from the other sideband



AM Modifications

- How to make AM more efficient?
  1. Remove the carrier
     - Double sideband-suppressed carrier (DSB-SC)
  2. Transmit only one sideband
     - Single sideband (SSB)
     - Vestigial sideband (VSB) (more than one, but less than two sidebands)
- What is the cost?
  - Increased system complexity and cost

We *trade off* system complexity for improved utilization of communication resources.

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DSB-SC Modulation

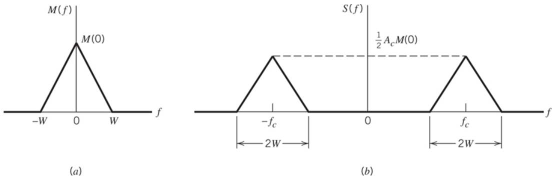
- Double sideband-suppressed carrier (DSB-SC) modulation consists of the product of the message signal and the carrier wave
- Considering the *sinusoidal carrier* wave

$$c(t) = A_c \cos 2\pi f_c t$$

the DSB-SC wave thus can be described as follows:

$$\boxed{s(t) = A_c m(t) \cos 2\pi f_c t}$$

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$



Check Yourself

Question: Why did we have the carrier at all?

A: To avoid phase reversal so that we could use simple envelope detector!

- A DSB-SC modulated signal $s(t)$ undergoes a phase reversal whenever the message signal crosses zero
- Then, the envelope of a DSB-SC modulated signal will be different from the message signal
- This means that simple demodulation using an envelope detection is not a viable option for DSB-SC modulation

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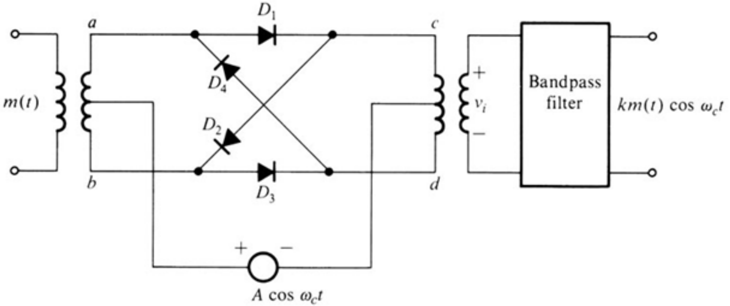
Ring Modulator

How can we multiply (modulate) two waves?



- **Positive half-cycle of the carrier:** diodes D1 and D3 conduct, and D2 and D4 are open, and $v_i(t) = m(t)$
- **Negative half-cycle:** diodes D1 and D3 are open, and D2 and D4 are conducting and $v_i(t) = -m(t)$ (note that terminal a is connected to d and terminal b is connected to c).

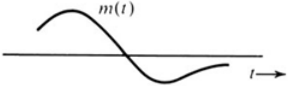
Ring Modulator

- Effectively, $m(t)$ is multiplied by a square pulse train $w_0(t)$
- The Fourier series of $w_0(t)$ is given as

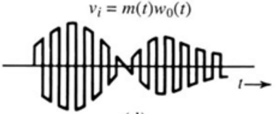
$$w_0(t) = \frac{4}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \cdots\right]$$

- $v_i(t)$ is then passed through a bandpass filter tuned to ω_c to give the modulated signal $s(t) = \frac{4}{\pi}m(t)\cos\omega_c t$

(b)



(c)

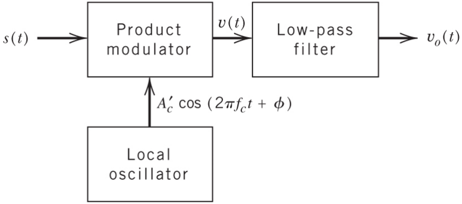


(d)

Synchronous Demodulation

How can we demodulate DSB-SC?

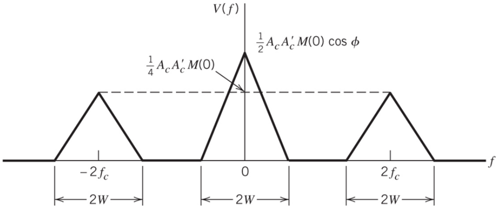
- **Synchronous demodulation** is also called **coherent detection** or



Why coherent? The receiver (demodulator) must know the carrier signal's both frequency (f_c) and phase (ϕ).

$$v(t) = s(t) \times A_c' \cos(\omega_c t + \phi) = A_c m(t) \cos \omega_c t \times A_c' \cos(\omega_c t + \phi)$$
$$= \frac{1}{2} A_c A_c' m(t) \cos \phi + \frac{1}{2} A_c A_c' m(t) \cos(2\omega_c t + \phi)$$

Synchronous Demodulation



- Passing $v(t)$ through a low pass filter yields

$$\boxed{v_o(t) = \frac{1}{2}A_cA_c'\cos\phi\, m(t)}$$

- If ϕ is constant (but $\phi \neq \frac{\pi}{2}$) the detector's output is an undistorted version of the transmitted message.
- If ϕ changes with time, the detector's output will changed randomly in time, which is not desired $\Rightarrow$ we need another receiver (see Costas Receiver)

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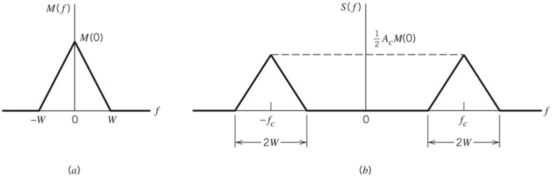
SSB Modulation

- DSB-SC improves the power efficiency (how?) but suffers from the spectral (bandwidth) efficiency
- Why DSB is not bandwidth efficient?
  - Baseband message $m(t)$ has a bandwidth of W while modulated message requires twice bandwidth $(2W)$ for transition

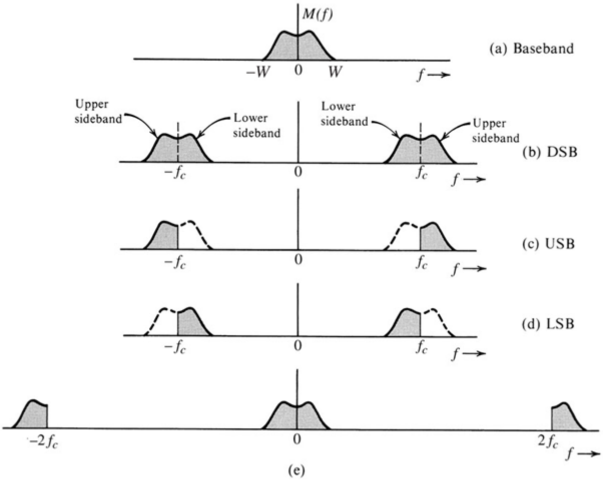


- **Question:** Any bandwidth efficient solutions?

1. **Single-sideband (SSB) modulation**: which removes either the lower or the upper sideband, i.e., transmits over bandwidth W
2. **Quadrature amplitude modulation (QAM)**: which sends two messages over bandwidth $2W$

SSB Modulation

- **Single-sideband (SSB):** transmit only one sideband either the lower or the higher. **LSB:** lower sideband **USB:** upper sideband



How to Generate SSB Signals?

1. Using Hilbert transform
2. Using selective filtering

Hilbert Transform

- The Hilbert transform of $x(t)$ (denoted by $x_h(t)$) is defined as

$$\boxed{x_h(t) = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau}$$

- Hilbert transform of a time-domain signal is another a time-domain signal (so, it is not transforming the domain)

- $$\boxed{X_h(f) = -j\,\mathrm{sign(f)X(f)}}$$

  (we know that $\mathrm{sign}(t) \rightleftharpoons \frac{1}{j\pi f}$. Using duality we get $\frac{1}{\pi t} \rightleftharpoons -j\mathrm{sign}(f)$)

- if $x(t)$ passes through a transfer function $H(f) = -j\mathrm{sign(f)}$, then the output is $x_h(t)$, the Hilbert transform of $x(t)$

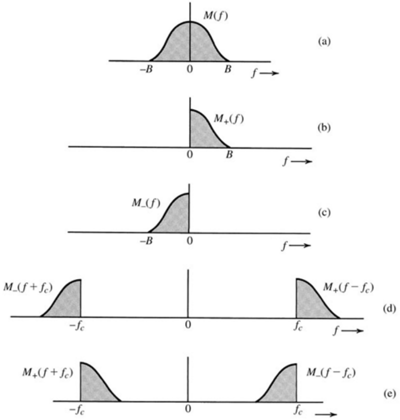
$$H(f) = -j\,\mathrm{sign(f)} = \begin{cases} -j = e^{-j\frac{\pi}{2}}, & f > 0 \\ +j = e^{j\frac{\pi}{2}}, & f < 0 \end{cases}$$

Hilbert Transform Example

Example: Show that the Hilbert transform of $\sin \omega_0 t$ is $-\cos \omega_0 t$ whereas the Hilbert transform $\cos \omega_0 t$ is $\sin \omega_0 t$

SSB in Frequency Domain

- $M_+(f) = M(f)u(f) = M(f) \times \frac{1}{2}[1 + \text{sign}(f)] = \frac{1}{2}[M(f) + jM_h(f)]$
- $M_-(f) = M(f)u(-f) = M(f) \times \frac{1}{2}[1 - \text{sign}(f)] = \frac{1}{2}[M(f) - jM_h(f)]$



SSB in Time Domain

- From Figure (d), we can see that **in frequency domain**

$$
\begin{aligned}
S_{\text{USB}}(f) &= M_+(f-f_c) + M_-(f+f_c) \\
&= \frac{1}{2}[M(f-f_c) + jM_h(f-f_c)] + \frac{1}{2}[M(f+f_c) - jM_h(f+f_c)] \\
&= \frac{1}{2}[M(f-f_c) + M(f+f_c)] - \frac{1}{2j}[M_h(f-f_c) - M_h(f+f_c)]
\end{aligned}
$$

- Thus the modulated signal **in time domain** can be constructed by

$$\boxed{s_{\text{USB}}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t}$$

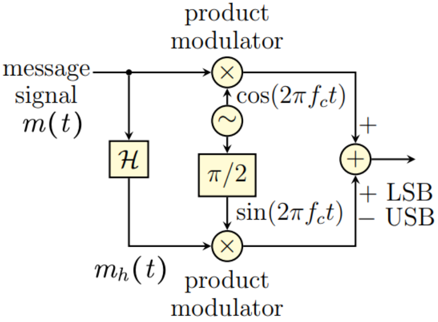
and similarly from Fig. (e) we get

$$\boxed{s_{\text{LSB}}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t}$$

  - $m(t)$ is the original real message signal, and $m_h(t)$, the **Hilbert transform** of that, is another real signal

SSB Modulation System

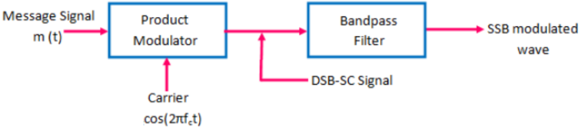
- Block diagram of SSB modulator

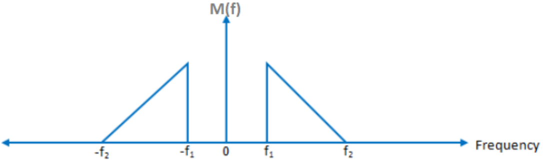


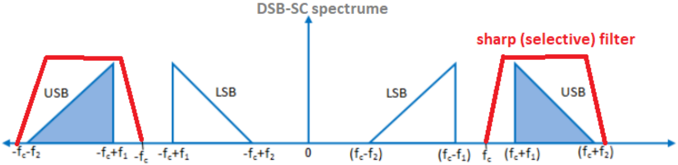
- The box $\mathcal{H}$ (Hilbert transformer) cab be implemented by a *phase shifter* which delays the phase of every positive spectral component by $\frac{\pi}{2}$

SSB Modulation-2nd Approach

- The second and the most commonly used method of generating SSB signals is *selective-filtering method* in which a DSB-SC signal is passed through a sharp cutoff filter to eliminate the undesired sideband.

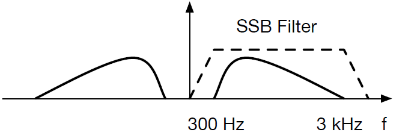






Which Signals and Which SSB Sideband?

SSB works satisfactorily for message with **an energy gap centered around zero frequency** (e.g., speech signal)



Which SSB Sideband?

- Transmitter and receiver must agree on use of LSB vs. USB
- SSB is common for *amateur radio*
  - Below 10 MHz: LSB
  - Above 10 MHz: USB
  - There are some exceptions
- SSB is also common for *shortwave radio* (3–30 MHz (100 to 10 metres))

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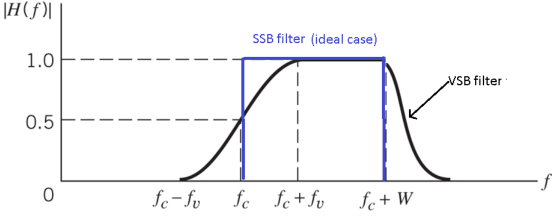
Vestigial Sideband (VSB) Modulation - Motivation

- SSB relies on being able to filter out one sideband and keep the other
- For audio this is possible because the voice spectrum drops off below 300 Hz, allowing space for a transition band (see the previous slide)
- This is not possible for other signals, like video, that have strong components at low frequencies (i.e., we will need an ideal filter)
- DSB-SC fits the characteristics of video but it is not bandwidth efficient
- To overcome these two practical limitations, we need a compromise method of modulation
- The solution is **Vestigial Sideband (VSB) Modulation**

VSB is a compromise method of modulation that lies somewhere between SSB and DSB-SC in its spectral characteristics

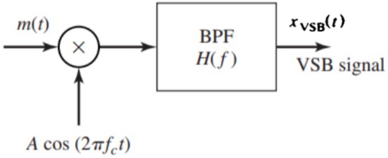
Vestigial Sideband Modulation (VSB)

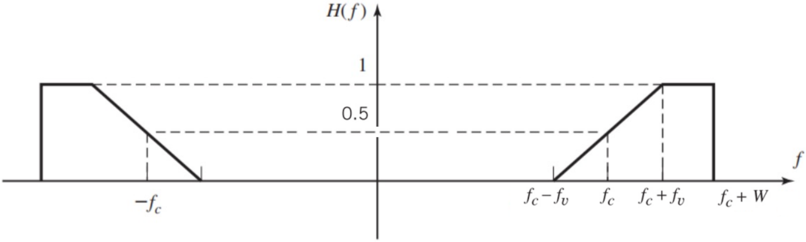
Vestigial Sideband (VSB) Modulation: a small portion (a *vestige*) of the unneeded sideband is used. This reduces DC distortion.



- $\boxed{B_T = f_v + W}$ where f_v is the vestige bandwidth and W is the message bandwidth. Typically, is 25% percent of W.
- VSB signals are generated using
  - standard AM or DSB-SC modulation
  - then passing modulated signal through a sideband shaping filter
- Demodulation uses either standard AM or DSB-SC demodulation, depending on whether or not a carrier tone is transmitted

VSB Modulator and Ideal VSB Filter

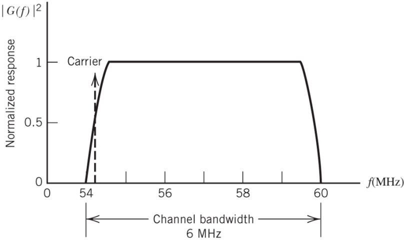




VSB in Commercial Systems

VSB modulation with envelope detection is used to modulate **image** in analog TV signals. (The audio signal is modulated using FM.)

- Channel bandwidth for TV broadcasting in Northern America is 6MHz
- The choice of VSB for analog TV was influenced by
  - Image signal has large BW with significant low frequency content
  - The circuitry used for VSB is simple (envelope detection) than SBB
- VSB is also applied to digital signals for the same reasons



Amplitude response for VSB modulated digital TV signal

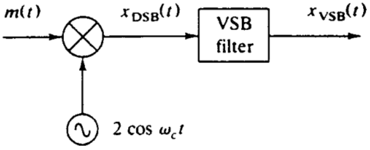
VSB Filter/Modulator/Demodulator

For distortionless demodulation of a VSB signal, the frequency response $H(f)$ of the VSB filter must satisfy

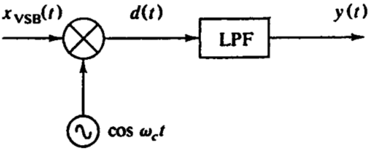
$$H(f - f_c) + H(f + f_c) = \text{constant} \qquad for \ |f| \leq W,$$

where W is the message bandwidth.

Proof: HW question!



(a) Modulator



(b) Demodulator

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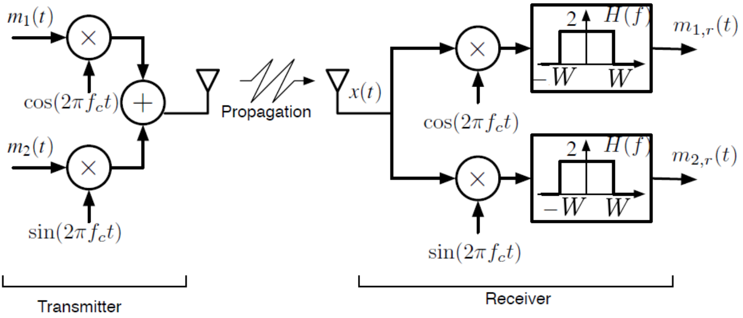
Quadrature Amplitude Modulation (QAM)
(2nd BW Efficient Transmission)

- DSB-SC modulates a message signal with bandwidth W to a transmitted signal with bandwidth $2W$
- SSB reduces the transmitted bandwidth to W, but
  - requires more complex modulator
  - reduces SNR (for a fixed carrier amplitude)
- Quadrature amplitude modulation uses the $2W$ transmitter bandwidth to send two independent signals:
  - QAM has the same spectral efficiency as SSB
  - but it does not need sharp bandpass filters

> QAM is used in almost all digital communication methods, including telephone modems, cable TV, satellite TV

QAM Modulator/Demodulator



- Two real messages: $m_1(t)$ and $m_2(t)$
- $m_1(t)$ is modulated on a cosine and $m_2(t)$ is modulated on a sine

QAM Modulator/Demodulator

Exercise: in the previous figure, write down the $x(t)$, and signals before each filter.

- $x(t)$ = ...
- top filter's input = ...
- bottom filter's input = ...
- $m_{1,r}(t)$ = ...
- $m_{2,r}(t)$ = ...

Question: What happens with a $90°$ phase shift at the receiver oscillators?

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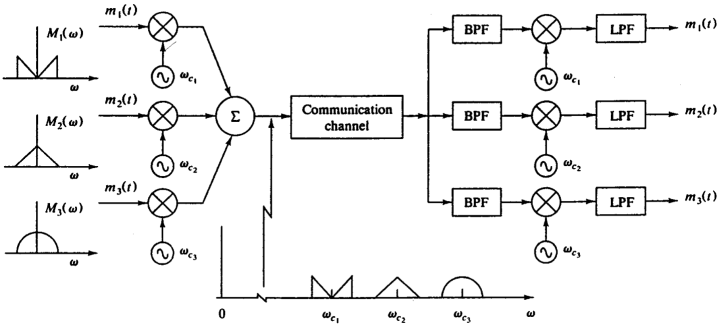
Frequency Division Multiplexing (FDM)

- Multiplexing is another motivation for modulation



Contents

Amplitude Modulation Summary

- Different ways to encode information in amplitude.
  Let $m(t)$ be the message
  - AM: $s(t) = [1 + k_a m(t)] \cos 2\pi f_c t$
  - DSB-SC $s(t) = m(t) \cos 2\pi f_c t$
  - SSB $s(t) = m(t) \cos 2\pi f_c t \pm m_h(t) \sin 2\pi f_c t$ (- for USB, + for LSB)
  - VSB $s(t)$ depends on the VSB filter
  - QAM $s(t) = m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t$
- Common issues
  - Synchronization
  - Bandwidth

Amplitude Modulation Lessons

The study of linear modulation schemes will teach us the following lessons:

- Fourier analysis provides a powerful tool for developing mathematical and physical insight into the spectral characterization of linear modulation strategies.
- The implementation of analog communications is significantly simplified by using AM, at the expense of transmitted power and channel bandwidth.
- The utilization of transmitted power and channel bandwidth is improved through well-defined modifications of an amplitude-modulated wave's spectral content at the expense of increased system complexity.

Next lecture: Encoding information in frequency or phase (FM, PM)