Lecture 9
Angle Modulation Part III

- FM Modulators
- FM Demodulation
- Advantages of FM
- Superheterodyne Receiver
- Summary of Analog Modulations
- Appendix

Contents

Check Yourself!

Example: We knowing that in general

- $s_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$,
- $s_{FM}(t) = A\cos[\omega_c t + k_f a(t)]$, $\quad a(t) \triangleq \int_{-\infty}^{t} m(\lambda) d\lambda$.

How would you get the equations for narrow-band PM and FM?

(Recall that $|k_p m(t)| \ll 1$ and $|k_f a(t)| \ll 1$ for narrow-band PM/FM)

Solution:

$$s_{NBPM}(t) = A\cos\omega_c t\ \cos(k_p m(t)) - A\sin\omega_c t\ \sin(k_p m(t))$$

$$\approx$$

$$s_{NBFM}(t) = A\cos\omega_c t\ \cos(k_f a(t)) - A\sin\omega_c t\ \sin(k_f a(t))$$

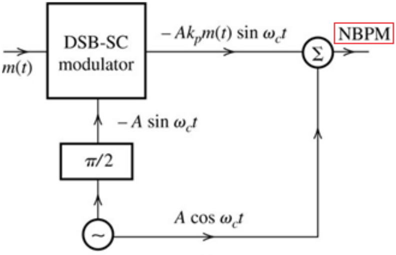
$$\approx$$

NBFM Generation (Modulator)

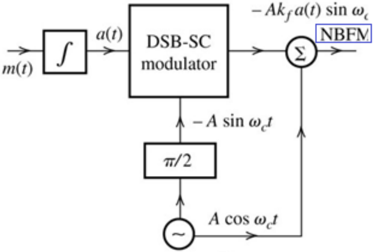
- Thus, narrow-band PM ($|k_p m(t)| \ll 1$) and FM ($|k_f a(t)| \ll 1$) are given by

$$\boxed{s_{NBPM}(t) \approx A\cos\omega_c t - Ak_p m(t)\sin\omega_c t]}$$

$$\boxed{s_{NBFM}(t) \approx A\cos\omega_c t - Ak_f a(t)\sin\omega_c t]}$$



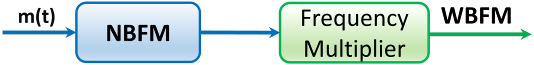
(a)



(b)

WBFM Generation (Modulator)

1. Direct Generation Method
   - Using a voltage controlled oscillator (VCO)
2. Indirect Method of Armstrong
   - First generate a NBFM
   - Then, convert it to WBFM by using a frequency multiplier



WBFM Modulation: Direct Generation Using VCO

- A voltage controlled oscillator (VCO) generates a signal whose instantaneous frequency is proportional to an input $m(t)$:

$$\omega_i(t) = \omega_c + k_f m(t)$$

- VCO can be constructed by using input voltage to control (vary) L or C of an LC parallel resonant circuit (oscillator)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- If the capacitance is varied by $m(t)$

$$C = C_0 - km(t)$$

(Example: a *reversed biased diode* acts as a capacitor whose capacitance varies with the bias voltage)

WBFM Modulation: Direct Generation Using VCO

- Then

$$\omega_0 = \frac{1}{\sqrt{LC_0\left[1 - \frac{km(t)}{C_0}\right]}} = \frac{1}{\sqrt{LC_0}}\left[1 - \frac{km(t)}{C_0}\right]^{-\frac{1}{2}}$$

$$\approx \frac{1}{\sqrt{LC_0}}\left[1 + \frac{km(t)}{2C_0}\right] \qquad \text{if} \qquad \frac{km(t)}{C_0} \ll 1$$

(note that $(1+x)^n \approx 1 + nx \qquad |x| \ll 1.$)

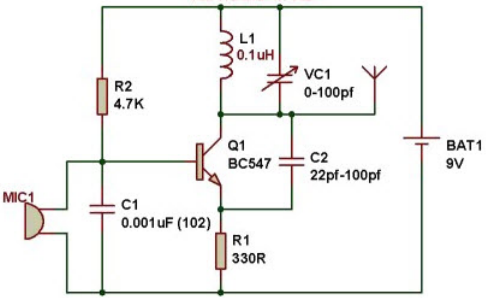
- Thus,

$$\omega_0 = \omega_c + k_f m(t)$$

where $\omega_c = \frac{1}{\sqrt{LC_0}}$ and $k_f = \frac{k\omega_c}{2C_0}$

How to Build an FM Transmitter?

- Simple FM transmitter circuit



- Required components: power supply of 9V, resistor, capacitor, inductor, trimmer capacitor, mic, transistor, and antenna
- **Q:** what is the role of each element in this circuit?

How to Build an FM Transmitter?

- **LC tank** (L_1, VC_1): to generate the radio frequency carrier waves (oscillator). The tank circuit stores the energy for oscillations.
- **variable capacitor** is used to change the resonant frequency (tuner)
- **transistor** is used for amplification.
- C_2: negative feedback to the oscillating tank circuit.
- **Antenna**: use 15cm to 1m long wire
- The input audio signal from the mic penetrated to the base of the transistor, which is amplifies to i_E and modulates the LC tank (carrier frequency) in FM format.

WBFM Modulation: Armstrong's Indirect Method

- A **frequency multiplier** can be realized using a nonlinear device. Suppose we have a nonlinear device with the response

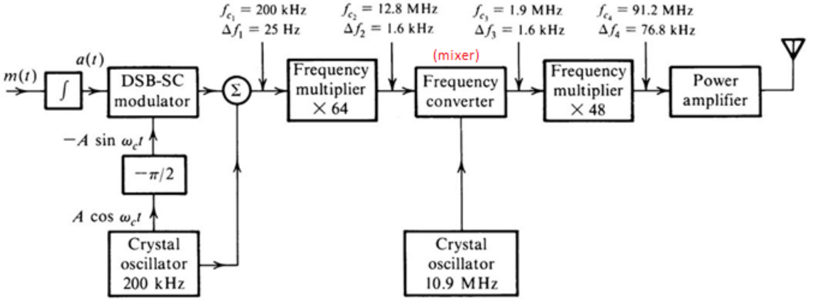
$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \cdots + a_n x^n(t)$$

- If $x(t) = A_c \cos[\omega_c t + ka(t)]$, then by using trigonometric identities for $n\theta$ the output can be represented as

$$y(t) = c_0 + c_1 \cos[\omega_c t + ka(t)] + \cdots + c_n \cos[n\omega_c t + nka(t)]$$

- i.e., the output will have spectra at $\omega_c, 2\omega_c, \dots, n\omega_c$ with frequency deviations $\Delta f, 2\Delta f, \dots, n\Delta f$.
- A bandpass filter centering at $n\omega_c$ can recover an FM signal whose frequency is multiplied by n

WBFM Modulation: Armstrong's Indirect Method



- Besides frequency multiplier, this circuit has another critical component, named frequency converter (a.k.a mixer). We will discuss this soon.
- The key is that by using a frequency converter, we can multiply Δf by a big number (64×48 in this figure) while carrier frequency is not multiplied by the same number and is kept in the desired range of FM frequencies.

WBFM Modulation: Armstrong's Indirect Method

- **Q:** find the following ratios $\Delta f_4/\Delta f_1$ and f_{c_4}/f_{c_1}
- **A:**
  - $\frac{\Delta f_4}{\Delta f_1} = \cdots$
  - $\frac{f_{c_4}}{f_{c_1}} = \cdots$

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Demodulation of FM Signal

FM demodulator can be realized by a differentiator followed by an envelope detector.

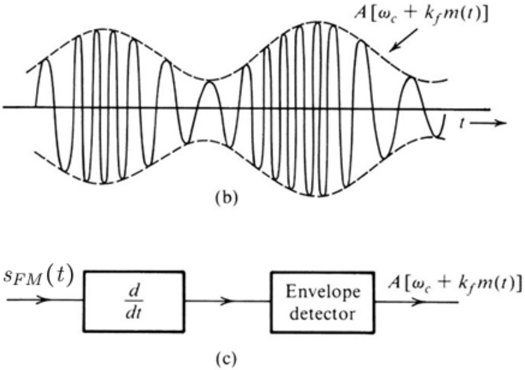
- Recall that for the FM signal is $s_{FM}(t) = A_c \cos\theta_i(t)$ where $\theta_i(t) = \omega_c t + k_f \int_0^t m(\lambda)d\lambda$
- Differentiating $s_{FM}(t)$ results in

$$\begin{aligned}\frac{ds_{FM}(t)}{dt} &= -A_c \frac{d\theta_i(t)}{dt}\sin\theta_i(t)\\ &= -A_c\big[\omega_c + k_f m(t)\big]\sin\theta_i(t)\end{aligned}$$

- Thus, an envelope detector yields an output proportional to

$$A_c\big[\omega_c + k_f m(t)\big]$$

Demodulation of FM Signal



(b) Output of a differentiator to the input FM wave. (c) FM demodulation by direct differentiation.

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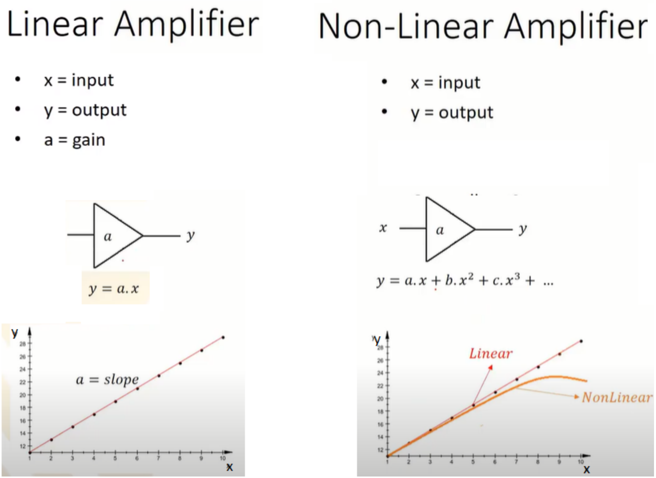
Advantages of FM

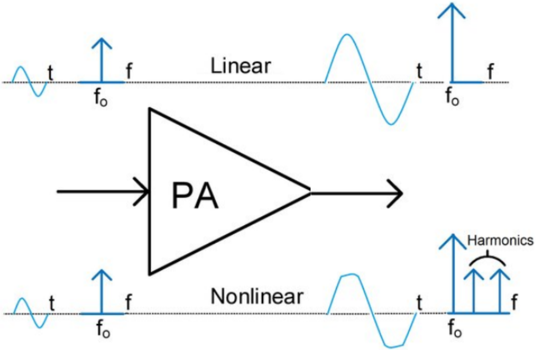
1. FM is less susceptible to amplifier nonlinearities.



Power Amplifiers Non-linearity

- Power amplifiers (PA) are large-signal amplifiers.
- **Q:** Why linearity is important?

  A: non-linear amplifier distorts the signal. It has uneven gain for different input values, which results in unwanted harmonics.



Amplifier Non-linearity in FM vs AM

- **Non-linearity in FM:** If input is $x(t) = A_c \cos(\omega_c t + \phi(t))$ and the output is

$$\begin{aligned} y(t) &= a_0 + a_1 x(t) + a_2 x^2(t) + \dots \\ &= c_0 + c_1 \cos[\omega_c t + \phi(t)] + c_2 \cos[2\omega_c t + 2\phi(t)] + \dots \end{aligned}$$

  The extra terms will be blocked by a bandpass filter as their spectrum is outside the carrier signal band.

- **Non-linearity in AM:** if the input is $x(t) = m(t)\cos\omega_c t$ and $y(t) = a_1 x(t) + a_3 x^3(t)$ then we get

$$\begin{aligned} y(t) &= a_1 m(t)\cos\omega_c t + a_3 m^3(t)\cos^3\omega_c t \\ &= \left[a_1 m(t) + \frac{3}{4} a_3 m^3(t)\right]\cos\omega_c t + \frac{1}{4} a_3 \cos 3\omega_c t \end{aligned}$$

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Advantages of FM

2. FM is less susceptible to additive noise and fading (change of amplitude).

- An additive noise changes the amplitude of the signal.
- Also, fading, by definition, changes the amplitude of the signal.
- **In AM:** the receiver uses the amplitude of the received signal to recover the original message, and an additive noise changes the receive signal amplitude. Hence, AM is susceptible to additive noise channel.
- **In FM:** the information is in the frequency of the carrier. Thus, the absolute strength of the received signal doesn't matter, which means FM has better immunity to noise and fading.

FM can adjust to rapid fading using automatic gain control (AGC).

Advantages of FM

3. FM is less vulnerable to interference from adjacent channels.

- Let us consider the simple case of the interference another sinusoid $I\cos(\omega_c + \omega)t$. The received signal $r(t)$ is

$$\begin{aligned} r(t) &= A\cos\omega_c t + I\cos(\omega_c + \omega)t \\ &= (A + I\cos\omega t)\cos\omega_c t - I\sin\omega t\sin\omega_c t \\ &= E_r(t)\cos[\omega_c t + \phi_r(t)] \end{aligned}$$

  and $\phi_r(t) = \tan^{-1}\frac{I\sin\omega t}{A+I\cos\omega t}$ and $E_r(t) = \sqrt{(A + I\cos\omega t)^2 + (I\sin\omega t)^2}$
- When the interfering signal is small in comparison to the carrier, i.e., $I \ll A$, then $\phi_r(t) \approx \frac{I}{A}\sin\omega t$
- Observe that, the interference output is inversely proportional to the carrier amplitude A.
- Thus, the larger the carrier amplitude A, the smaller the interference effect.

Interference in AM

- An AM signal with an interfering sinusoid $I\cos(\omega_c + \omega)t$ is given by

$$\begin{aligned} r(t) &= (A + m(t))\cos\omega_c t + I\cos(\omega_c + \omega)t \\ &= (A + m(t) + I\cos\omega t)\cos\omega_c t - I\sin\omega t\sin\omega_c t \end{aligned}$$

- The envelope of this signal is

$$\begin{aligned} E_r(t) &= \sqrt{(A + m(t) + I\cos\omega t)^2 + (I\sin\omega t)^2} \\ &\approx A + m(t) + I\cos\omega t \qquad\qquad I << A \end{aligned}$$

- Hence, the interference signal at the envelope detector output is $I\cos\omega t$, which is independent of the carrier amplitude A.

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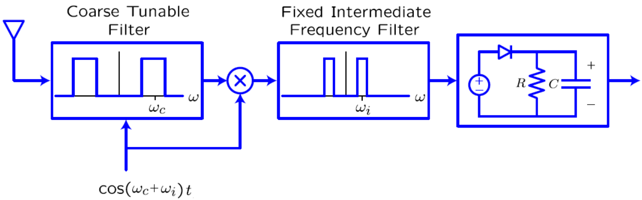
Superheterodyne Receiver: Big Picture

Edwin H. Armstrong invented the superheterodyne receiver in 1918, which made broadcast AM practical.

- Superhet receiver uses frequency mixing to convert a received signal to a fixed intermediate frequency (IF) which can be more conveniently processed (better sensitivity and selectivity) than the original carrier frequency.
- The key circuit is the mixer, which acts as a simple amplitude modulator to produce sum and difference frequencies.



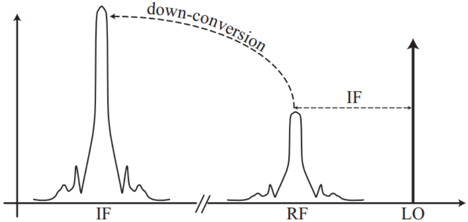
Historical Note

E. H. ARMSTRONG
The discoverer of the "feed-back" circuit, in the uniform of a major in the Signal Corps during the war

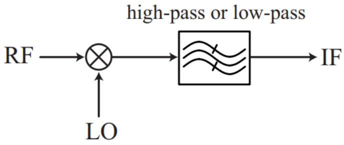
- Armstrong also invented and patented the "regenerative" (**positive feedback**) circuit for amplifying radio signals (while he was a junior at Columbia University).
- He also invented **wide-band FM**.

Mixer

- A mixer translates a frequency to another (**up-conversion** mixer or **down-conversion** mixer)
- An ideal mixer is usually represented by a multiplier symbol
- A real mixer cannot be driven by arbitrary inputs. Instead one port (the "LO" port) is driven by a local oscillator with a fixed amplitude sinusoid



Mixer



- How does it work? Recall the trigonometric identity

$$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$

- When mixing, the received signal of frequency f_{RF} will be translated into two new frequencies: $f_{LO} + f_{RF}$ and $f_{LO} - f_{RF}$
- We usually select one of these new frequencies by filtering the output of the mixer

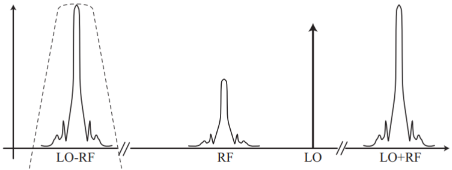


Image Frequencies

- For any mixer, there is a second frequency that results in the same IF. This is undesirable frequency is called image frequency.
- More specifically,
  - If $f_{LO} < f_{RF}$, the both f_{RF} and $f_{RF} - 2f_{IF}$ will result in f_{IF} (why?)
  - If $f_{LO} > f_{RF}$, the both f_{RF} and $f_{RF} + 2f_{IF}$ will result in f_{IF} (why?)

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Numerical Example

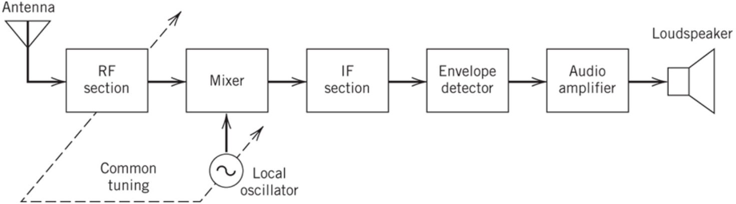
- Demonstrate that for $f_{LO} = 89.3$ MHz, both $f_{RF1} = 100$ MHz and $f_{RF2} = 78.6$ MHz could yield $f_{IF} = 10.7$ MHz.

Superheterodyne Receiver

- *Demodulation* is not the only task of the receiver. The receiver should also have the following functionalities
  - **Carrier-frequency tuning:** to select the desired signal (radio or TV station)
  - **Filtering:** to pick up only the desired signal
  - **Amplification:** to compensate for the signal power loss due to transmission

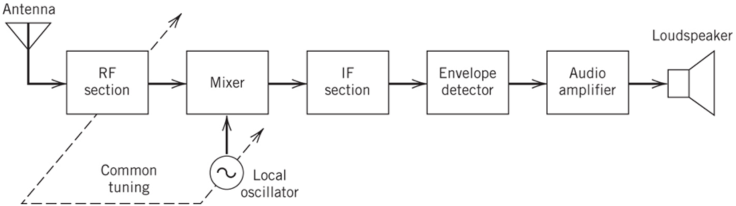
The main difficulty is to have a tunable high-quality (sharp) filter.

- A superhet receiver does all three tasks + demodulation
- All analog radio and TV receivers are of the superhet type
- The block diagram of the superhet receiver (for AM) is shown below



Superheterodyne Receiver

- Below is the block diagram of a superheterodyne receiver for AM signals.
  - What modifications are required to adapt it for FM reception?
  - How about PM reception?



Superheterodyne Receiver

- Superhet receiver uses frequency mixing to convert a received signal to a "fixed" intermediate frequency (IF)
- The IF signal can be more conveniently processed (better sensitivity and selectivity) than the original carrier frequency, as its frequency is fixed and lower than f_c
- The key concept is to have a fixed IF and the key circuit is the mixer

Typical frequency parameters of AM and FM radio receivers

	AM Radio	FM Radio
RF carrier range	535 - 1605 kHz	88 - 108 MHz
IF mid-frequency	455 kHz	10.7 MHz
IF bandwidth	10 kHz	200 kHz

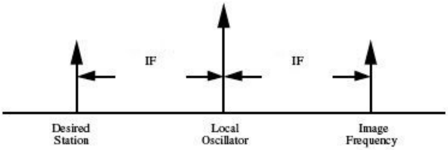
Superheterodyne Receiver

- The result of heterodyning is to produce IF carrier

$$f_{IF} = |f_{RF} - f_{LO}|$$

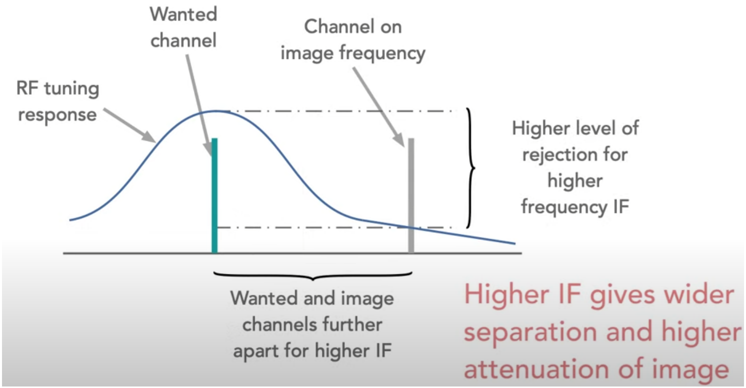
But, two different RF (input frequencies) result to the same IF. The unwanted one is called *image frequency*. $|f_{RF} - f_{\text{im}}| = 2f_{IF}$

- The image frequency of a given $f_{RF} < f_{LO}$ is $f_{RF} + 2f_{IF}$
- The image frequency of a given $f_{RF} > f_{LO}$ is $f_{RF} - 2f_{IF}$
- The practical cure for image interference is to employ a highly selective RF section (i.e., not to let image come in to the mixer)



Removing Image Frequency

- How does a superheterodyne receiver eliminate image frequency?



Example: Superhet Receiver

Example: Consider a superhet receiver in an AM system. The mixer translates the input frequency f_c to a fixed IF frequency of 455kHz by using a local oscillator of frequency f_{LO}. The broadcast frequencies range from 540 to 1600 kHz.

(a) Determine the range of tuning must be provided by the local oscillator when

i) $f_{LO} > f_c$ (superheterodyne receiver), and

ii) $f_{LO} < f_c$

(b) Explain why the usual AM receiver uses the superhet system.

- (a) i) $995kHz < f_{LO} < 2055kHz$,

  ii) $85kHz < f_{LO} < 1145kHz$
- (b) hint: look at the frequency ratios

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Analog Modulation Summary

- Comparison of relative merit of different modulation techniques

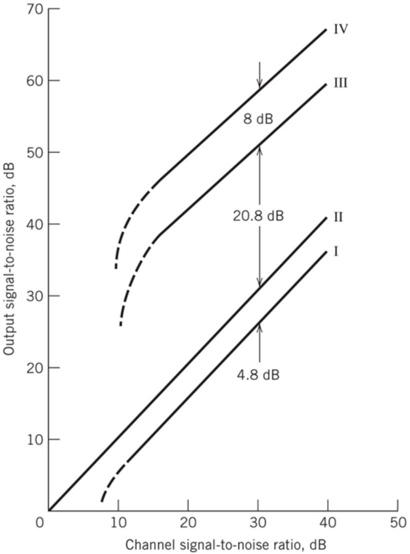
Scheme	Complexity	Trans. BW (B_T)	Fidelity (SNR$_o$)
AM+C	Low	$2W$	$\frac{\mu^2}{2+\mu^2}\gamma < \frac{1}{3}\gamma$
DSB-SC	High	$2W$	γ
SSB	High	W	γ
VSB	High	$W < B_T < 2W$	γ
FM	Moderate	$\approx 2(1+\beta)W$	$\frac{3}{2}\beta^2\gamma$
PM	Moderate	$\approx 2(1+\beta)W$	$\frac{1}{2}\beta^2\gamma$

- SNR$_o$ (last column) is the topic of Chapter 6 (we have not covered it)
  - Modulating signal is a tone ($m(t) = \cos 2\pi f_m t$)
  - $\gamma \triangleq \frac{S_i}{\eta W}$ and S_i is the input signal power
  - In FM and PM, higher bandwidth $\Rightarrow$ better fidelity

Analog Modulations: Noise Performance

Noise performance in different CW modulations.

- Curve I: AM, $\mu = 1$
- Curve II: DSB-SC
- Curve III: FM, $\beta = 2$
- Curve IV: FM, $\beta = 5$



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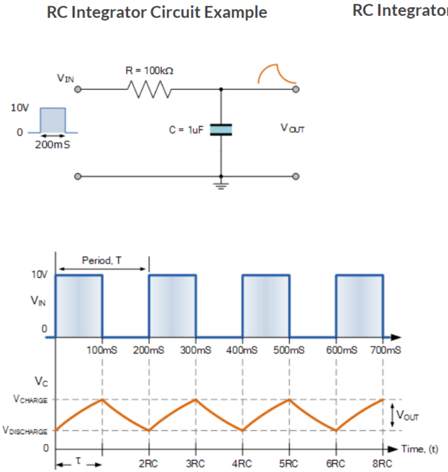
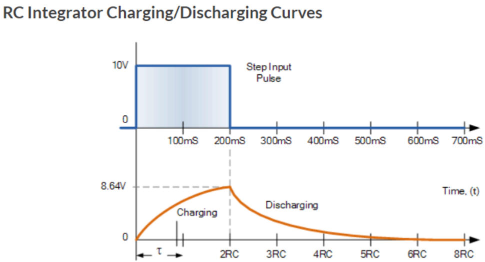
Analog Modulations: Fun Facts

- AM method of audio transmission was first carried out in the mid 1870s.
- FM radio was developed in the 1930s.
- FM radio became popular in the 1970s and early 80s. By the 1990s most music stations switched from AM and adopted FM due to better sound quality.
- Larger wavelength AM waves can travel farther than FM radio waves which cannot travel through solid objects.
- AM radio uses lower frequencies than FM (bout 100 times). This means that it is more capable of bouncing off of the upper atmosphere and curving around obstacles.
- Saving AM radio in cars!
  - https://nrb.org/trump-to-nrb-i-will-do-my-part-to-protect-am-radio-in-our-cars/
  - https://abcnews.go.com/US/wireStory/save-signal-politicians-close-votes-needed-radio-car-108015528

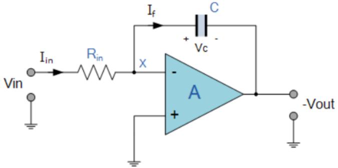
Integrator Circuit (Passive)

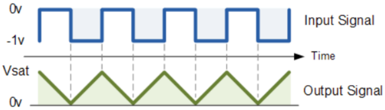




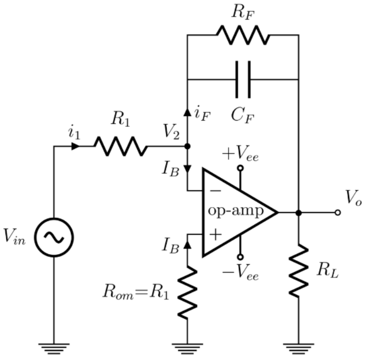
Integrator Circuit (Active)

Op-amp Integrator Circuit (Ideal)



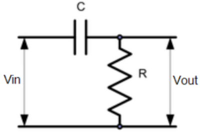


Practical

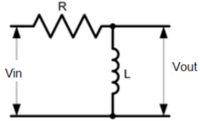


Differentiator Circuit

Passive differentiator



Capacitive differentiator



Inductive differentiator

Op-amp Differentiator Circuit