Lecture 7
Angle Modulation I

- Angle Modulation

- Summary

Contents

CW Modulation Schemes

- **CW Modulation:** is classified into two broadly defined families:
  - Amplitude Modulation (chapter 3)
    - Amplitude modulation (AM)
    - Double sideband-suppressed carrier (DSB-SC)
    - Single sideband (SSB)
    - Vestigial sideband (VSB)
  - Angle Modulation (chapter 4)
    - Frequency Modulation (FM)
    - Phase modulation (PM)

AM/PM/FM: What is the Difference?

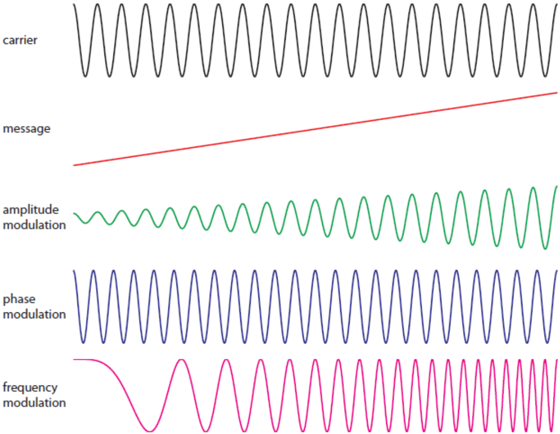
- In AM, **amplitude** of carrier is varied linearly with message.
- In FM, **frequency** of carrier is varied linearly with message.
- In PM, **phase** of carrier is varied linearly with message.
  (or equivalently, frequency of carrier is varied with the derivative of the message)

- Unlike amplitude modulation, angle modulation schemes are ***nonlinear***

Why nonlinear? They do not obey superposition, i.e.,

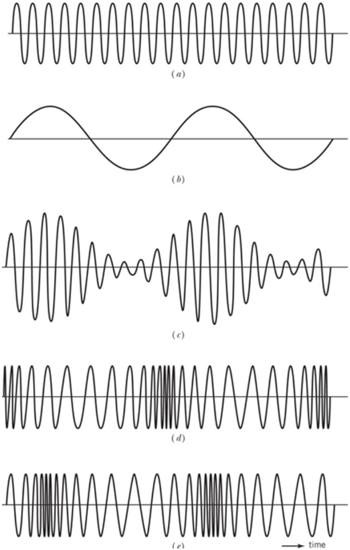
$$m(t) = m_1(t) + m_2(t) \quad \not\Longrightarrow \quad s(t) = s_1(t) + s_2(t)$$

AM/PM/FM: Illustrative Example 1



AM/PM/FM: Illustrative Example 2

- (a) Carrier Wave
- (b) Message
- (c) AM Signal
- (d) PM Signal
- (e) FM Signal



Instantaneous Frequency

- For generalized sinusoid

$$\phi(t) = A\cos(\theta(t))$$

  $\theta(t)$ is the generalized angle (can be any function of t).

- For true sinusoid

$$\theta(t) = \omega_c t + \theta_0$$

  This is linear with slope ω_c and offset θ_0.

- Instantaneous frequency is derivative of generalized angle:

$$\omega_i(t) = \frac{d\theta(t)}{dt} = \dot{\theta}(t)$$

Example: What is the instantaneous frequency of $10\cos(20\pi t + \pi t^2)$ in Hertz.

A: $\omega_i(t) = \dot{\theta}(t) = 20\pi + 2\pi t$ radian/second, then
$f_i(t) = \frac{1}{2\pi}\omega_i(t) = 10 + t$ Hertz

Instantaneous Frequency

- By Fundamental Theorem of Calculus

$$\theta(t) = \int_{-\infty}^{t} \omega_i(u)du = \theta(0) + \int_{0}^{t} \omega_i(u)du$$

- We can modulate a generalized sinusoid by using a signal $m(t)$ to vary either $\theta(t)$ or $\omega_i(t)$.
- In either case, the frequency of the modulated signal changes as a function of $m(t)$.

Phase Modulation (PM)

- In PM, phase is varied *linearly* with $m(t)$:

$$\theta_i(t) = \omega_c t + k_p m(t)$$

Hence,

$$s_{PM}(t) = A_c \cos(\theta_i) = A_c \cos[\omega_c t + k_p m(t)]$$

The instantaneous frequency is

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} = \omega_c + k_p \frac{dm(t)}{dt}$$

If $m(t)$ varies rapidly, then the frequency deviations are larger.

- k_p is a constant (to be designed)

Frequency Modulation (FM)

- In FM, frequency is varied *linearly* with $m(t)$:

$$\omega_i(t) = \omega_c + k_f m(t)$$

Then, the angle is

$$\theta_i(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\lambda) d\lambda$$

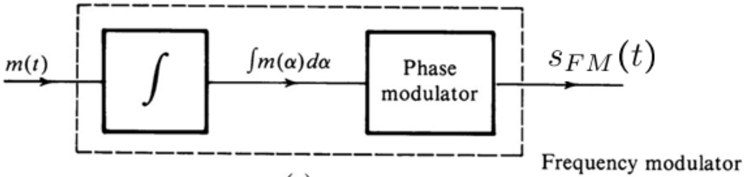
and

$$s_{FM}(t) = A_c \cos(\theta_i) = A_c \cos[\omega_c t + k_f \int_{-\infty}^{t} m(\lambda) d\lambda]$$

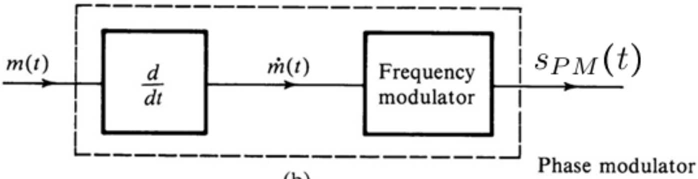
- k_f is a constant (to be designed)

Relationship Between FM and PM

- Phase modulation of $m(t)$ = frequency modulation of $\dot{m}(t)$.
- Frequency modulation of $m(t)$ = phase modulation of $\int m(\lambda)d\lambda$



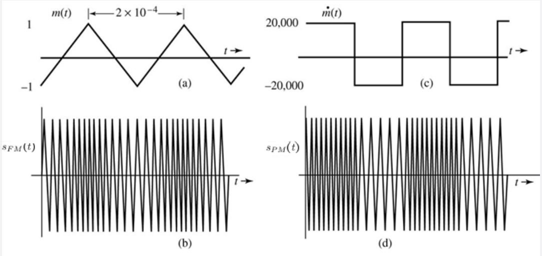
(a)



(b)

FM/PM Example

Example: Sketch FM and PM waves for $m(t)$ shown in Fig. (a), for $k_f = 2\pi \times 10^5$, $k_p = 10\pi$, and $f_c = 100$MHz. Plot $s_{FM}, \dot{m}(t), s_{PM}$, and find instantaneous frequency for the FM case.



- Instantaneous frequency: $f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$

  Then, $f_{i,\min} = 99.9$ MHz and $f_{i,\max} = 100.1$ MHz

FM/PM Example

- Instantaneous frequency: $f_{i,\min} = 99.9$ MHz and $f_{i,\max} = 100.1$ MHz
- **Q:** Is the modulated signal confined to the frequency band 99.9-100.1?

  A: No! The bandwidth is approximately 250 KHz (we will see this later).

Angle Modulation: Summary

- In FM, **frequency** is varied linearly with $m(t)$
- In PM, **phase** is varied linearly with $m(t)$

- In FM $\boxed{\omega_i(t) = \omega_c + k_f m(t)}$ where k_f is a constant.
- In PM $\boxed{\omega_i(t) = \omega_c + k_p \dot{m}(t)}$ where k_p is a constant.

FM vs. PM

- In PM

$$\theta_i(t) = \omega_c t + k_p m(t)$$

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} = \omega_c + k_p \frac{dm(t)}{dt}$$

$$s_{PM}(t) = A_c \cos(\theta_i) = A_c \cos[\omega_c t + k_p m(t)]$$

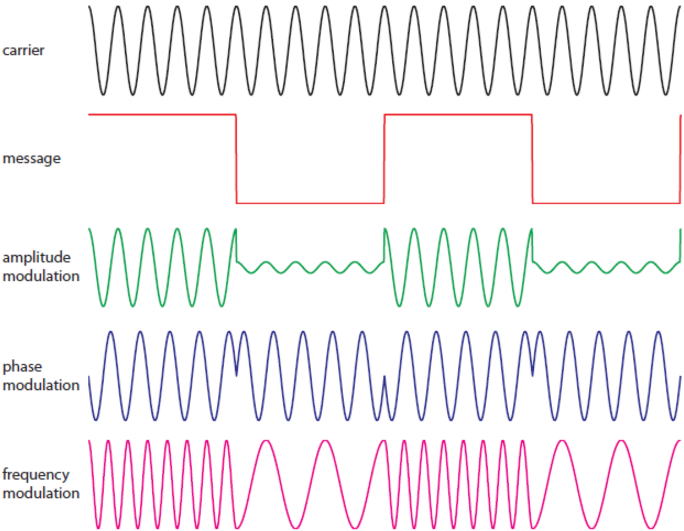
- In FM

$$\theta_i(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\lambda) d\lambda$$

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} = \omega_c + k_f m(t)$$

$$s_{FM}(t) = A_c \cos(\theta_i) = A_c \cos[\omega_c t + k_f \int_{-\infty}^{t} m(\lambda) d\lambda]$$

AM/PM/FM: Illustrative Example 3



Contents

Analog Modulation Schemes

Assuming the message is $m(t)$ and the frequency of the carrier is ω_c, we have

- **Amplitude Modulation** (different ways to encode information in amplitude of carrier)
  - AM: $s(t) = [1 + k_a m(t)] \cos \omega_c t$
  - DSB-SC $s(t) = m(t) \cos \omega_c t$
  - SSB $s(t) = m(t) \cos \omega_c t \pm m_h(t) \sin \omega_c t$ (- for USB, + for LSB)
  - VSB $s(t)$ depends on the filter
  - QAM $s(t) = m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t$
- **Angle Modulation** (different ways to encode information in carriers angle of carrier)
  - PM: $s(t) = A_c \cos[\omega_c t + k_p m(t)]$
  - FM $s(t) = A_c \cos[\omega_c t + k_f \int_{-\infty}^{t} m(\lambda) d\lambda]$
- k_a, k_p, and k_f are parameters (constants) to design