ECE 8700: Communication Systems Engineering

Module 3
Digital Modulation Schemes

Mojtaba Vaezi
http://www.ece.villanova.edu/ mvaezi

Villanova University

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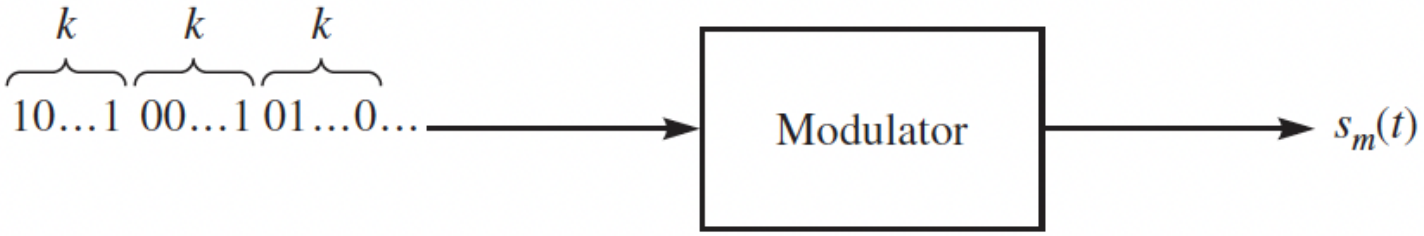
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1 Representation of Digitally Modulated Signals

1.1 Bit $\overset{map}{\longleftrightarrow}$ Symbol

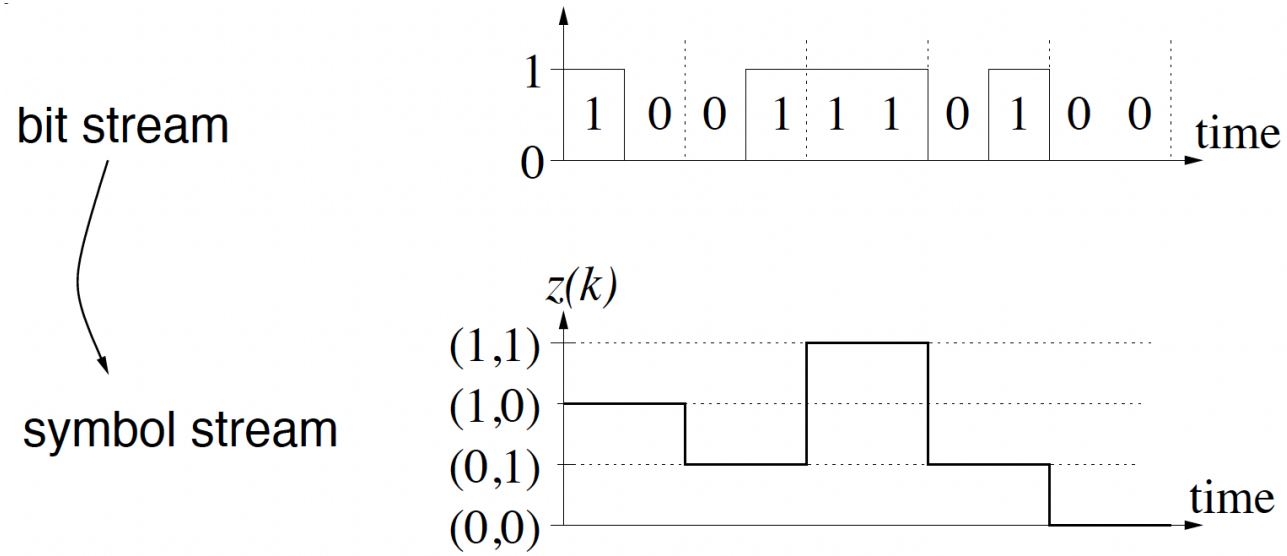
- In binary communication, 0 and 1 bits are the information.
- The modulator in a digital communication system maps a sequence of k binary symbols, which, in the case of equiprobable symbols, carries k bits of information.
- These binary symbols are mapped into a set of corresponding signal waveforms $s_m(t)$, where $1 \leq m \leq M$ and $M = 2^k$.



- We assume that these signals are transmitted every T_s seconds, where T_s is called the **signaling interval**, or symbol duration.

Bit $\overset{map}{\longleftrightarrow}$ Symbol

- **Example:** For $k = 2$ bits per symbol (4-ary modulation), symbol period T_s is twice of bit period T_b



Why Bit $\overset{map}{\longleftrightarrow}$ Symbol?

– We know that we cannot transmit more than 2 symbols/s/Hz.

– It is possible to save on bandwidth by packing multiple bits into a **symbol**.

Bit $\overset{map}{\longleftrightarrow}$ Symbol

Combining k bits generates $M = 2^k$ symbols.

- T_b: Bit duration.
- R_b: Bit rate, given by $R_b = \frac{1}{T_b}$.
- T_s: Symbol duration, where $T_s = kT_b$ or equivalently, $T_b = \frac{T_s}{k} = \frac{T_s}{\log_2 M}$.
- R_s Symbol rate: $R_s = \frac{R_b}{k} = \frac{R_b}{\log_2 M}$.
- $\mathcal{E}_s$ or $\mathcal{E}_{\text{avg}}$: Average symbol energy, given by $\mathcal{E}_s = k\mathcal{E}_b$.
- $\mathcal{E}_b$ or $\mathcal{E}_{\text{b,avg}}$: $\mathcal{E}_b = \frac{\mathcal{E}_{\text{avg}}}{k} = \frac{\mathcal{E}_{\text{avg}}}{\log_2 M}$ (for equiprobable symbols)
- $P_{\text{avg}} = \frac{\mathcal{E}_b}{T_b} = R_b\mathcal{E}_b = \frac{\mathcal{E}_{\text{avg}}}{T_s} = R_s\mathcal{E}_{\text{avg}}$ (for equiprobable symbols)

- If the energy content of $s_m(t)$ is denoted by $\mathcal{E}_m$, then the *average signal energy* is given by

$$\mathcal{E}_{\text{avg}} = \sum_{m=1}^{M} p_m \mathcal{E}_m$$

  where p_m is the probability of the mth signal (message probability).

- In the case of equiprobable messages, $p_m = 1/M$, and therefore,

$$\mathcal{E}_{\text{avg}} = \frac{1}{M} \sum_{m=1}^{M} \mathcal{E}_m = \frac{1}{M} \sum_{m=0}^{M-1} |s_m|^2$$

  Obviously, if all signals have the same energy, then $\mathcal{E}_m = \mathcal{E}_{\text{avg}} = \mathcal{E}$.

- The average energy for transmission of 1 bit, or *average energy per bit*, denoted by $\mathcal{E}_{b,\text{avg}}$ or simply $\mathcal{E}_b$, when the signals are equiprobable is given by

$$\mathcal{E}_b = \frac{\mathcal{E}_{\text{avg}}}{\log_2 M} = \frac{1}{M \log_2 M} \sum_{m=0}^{M-1} |s_m|^2$$

- The average power sent by the transmitter is

$$P_{\text{avg}} = \frac{\mathcal{E}_b}{T_b} = R_b \mathcal{E}_b = \frac{\mathcal{E}_{\text{avg}}}{T_s} = R_s \mathcal{E}_{\text{avg}}$$

Observations:

– Faster data rate needs more power.

– Lower transmitted power means slower connection. Why?

Real-World Example

Wi-Fi and Cellular Networks:

– Close to a router/tower ⇒ high power ⇒ fast connection.

– Far from a router/tower ⇒ low power ⇒ slower data rate (e.g., 3G instead of 5G).

2 Memoryless Modulation Methods

2.1 Pulse Amplitude Modulation (PAM)

- In digital PAM, the signal waveforms may be represented as:

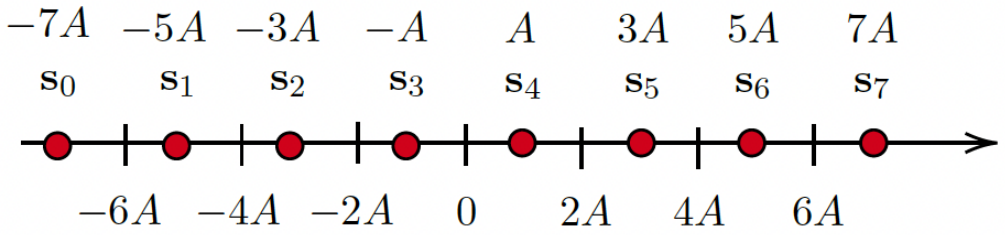
$$s_m(t) = A_m p(t), \quad 1 \le m \le M$$

  - $p(t)$ is a pulse of duration T, and
  - $\{A_m, 1 \le m \le M\}$ denotes the amplitudes of M possible symbols.

- Usually, the signal amplitudes A_m take the discrete values:

$$A_m = 2m - 1 - M, \quad m = 1, 2, \ldots, M$$

  i.e., the amplitudes are $\pm 1, \pm 3, \pm 5, \ldots, \pm(M-1)$.



Example: 8-PAM

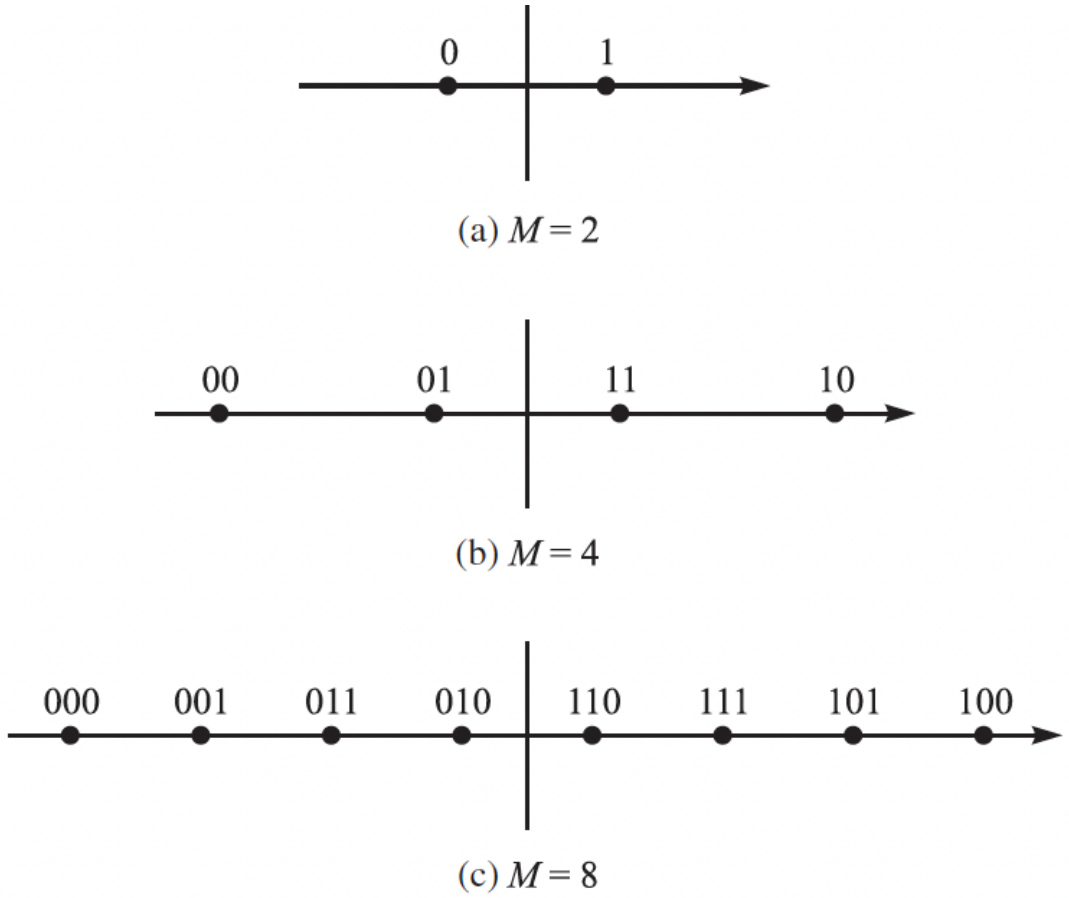


Figure 1: Constellation for M-PAM signaling.

- The energy in signal $s_m(t)$ is given by

$$\mathcal{E}_m = \int_{-\infty}^{\infty} A_m^2 p^2(t)\, dt = A_m^2 \mathcal{E}_p$$

where $\mathcal{E}_p$ is the energy in $p(t)$. From this[1],

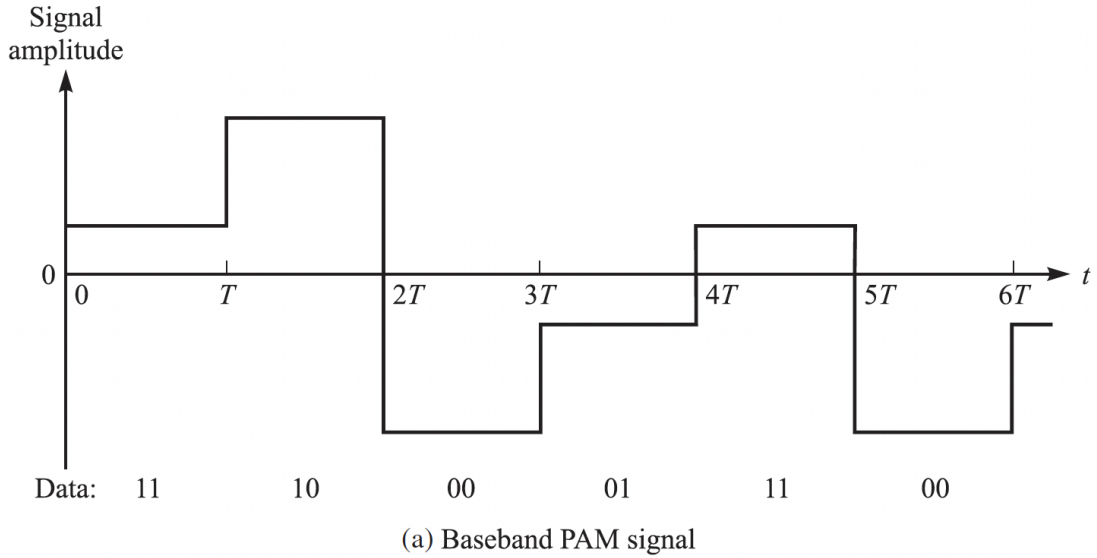
$$\begin{aligned}
\mathcal{E}_{\text{avg}} &= \frac{\mathcal{E}_p}{M} \sum_{m=1}^{M} A_m^2 \\
&= \frac{2\mathcal{E}_p}{M} \left(1^2 + 3^2 + 5^2 + \cdots + (M-1)^2\right) \\
&= \frac{2\mathcal{E}_p}{M} \times \frac{M(M^2-1)}{6}
\end{aligned}$$

and

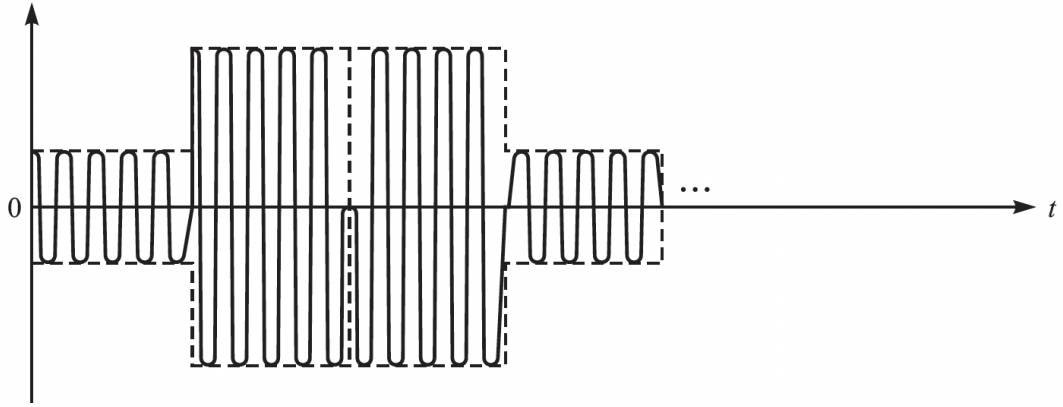
$$\mathcal{E}_b = \frac{(M^2-1)}{3\log_2 M} \mathcal{E}_p$$

[1]The formula for the summation of the squares of the first n odd numbers is:

$$\sum_{k=1}^{n} (2k-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$



(a) Baseband PAM signal



(b) Bandpass PAM signal

Figure 2: Example of (a) baseband and (b) carrier-modulated PAM signals.

- **Bandpass PAM** signals are carrier-modulated bandpass signals with lowpass equivalents of the form $A_m g(t)$, where A_m and $g(t)$ are real.

$$\begin{aligned} s_m(t) &= \Re\left\{ s_{m_l}(t) e^{j2\pi f_c t} \right\} \\ &= \Re\left\{ A_m g(t) e^{j2\pi f_c t} \right\} \\ &= A_m g(t) \cos(2\pi f_c t) \end{aligned}$$

  where f_c is the carrier frequency.

- This, in the generic form of PAM signaling we substitute:

$$p(t) = g(t) \cos(2\pi f_c t)$$

  and $g(t)$ is the pulse shaping function.

- Pulse duration T is usually assumed to be a multiple of $\frac{1}{f_c}$.

- The bandpass digital PAM is also called amplitude-shift keying (ASK).

Example 1. *M=4*

$$\begin{cases} s_1(t) = -3A\ g(t)\cos(2\pi f_c t) \\ s_2(t) = -A\ g(t)\cos(2\pi f_c t) \\ s_3(t) = +A\ g(t)\cos(2\pi f_c t) \\ s_4(t) = +3A\ g(t)\cos(2\pi f_c t) \end{cases}$$

The amplitude difference between two adjacent signals = 2A.

PAM in Signal/Vector Space

- PAM signals are one-dimensional ($N = 1$) since all are multiples of the same basic signals. Then, we get

$$\phi(t) = \frac{p(t)}{\sqrt{\mathcal{E}_p}}$$

  as the basis for the general PAM $s_m(t) = A_m p(t)$.

- Also, the basis for the **bandpass** PAM signal, i.e., when $p(t) = g(t)\cos(2\pi f_c t)$,

is

$$\phi(t) = \sqrt{\frac{2}{\mathcal{E}_g}} g(t) \cos(2\pi f_c t)$$

- Then, the signal space is for baseband and modulated PAM is

$$s_m(t) = A_m \sqrt{\mathcal{E}_p}\phi(t), \qquad \texttt{for baseband PAM}$$

$$s_m(t) = A_m \sqrt{\frac{\mathcal{E}_g}{2}}\phi(t), \qquad \texttt{for bandpass PAM}$$

- Also, vector representations are given by

$$\mathbf{s}_m = A_m \sqrt{\mathcal{E}_p}, \qquad \texttt{for baseband PAM}$$

$$\mathbf{s}_m = A_m \sqrt{\frac{\mathcal{E}_g}{2}}, \qquad \texttt{for bandpass PAM}$$

where $A_m = \pm 1, \pm 3, \ldots, \pm(M-1)$.

- The **Euclidean distance** between any pair of signal points is

$$d_{mn} = \sqrt{\|s_m - s_n\|^2} = |A_m - A_n| \sqrt{\mathcal{E}_p}$$

- For adjacent signal points $|A_m - A_n| = 2$, and hence the **minimum distance** of the constellation is given by

$$d_{\min} = 2\sqrt{\mathcal{E}_p}$$

- We can express the **minimum distance** of an M-ary PAM system in terms of its $\mathcal{E}_{\text{b}}$

$$d_{\min} = \sqrt{\frac{12 \log_2 M}{M^2 - 1} \mathcal{E}_{\text{b}}}$$

- In the special case of $M = 2$, or binary signals, the PAM waveforms have the special property that

$$s_1(t) = -s_2(t).$$

- Hence, these two signals have the same energy and a cross-correlation coefficient of -1.
- Such signals are called **antipodal**. This case is sometimes referred to as *binary antipodal signaling*.

2.2 Single-Sideband (SSB) PAM

- The carrier-modulated PAM signal represented above is a double-sideband (DSB) signal, which requires twice the channel bandwidth of the equivalent lowpass signal for transmission.
- We may used single-sideband (SSB) PAM, which has the representation

$$s_m(t) = \Re\left\{A_m\big(g(t) \pm j\hat{g}(t)\big)e^{j2\pi f_c t}\right\}, \quad m = 1, 2, \ldots, M$$

  where $\hat{g}(t)$ is the Hilbert transform of $g(t)$.
- The bandwidth of the SSB signal is one-half that of the DSB signal.

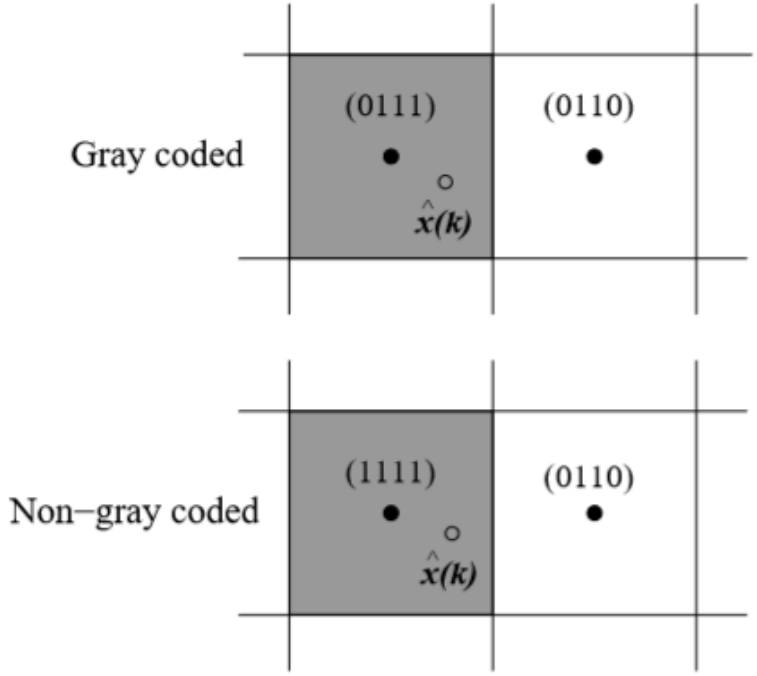
2.3 Gray Coding



Frank Gray (1887–1969) was a physicist and researcher at Bell Labs. [picture source: Wiki]

- The mapping or assignment of k information bits to the $M = 2^k$ possible signal amplitudes may be done in a number of ways.
- The preferred assignment is one in which the adjacent symbols differ by one binary digit.
- This mapping is called `Gray coding`.

Gray coding is important in the demodulation of the signal because the most likely errors caused by noise involve the erroneous selection of an adjacent amplitude to the transmitted signal amplitude. In such a case, only a single bit error occurs in the k-bit sequence.



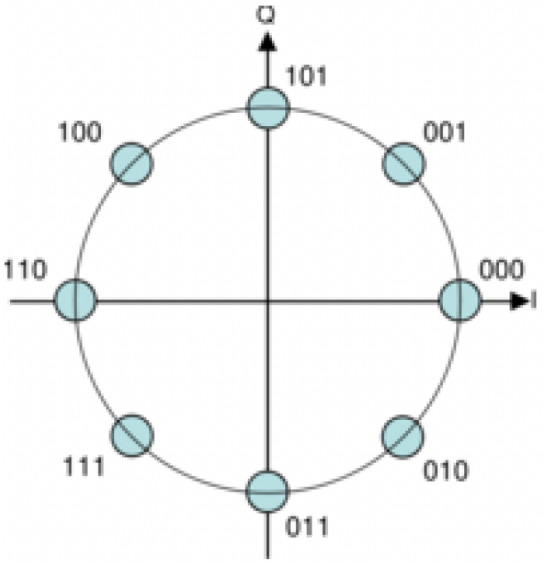
Symobl (0110) was sent but received sample in neighbor region due to noise

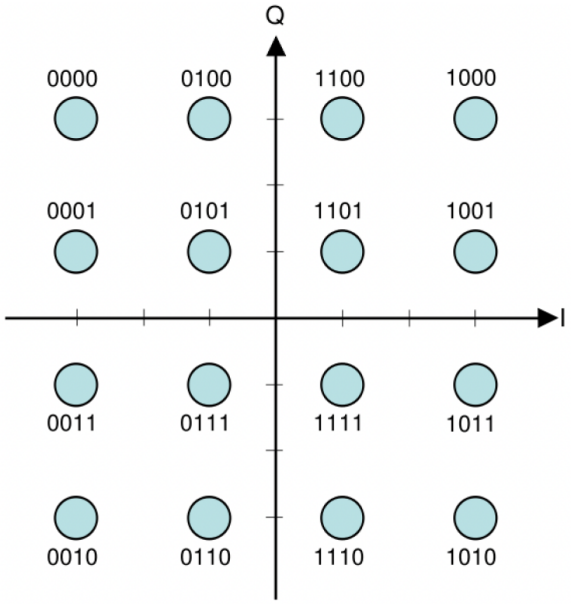


Example: 8-PAM

Memoryless Modulation Schemes

- Example of Gray coding for other modulations





2.4 Phase Modulation

- Digital phase modulation is usually called phase-shift keying (PSK).
- The M signal waveforms can be represented by M phases (MPSK) as $0 \leq t \leq T$

$$\begin{aligned} s_m(t) &= \Re\left\{g(t)e^{j\theta_m}e^{j2\pi f_c t}\right\}, \\ &= g(t)\cos\left(2\pi f_c t + \frac{2\pi}{M}(m-1)\right) \end{aligned}$$

where $\theta_m = \frac{2\pi(m-1)}{M}$ and $m = 1, 2, \ldots, M$.

Example 2. *M=4*

$$\begin{cases} s_1(t) = g(t)\cos(2\pi f_c t) \\ s_2(t) = g(t)\cos\left(2\pi f_c t + \frac{\pi}{2}\right) \\ s_3(t) = g(t)\cos(2\pi f_c t + \pi) \\ s_4(t) = g(t)\cos\left(2\pi f_c t + \frac{3\pi}{2}\right) \end{cases}$$

- Example: binary PSK (BPSK)

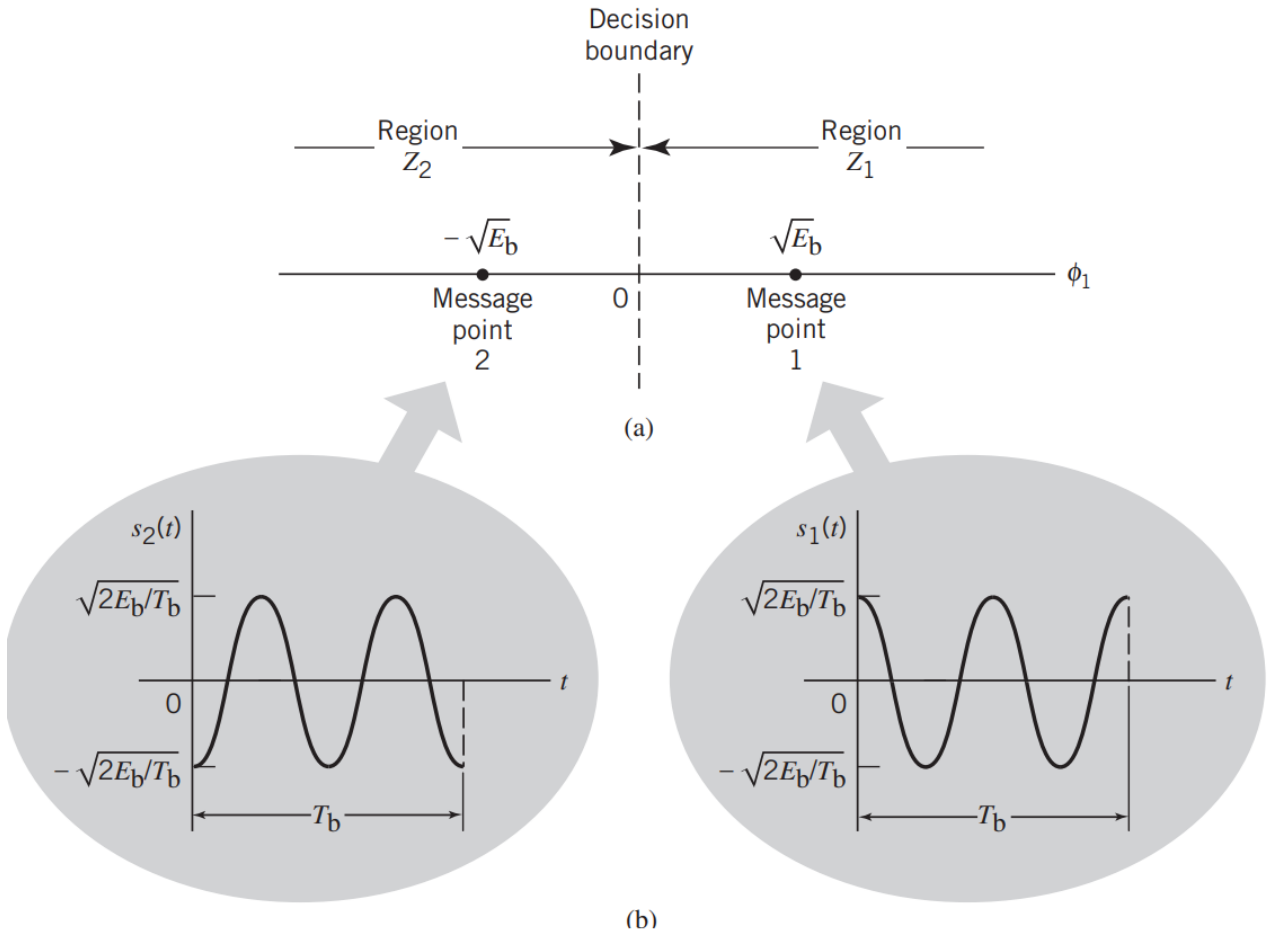


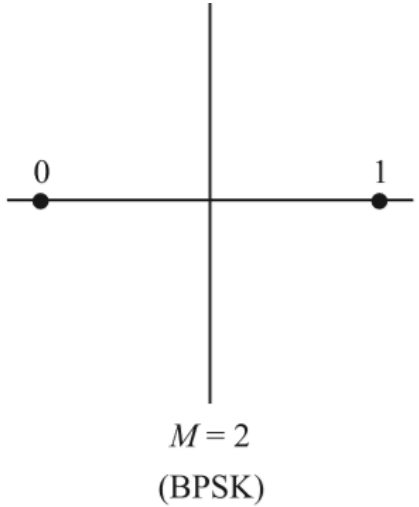
Figure 3: (a) Signal-space diagram for coherent binary PSK system. (b) The waveforms depicting the transmitted signals $s_1(t)$ and $s_2(t)$

Memoryless Modulation Schemes

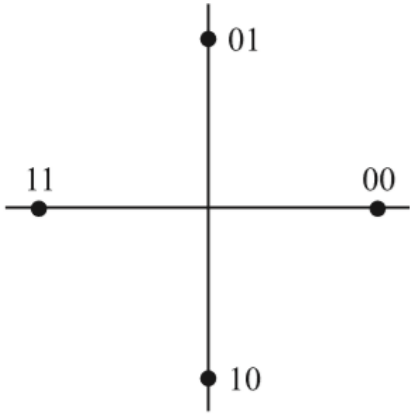
- Phase-modulated signals have equal energy as they are all on a circle.
- The energy of the signal is:

$$\mathcal{E}_{\text{ave}} = \mathcal{E}_m = \int_0^T s_m^2(t)dt = \frac{1}{2}\int_0^T g^2(t)dt = \frac{1}{2}\mathcal{E}_g$$

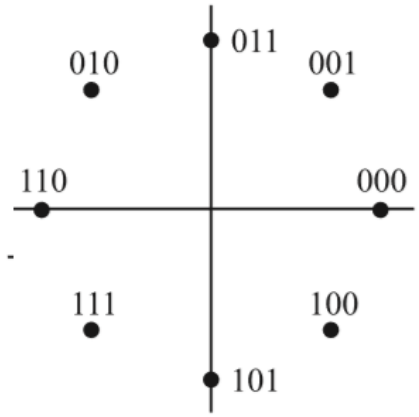
- Signal space diagrams for BPSK, QPSK, and 8-PSK, are shown here.



$M = 2$
(BPSK)



$M = 4$
(QPSK)



$M = 8$
(Octal PSK)

- Noting that $g(t)\cos 2\pi f_c t$ and $g(t)\sin 2\pi f_c t$ are orthogonal, the basis functions $\phi_1(t)$ and $\phi_2(t)$ can be used for QAM signal expansion.

$$\phi_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}}\, g(t)\cos 2\pi f_c t$$

$$\phi_2(t) = -\sqrt{\frac{2}{\mathcal{E}_g}}\, g(t)\sin 2\pi f_c t$$

- These basis can be used for the expansion of $s_m(t)$

$$s_m(t) = \sqrt{\frac{\mathcal{E}_g}{2}}\cos\frac{2\pi}{M}(m-1)\phi_1(t) + \sqrt{\frac{\mathcal{E}_g}{2}}\sin\frac{2\pi}{M}(m-1)\phi_2(t)$$

Vector Representation of MPSK Signals

- Define the mth signal as a vector

$$\mathbf{s}_m = [s_{m1}, s_{m2}]$$

- Then

$$\begin{aligned}\mathbf{s}_m &= \left[\frac{\|g(t)\|}{\sqrt{2}}\cos\theta_m, \quad \frac{\|g(t)\|}{\sqrt{2}}\sin\theta_m\right] \\ &= \left[\sqrt{\frac{\mathcal{E}_g}{2}}\cos\frac{2\pi}{M}(m-1), \quad \sqrt{\frac{\mathcal{E}_g}{2}}\sin\frac{2\pi}{M}(m-1)\right]\end{aligned}$$

  where $m = 1, 2, \ldots, M$.

- The energy of $s_m(t)$ is

$$\mathcal{E}_m = \|\mathbf{s}_m\|^2 = \frac{1}{2}\mathcal{E}_g$$

- The distance between two signals in MPSK is given by:

$$\begin{aligned} d_{mn} &= \|\mathbf{s}_m - \mathbf{s}_n\| \\ &= \frac{\|g(t)\|}{\sqrt{2}}\sqrt{(\cos\theta_m - \cos\theta_n)^2 + (\sin\theta_m - \sin\theta_n)^2} \\ &= \|g(t)\|\sqrt{1+\cos(\theta_m - \theta_n)} \\ &= \sqrt{\mathcal{E}_g\left(1 - \cos\frac{2\pi}{M}(m-n)\right)} \end{aligned}$$

- The minimum Euclidean distance is achieved for adjacent signals, i.e., when $|m-n| = 1$, and is:

$$d_{m,m+1} = \sqrt{\mathcal{E}_g\left(1-\cos\frac{2\pi}{M}\right)} = \sqrt{2\mathcal{E}_g \sin^2\frac{\pi}{M}}$$

- This shows that as the signal space becomes more crowded (i.e., as M increases), the distance between signals decreases, leading to a higher error rate.

Note that $g(t)e^{j\theta_m}$ is not real; hence spectrum is not symmetric and there is no SSB-PM.

2.5 Quadrature Amplitude Modulation (QAM)

- The bandwidth efficiency of SSB-PAM can also be achieved by simultaneously mapping two separate k-bit symbols from the information sequence on two quadrature carriers $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, called **in-phase (I)** and **quadrature-phase (Q)** carriers.
- The resulting modulation technique is called quadrature PAM, or (QAM).

$$s_m(t) = A_{mi} \underbrace{g(t)\cos(2\pi f_c t)}_{\tilde{\phi}_1(t)} - A_{mq} \underbrace{g(t)\sin(2\pi f_c t)}_{\tilde{\phi}_2(t)}, \quad m = 1, 2, \ldots, M$$

where

- A_{mi} and A_{mq} are the signal amplitudes of the I and Q carriers, and
- $g(t)$ is the signal pulse.

$$\Rightarrow \quad \mathbf{s}_m = \left[\frac{A_{mi}}{\sqrt{2}}\|g(t)\|, \quad \frac{A_{mq}}{\sqrt{2}}\|g(t)\|\right]$$

2.5.1 Alternative Representation: QAM = PM (PSK) + PAM (ASK)

QAM signal waveforms may be viewed as a combination of amplitude modulation (r_m) and phase modulation (θ_m), i.e., PAM-PSK.

- With a PAM-PSK representation, the QAM signal waveforms is expressed as:

$$\begin{aligned} s_m(t) &= \Re\left\{r_m e^{j\theta_m} e^{j2\pi f_c t}\right\} \\ &= r_m \cos(2\pi f_c t + \theta_m) \end{aligned}$$

where

$$r_m = \sqrt{A_{mi}^2 + A_{mq}^2},$$

$$\theta_m = \tan^{-1}\left(\frac{A_{mq}}{A_{mi}}\right).$$

- Any combination of M_1-level PAM and M_2-phase PSK can form an $M = M_1 M_2$ combined PAM-PSK signal constellation.
  - Examples of such constellations are shown below.

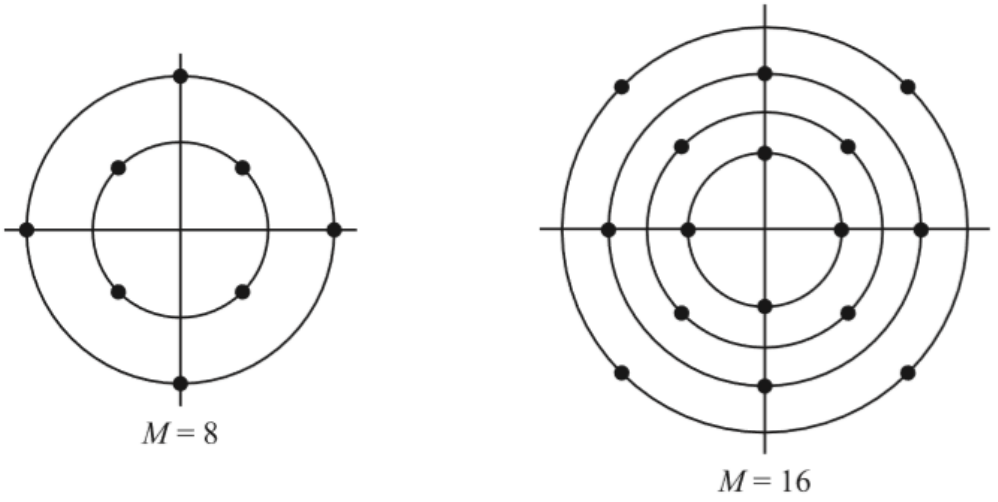


Figure 4: Two combinations of M_1-level PAM and M_2-phase PSK: $M_1 = 2$ and $M_2 = 4$ (left), and $M_1 = M_2 = 4$ (right).

- If $M_1 = 2^{m_1}$ and $M_2 = 2^{m_2}$, then the combined PAM-PSK constellation transmits

$$m_1 + m_2 = \log_2(M_1 M_2) \quad \text{binary digits.}$$

- The **symbol rate** of this transmission is given by:

$$\frac{R_b}{m_1 + m_2}.$$

- Similar to PSK, the basis functions $\phi_1(t)$ and $\phi_2(t)$ can be used for QAM signal expansion.

$$\phi_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}}\, g(t) \cos 2\pi f_c t$$

$$\phi_2(t) = -\sqrt{\frac{2}{\mathcal{E}_g}}\, g(t) \sin 2\pi f_c t$$

- Using this orthonormal basis, the QAM signal representation is:

$$s_m(t) = A_{mi}\sqrt{\frac{\mathcal{E}_g}{2}}\,\phi_1(t) + A_{mq}\sqrt{\frac{\mathcal{E}_g}{2}}\,\phi_2(t).$$

- In the vector form we get

$$\mathbf{s}_m = \left[A_{mi}\sqrt{\frac{\mathcal{E}_g}{2}}, \quad A_{mq}\sqrt{\frac{\mathcal{E}_g}{2}}\right].$$

- The dimensionality of the signal space for QAM is $N = 2$.

2.5.2 Rectangular/Square QAM

- In the special case where the signal amplitudes A_m take the set of discrete values

$$\{(2m - 1 - M),\ m = 1, 2, \ldots, M\},$$

the signal space diagram is rectangular, as shown in figure below.

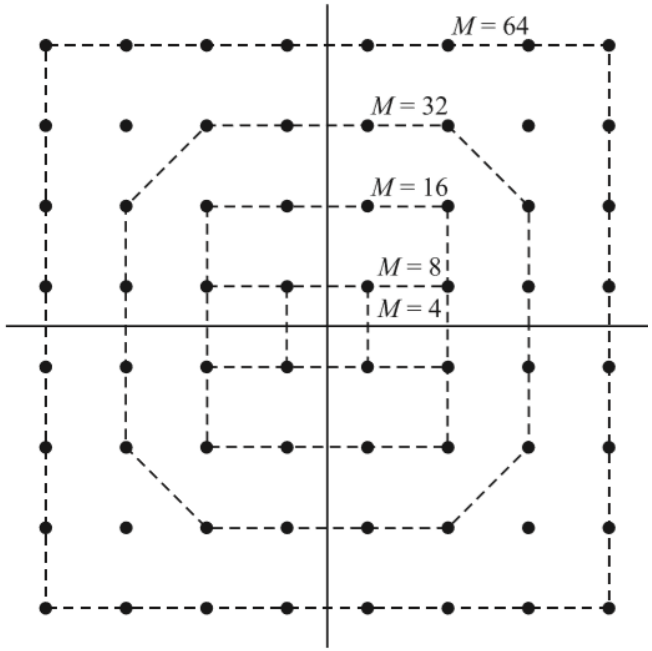
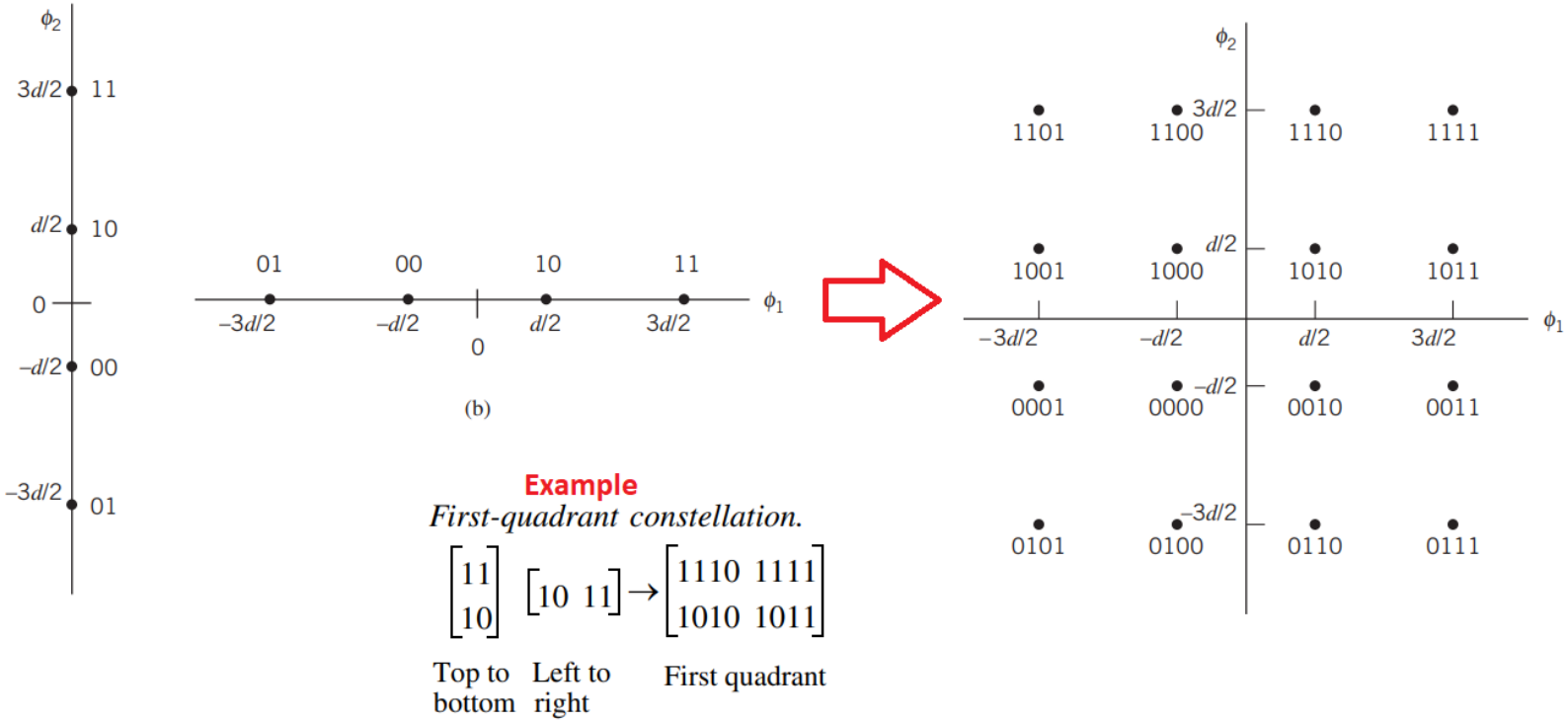


Figure 5: Signal space diagrams for several rectangular/square QAM.

Memoryless Modulation Schemes

Rectangular/square QAM can be viewed as a combination of two PAM signals modulated onto the I and Q carriers.

- Square M-QAM is a combination of two $\sqrt{M}$-PAM constellations.
- Rectangular M-QAM,with $M = \sqrt{M_1 M_2}$, is a combination of $\sqrt{M_1}$-PAM and $\sqrt{M_2}$-PAM.



Example

First-quadrant constellation.

$$\begin{bmatrix} 11 \\ 10 \end{bmatrix} \quad [10\ 11] \rightarrow \begin{bmatrix} 1110 & 1111 \\ 1010 & 1011 \end{bmatrix}$$

Top to bottom | Left to right | First quadrant

- Euclidean distance between $\mathbf{s}_m$ and $\mathbf{s}_n$ for "any" QAM signaling

$$d_{mn} = \|\mathbf{s}_m - \mathbf{s}_n\| = \sqrt{\frac{\mathcal{E}_g}{2}}\sqrt{|A_{mi} - A_{ni}|^2 + |A_{mq} - A_{nq}|^2}$$

- Minimum Euclidean distance for rectangular/square QAM

$$d_{\min} = \min_{m \neq n} \sqrt{\frac{\mathcal{E}_g}{2}}\sqrt{\underbrace{|A_{mi} - A_{ni}|^2}_{=4} + \underbrace{|A_{mq} - A_{nq}|^2}_{=0}} = \sqrt{2\mathcal{E}_g}$$

- Average symbol energy (of square QAM)

$$\begin{aligned}
\mathcal{E}_{\text{avg}} &= \frac{1}{M}\frac{\mathcal{E}_g}{2}\sum_{m=1}^{\sqrt{M}}\sum_{n=1}^{\sqrt{M}}\left(A_m^2 + A_n^2\right) \\
&= \frac{\mathcal{E}_g}{2M}\frac{2M(M-1)}{3} \\
&= \frac{M-1}{3}\mathcal{E}_g
\end{aligned}$$

- Average bit energy

$$\mathcal{E}_{\mathrm{b}} = \frac{M-1}{3\log_2 M}\mathcal{E}_g$$

- Since we had $d_{\min} = \sqrt{2\mathcal{E}_g}$, we get

$$d_{\min} = \sqrt{\frac{6\log_2 M}{M-1}\mathcal{E}_{\mathrm{b}}}$$

Example 3. *What is the average bit energy when $M_1 \neq M_2$?*

- **Summary**

  Bandpass PAM, PSK, and QAM signals share the common representation:

$$s_m(t) = \mathrm{Re}\{A_m g(t) e^{j2\pi f_c t}\}$$

  where A_m varies based on the signaling scheme.

- Amplitude
  - **PAM**: A_m is real and typically takes values $\pm 1, \pm 3, \ldots, \pm(M-1)$.
  - **M-ary PSK**: A_m is complex and given by $e^{j\frac{2\pi}{M}(m-1)}$.
  - **QAM**: A_m is a general complex number $A_m = A_{mi} + jA_{mq}$.
- Unified Perspective
  - PAM and PSK are special cases of QAM.
  - In QAM, both amplitude and phase carry information.
  - In PAM and PSK, only amplitude or phase carries information.

- Modulator Structure
  - Includes a `vector mapper` followed by a `vector-to-signal mapper` (one-dimensional for PAM, two-dimensional for PSK/QAM).

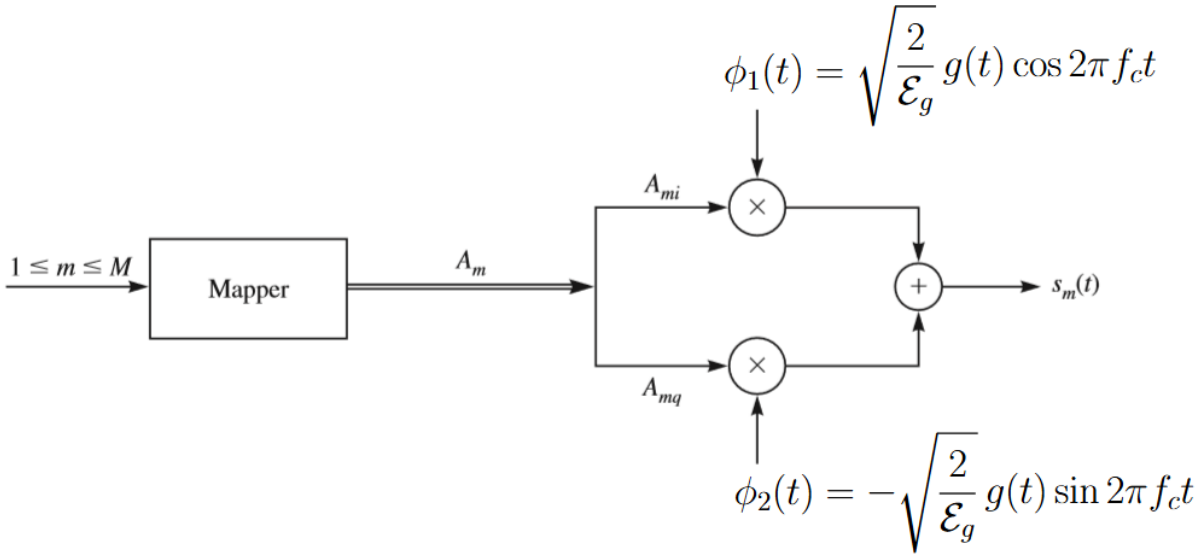


Figure 6: A general QAM modulator.

2.5.3 Applications in Standards

- When M is an even power of 2, we get a square QAM

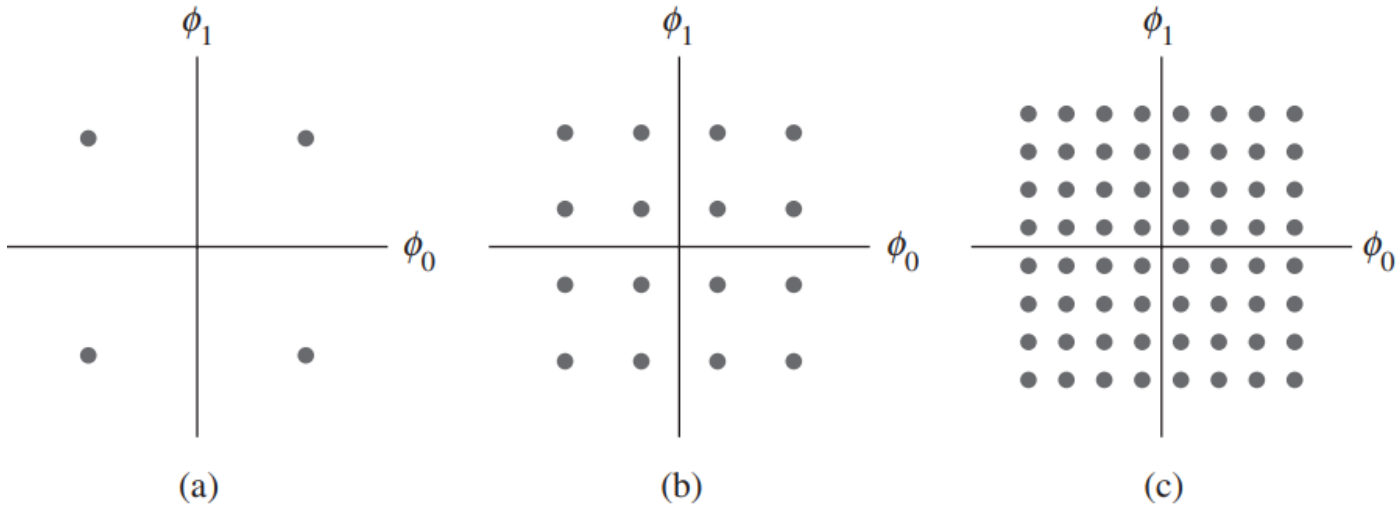


Figure 7: **Square MQAM** constellations for (a) M = 4, (b) M = 16, and (c) M = 64.

- **Cable Modems**:
  - The DOCSIS 1.0 standard uses 64-QAM and 256-QAM for for uplink and QPSK and 16-QAM downlink.
  - Modern DOCSIS versions (DOCSIS 3.1, 4.0) support higher-order QAM (e.g., **4096-QAM**) for improved spectral efficiency.

- **Wireless Modems**:
  - Wi-Fi Standards (e.g., IEEE 802.11ac/ax) employ BPSK, QPSK, 16-QAM, 64-QAM, 256-QAM, and even 1024-QAM for high-speed data transmission.
- **Cellular Networks**:
  - 4G LTE (Long-Term Evolution): Uses adaptive modulation from QPSK to 256-QAM.
  - 5G NR (New Radio): Supports modulation up to **1024-QAM** for enhanced data rates.
- **Satellite and IoT Communications**:
  - Often use lower-order modulations like **BPSK** and **QPSK** due to power and bandwidth constraints.

- When M is an odd power of 2

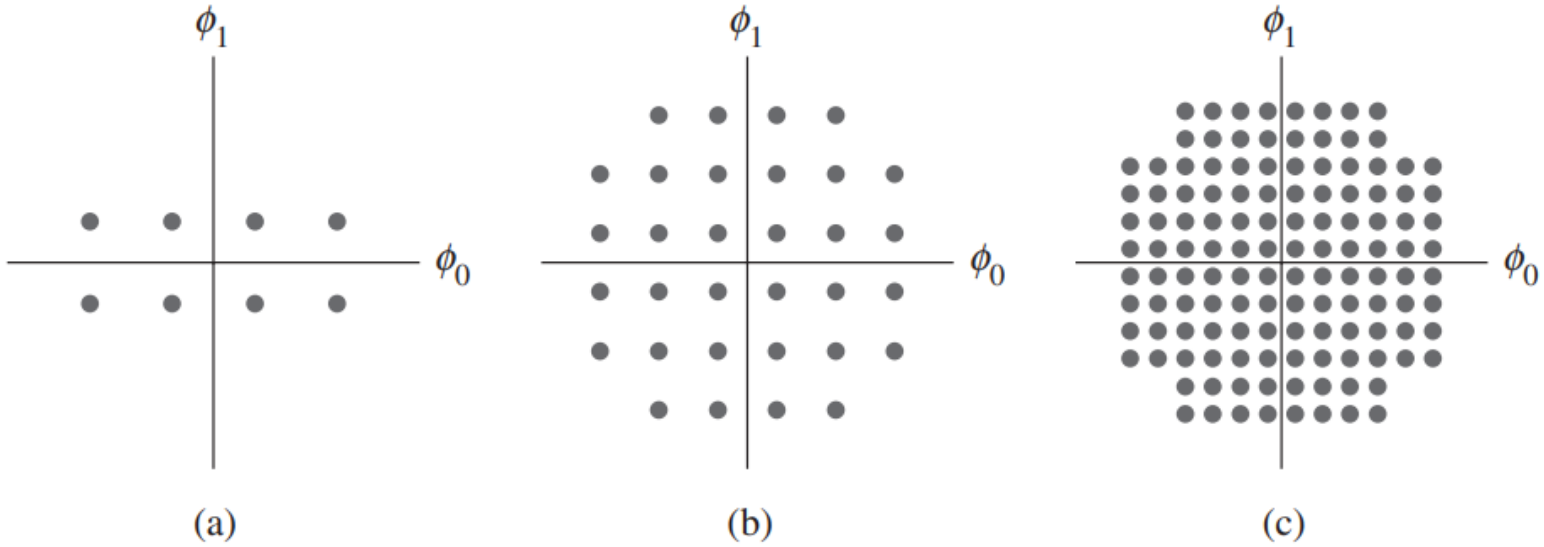


Figure 8: **Cross MQAM** constellations for (a) M = 8, (b) m = 32, and (c) M = 128.

Questions:

- Why are the corner points of the grid removed in Figures (b) and (c)?
- Find a few additional 8-point grids and compare their average power with the one in Figure (a).

2.5.4 Amplitude Phase Shift Keying (APSK)

- Constellations are characterized by points equally spaced on concentric rings in the signal space.

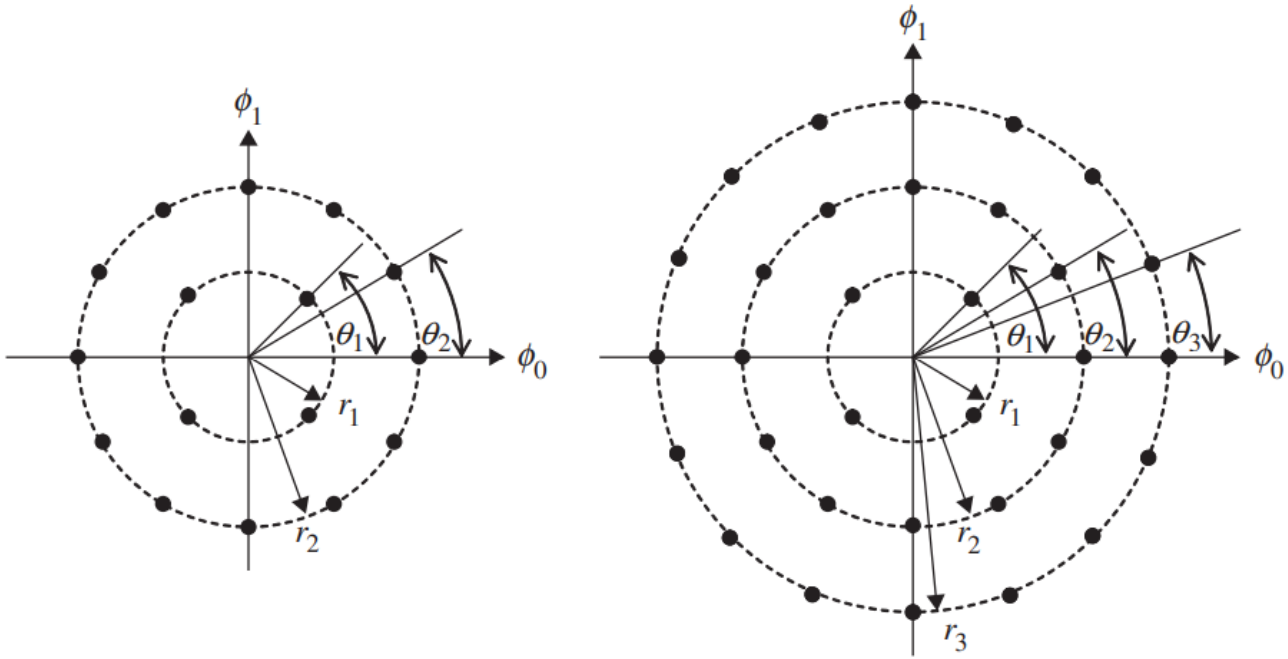
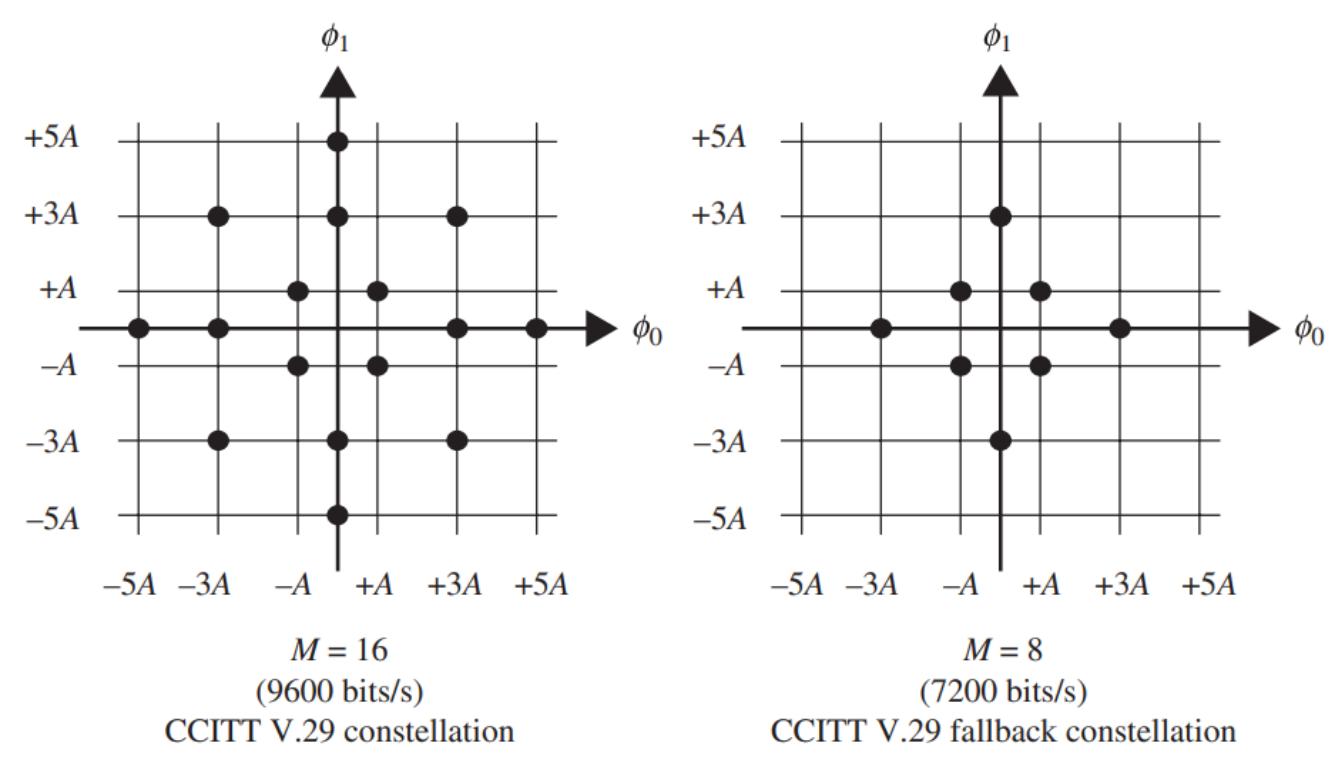


Figure 9: **APSK** constellations for M = 16 and 32.

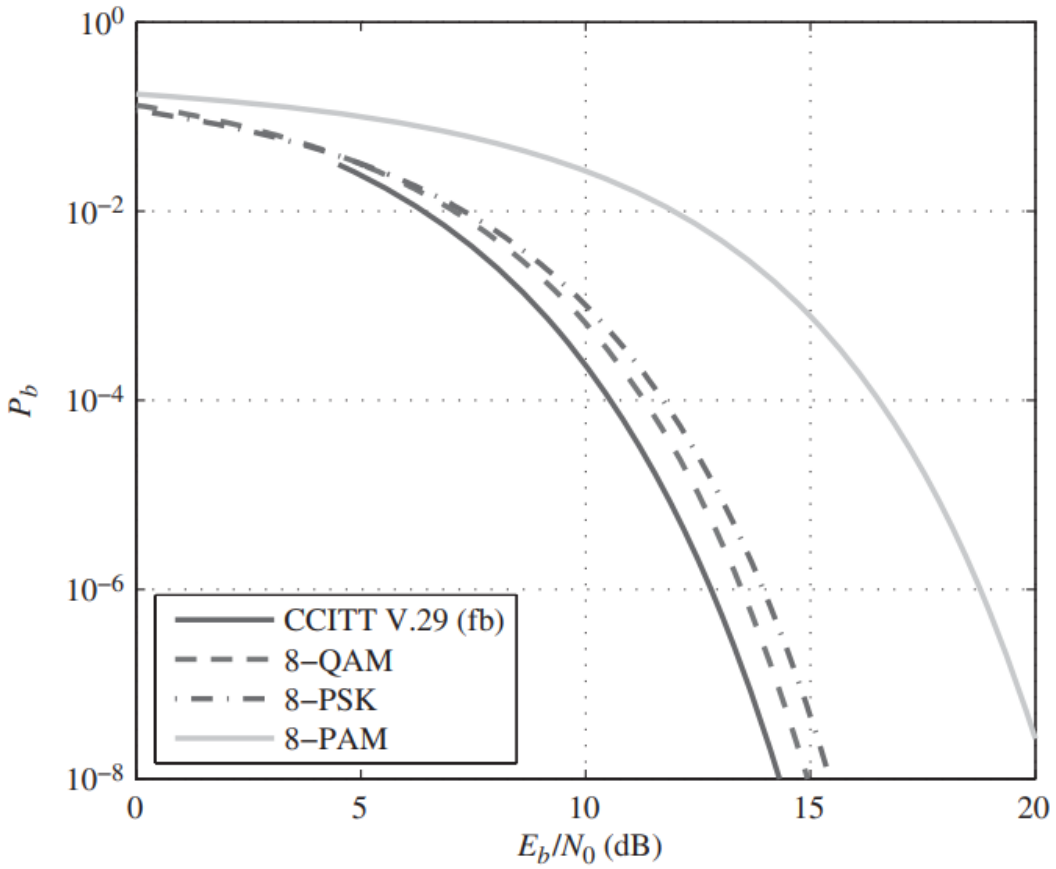
- Key Standards Incorporating APSK
  - **DVB-S2** (Digital Video Broadcasting - Satellite - 2nd Generation)
    Uses **16APSK** and **32APSK** for higher-order modulation, compared to traditional QPSK or 8PSK.
  - **DVB-S2X** (Extended Version of DVB-S2)
    As an extension of DVB-S2, uses **64APSK**, **128APSK**, and **256APSK**.
  - Satellite Broadband and Military Communications
    APSK is also used in proprietary and military satellite communication systems due to its robustness and efficiency in challenging environments such non-linear channels, such as those affected by satellite transponder non-linearities.

Example 4. *APSK constellation used in dial-up modem standard is defined by the CCITT6 V.29 standard.*



- *The default is 16-point constellation*
- *When SNR is not high enough to allow reliable communications using 16 points, the 8 pint constellation is used as a fallback constellation.*

Memoryless Modulation Schemes



the signal-to-noise ratio required to achieve $P_b = 10^{-6}$

Constellation	E_b/N_0 (dB)
CCITT V.29 backup	12.8
8 QAM	13.5
8 PSK	14.0
8 PAM	18.8

2.5.5 Constellations Optimized to Minimize Probability of Error

- We will see in Chapter 4 that the probability of error for M-QAM can be expressed in terms of the Euclidean distances, normalized to the average energy, between the constellation points.
- To minimize the probability of error, the constellation points should be positioned to maximize the normalized Euclidean distances between the points.
- **Optimized Constellations**:
  - Constellations that achieve this for $M = 4, 8, 16$ are illustrated in Figure 10.

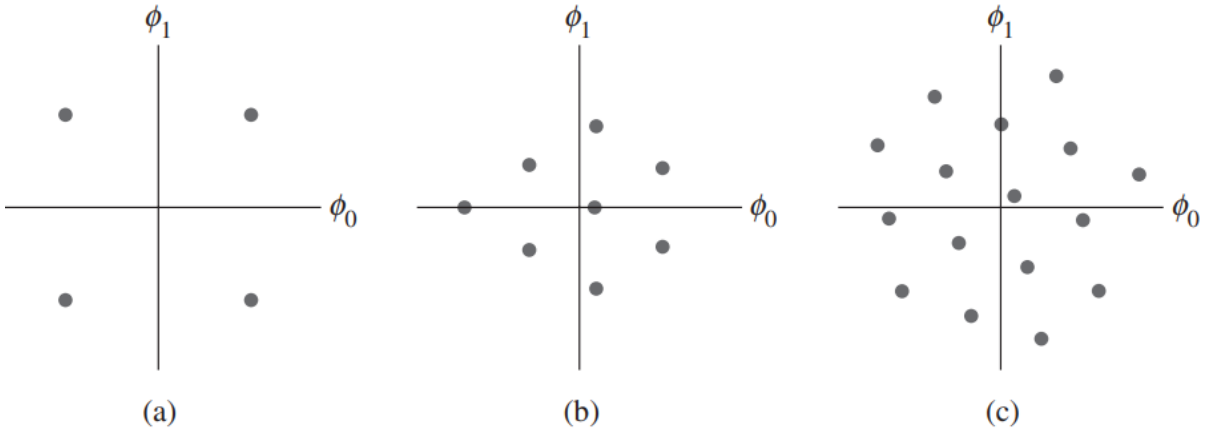


Figure 10: Minimum probability of error constellations for (a) M = 4, (b) M = 8, and (c) M = 16.

– When constellation points are restricted to lie on a **regular grid** or in **concentric circles**, the same optimization principles apply.

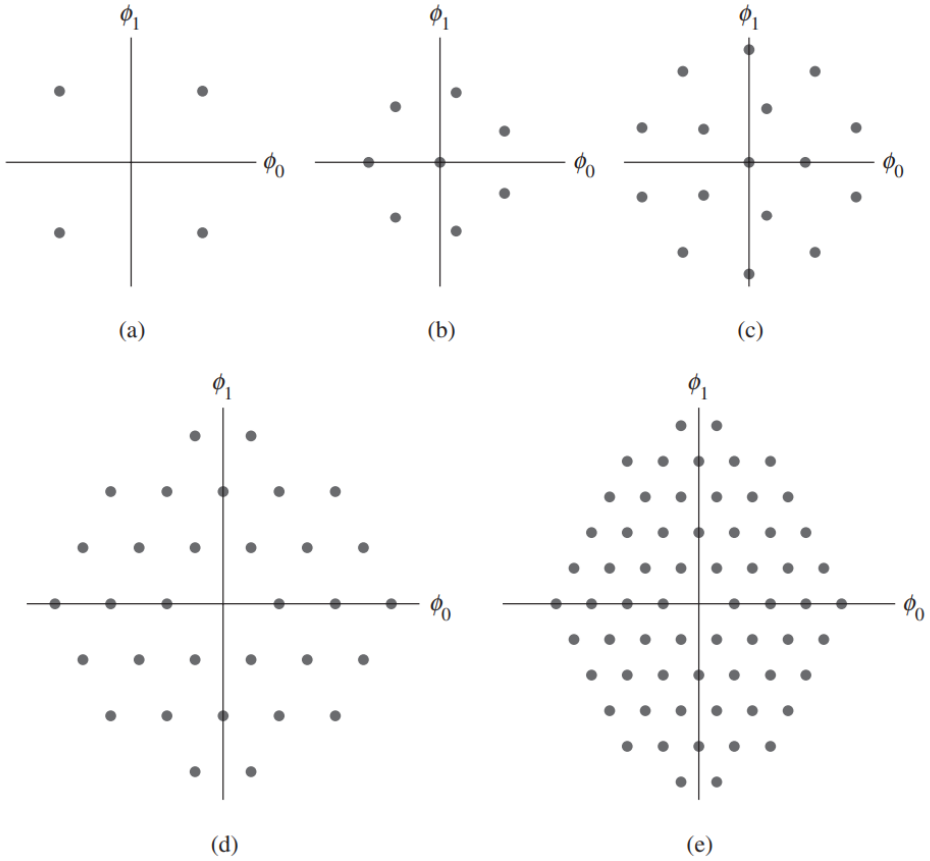


Figure 11: Minimum probability of error constellations when **the points are constrained to lie on a grid or concentric circles** for (a) M = 4, (b) M = 8, (c) M = 16, (d) M = 32, (e) M = 64.

2.6 Multidimensional Signaling

- Modulation we have considered so far are one or two dimensional
  - **PAM**: one-dimensional
  - **PM**: two-dimensional
  - **QAM**: two-dimensional
- How to create three or higher dimensional signal?

How to create three or higher dimensional signal?

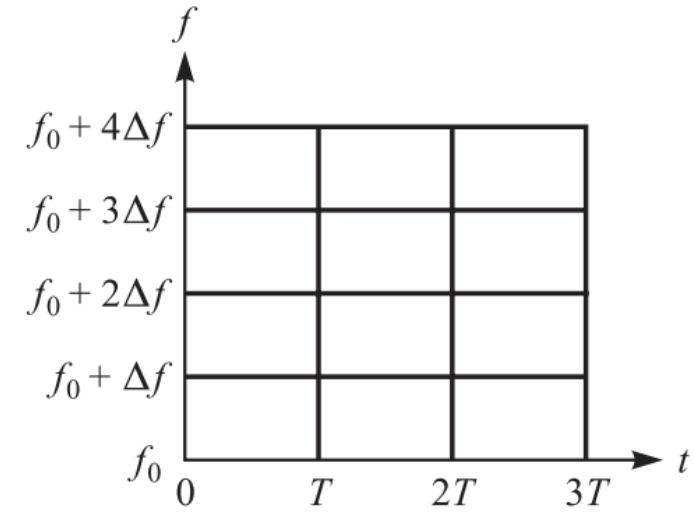
① **Subdivision of time**

Example. N time slots can be used to form $2N$ vector basis elements (each has I/Q bearers).

② **Subdivision of frequency**

Example. N frequency subbands can be used to form $2N$ vector basis elements (each has two quadrature bearers).

③ **Subdivision of both time and frequency**



2.6.1 Frequency Shift Keying (FSK)

- FSK subdivides frequency. Bandpass FSK is given by

$$s_m(t) = \sqrt{\frac{2\mathcal{E}}{T}} \cos\left(2\pi f_c t + 2\pi(m\Delta f)t\right) \qquad 1 \le m \le M, \quad 0 \le t \le T \qquad (1)$$

- The transmission energy $\mathcal{E}_m$ of an FSK signal is given by:

$$\mathcal{E}_m = \int_0^T |s_m(t)|^2 \, dt = \mathcal{E},$$

  where $\mathcal{E}$ represents the equal transmission power for each channel symbol.

- Lowpass equivalent

$$s_{m,\ell}(t) = \sqrt{\frac{2\mathcal{E}}{T}} e^{i2\pi(m\Delta f)t} \quad \text{and} \quad \|s_{m,\ell}(t)\| = \sqrt{2\mathcal{E}}$$

- QAM is a **linear modulation** whereas FSK is a **nonlinear modulation**.
  - the sum of two QAM (and ASK and PSK) signals is another QAM signal. (low-pass equivalent signal is $A_m g(t)$. If we add two, we get another signal.)
  - FSK signaling does not satisfy this property, and therefore it belongs to the class of **nonlinear modulation** schemes.

$$\rho_{mn,\ell} = \frac{\langle s_{m,\ell}(t), s_{n,\ell}(t)\rangle}{\|s_{m,\ell}(t)\| \cdot \|s_{n,\ell}(t)\|} = \frac{1}{T}\int_0^T e^{i2\pi(m-n)\Delta f t} dt$$

$$= \text{sinc}[T(m-n)\Delta f] e^{i\pi T(m-n)\Delta f}$$

$$\frac{\langle s_m(t), s_n(t)\rangle}{\|s_m(t)\|\|s_n(t)\|} = \Re\{\rho_{mn,\ell}\} = \frac{\sin(\pi T(m-n)\Delta f)}{\pi T(m-n)\Delta f}\cos(\pi T(m-n)\Delta f)$$

$$= \text{sinc}(2T(m-n)\Delta f)$$

– $\Delta f = \frac{k}{2T} \quad \Longrightarrow \quad \Re\{\rho_{mn,\ell}\} = 0$ for $m \neq n$.

– Thus, the minimum frequency separation between adjacent signals for **orthogonality** is

$$\Delta f = \frac{1}{2T}.$$

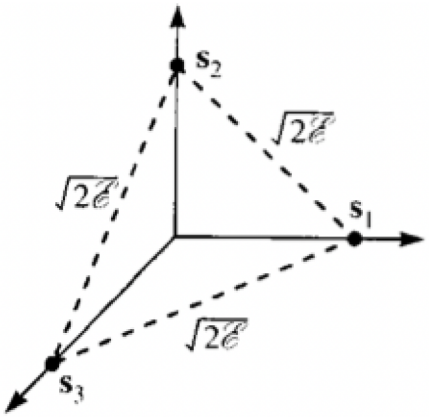
– This guarantees the orthogonality of both base-band and band-pass signals.

- Vectorization of FSK signals under orthogonality conditions

$$\phi_m(t) = \frac{1}{\sqrt{\mathcal{E}}} s_m(t) \quad \text{and} \quad \mathbf{s}_m = \left[0, \ldots, 0, \underbrace{\sqrt{\mathcal{E}}}_{\text{m-th position}}, 0, \ldots, 0\right]^{\mathrm{T}}$$

Signal Space Diagram for FSK

$$\left[\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_M\right] = \begin{bmatrix} \mathcal{E} & 0 & 0 & \cdots & 0 \\ 0 & \mathcal{E} & 0 & \cdots & 0 \\ 0 & 0 & \mathcal{E} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \mathcal{E} \end{bmatrix}$$



$$d_{mn} = \|s_m - s_n\| = \sqrt{2\mathcal{E}}$$

- **Example:** binary FSK

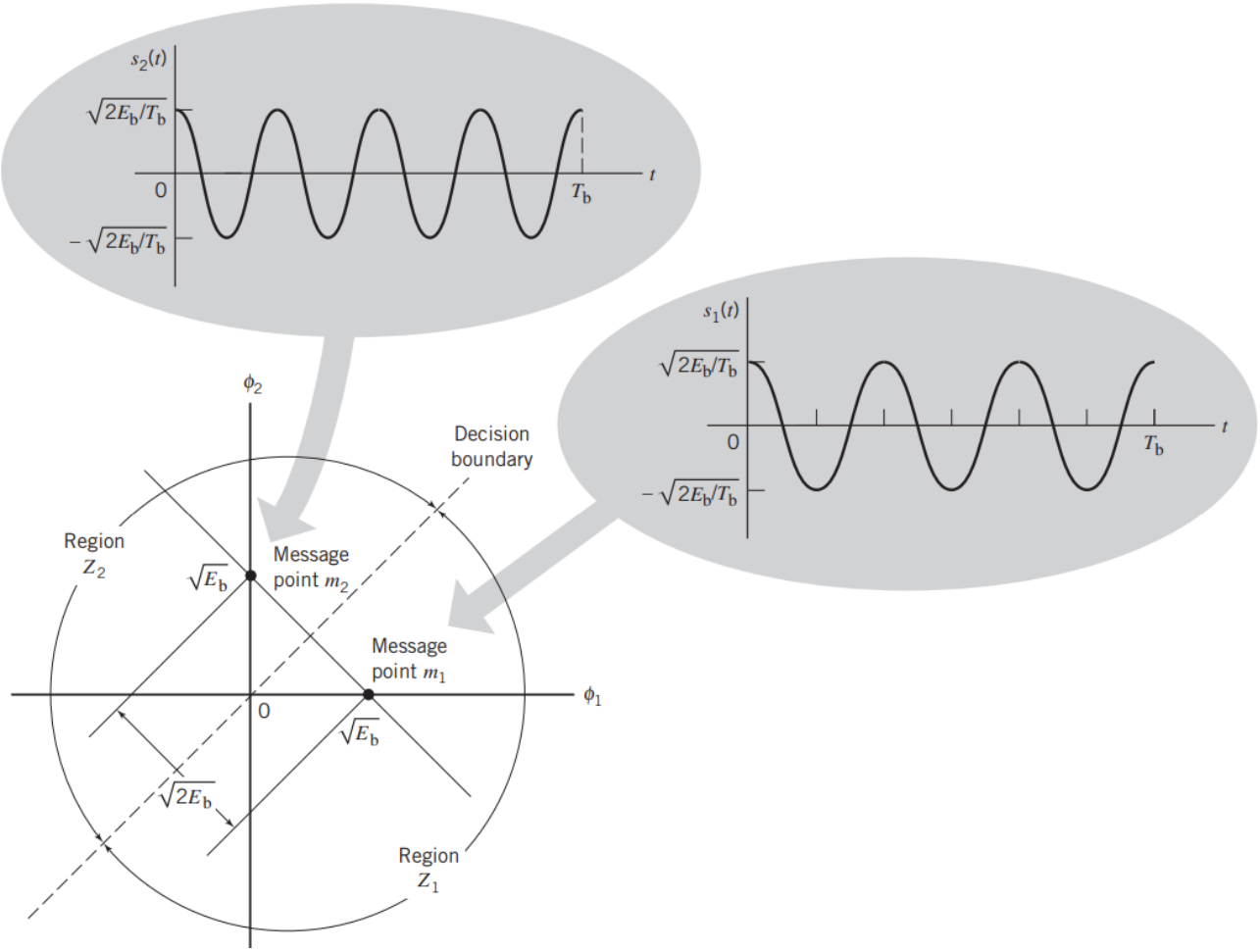


Figure 12: Signal-space diagram for binary FSK system. The diagram also includes example waveforms of the two modulated signals $s_1(t)$ and $s_2(t)$. In binary case, $E_b = \mathcal{E}$ and $T_b = T$.

2.6.2 Hadamard Signals

Definition 1. *The* ***Hadamard matrix*** *of size* $2^n \times 2^n$ *can be recursively defined as*

$$H_n = \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{bmatrix}$$

with initial value $H_0 = [1]$.

Example:

$$H_1 = \left[\begin{array}{c|c} 1 & 1 \\ \hline 1 & -1 \end{array}\right] \quad \textit{and} \quad H_2 = \left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ \hline 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{array}\right]$$

- Hadamard matrices are symmetric matrices whose rows (and, columns) are orthogonal.
- **Hadamard signals** are orthogonal signals which are constructed from Hadamard matrices.

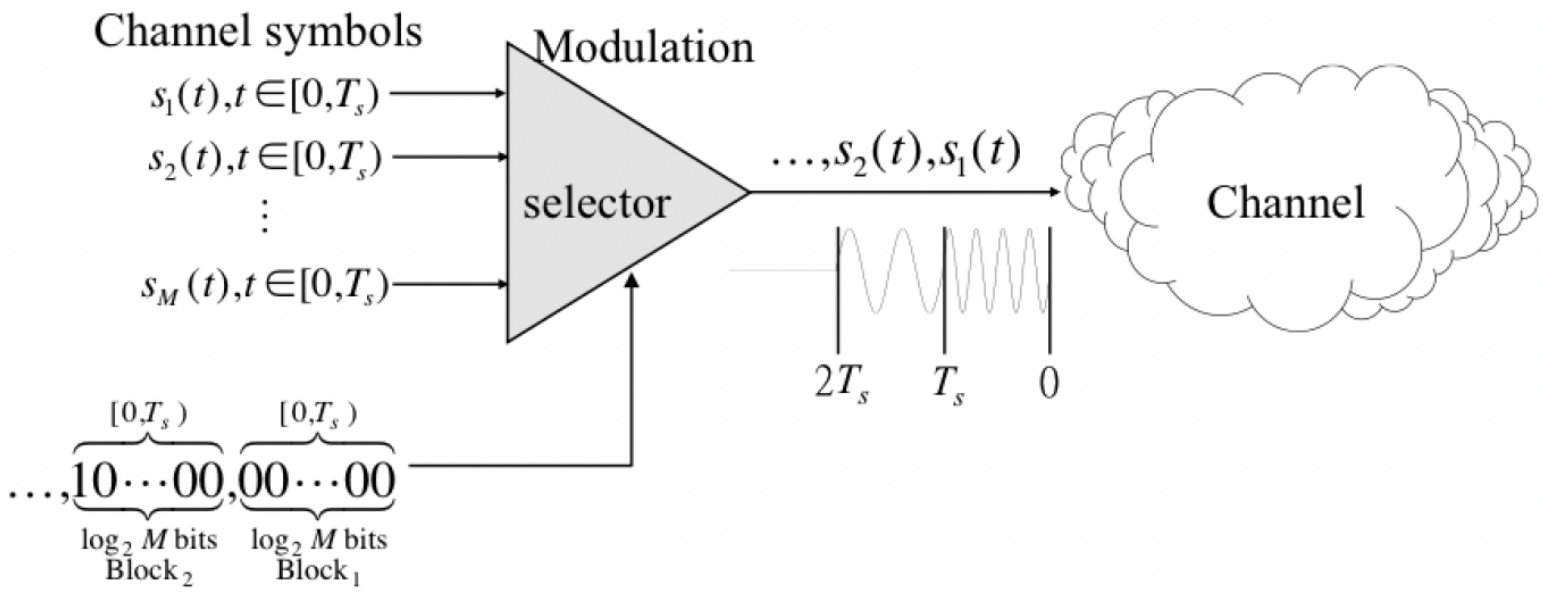
- For instance, using H_2 would result in the set of orthogonal signals:

$$\begin{aligned}
\mathbf{s}_1 &= \left[\sqrt{\mathcal{E}} \quad \sqrt{\mathcal{E}} \quad \sqrt{\mathcal{E}} \quad \sqrt{\mathcal{E}}\right], \\
\mathbf{s}_2 &= \left[\sqrt{\mathcal{E}} \quad -\sqrt{\mathcal{E}} \quad \sqrt{\mathcal{E}} \quad -\sqrt{\mathcal{E}}\right], \\
\mathbf{s}_3 &= \left[\sqrt{\mathcal{E}} \quad \sqrt{\mathcal{E}} \quad -\sqrt{\mathcal{E}} \quad -\sqrt{\mathcal{E}}\right], \\
\mathbf{s}_4 &= \left[\sqrt{\mathcal{E}} \quad -\sqrt{\mathcal{E}} \quad -\sqrt{\mathcal{E}} \quad \sqrt{\mathcal{E}}\right].
\end{aligned}$$

- The above are the Hadamard multidimensional orthogonal signals for $M = 4$
- This set of orthogonal signals may be used to modulate any four-dimensional orthonormal basis $\{\phi_j(t)\}_{j=1}^4$ to generate signals of the form:

$$s_m(t) = \sum_{j=1}^{4} s_{mj}\phi_j(t), \quad 1 \le m \le 4$$

3 Signaling Schemes with Memory



- **Memoryless modulation:** $s_{m_i}(t)$; $m_i \in \{1, 2, \ldots, M\}$,

$$m_i = \text{function of Block}_i.$$

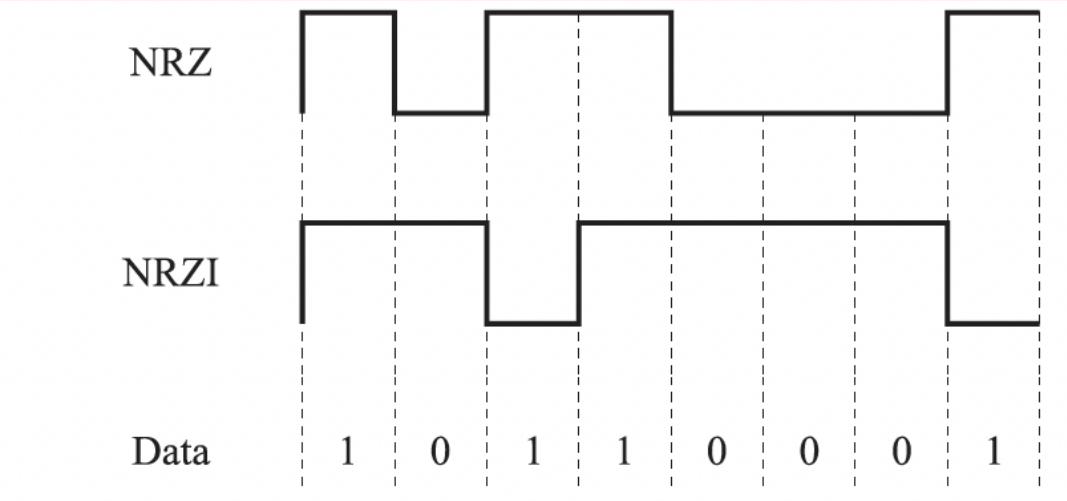
- **Modulation with memory:** $s_{m_i}(t)$,

$$m_i = \text{function of } (\text{Block}_i, \text{Block}_{i-1}, \ldots, \text{Block}_{i-(L-1)}).$$

Signaling Schemes with Memory

- **Why Introduce Memory in Signals?**
  - to shape the spectrum of the transmitted signal, ensuring it aligns with the spectral characteristics of the channel.
  - spectral shaping helps
    - ◇ optimize signal transmission,
    - ◇ reduce interference, and
    - ◇ increases robustness to phase ambiguity and frequency offsets.
- **Examples:** Differential BPSK/QPSK (DBPSK/DQPSK)
- **Application:** Wireless LAN standards like IEEE 802.11 (Wi-Fi)

Example 5. ***Differential Encoding:***



- **NRZ (Non-Return-to-Zero)**:
  - Binary 1: Rectangular pulse of polarity A.
  - Binary 0: Rectangular pulse of polarity $-A$.
  - Memoryless, equivalent to binary PAM or binary PSK in carrier-modulated systems.
- **NRZI (Non-Return-to-Zero, Inverted)**:
  - Transitions occur only when a **1** is transmitted.

– Amplitude remains unchanged for **0**.

– Uses **differential encoding**, defined as:

$$b_k = a_k \oplus b_{k-1},$$

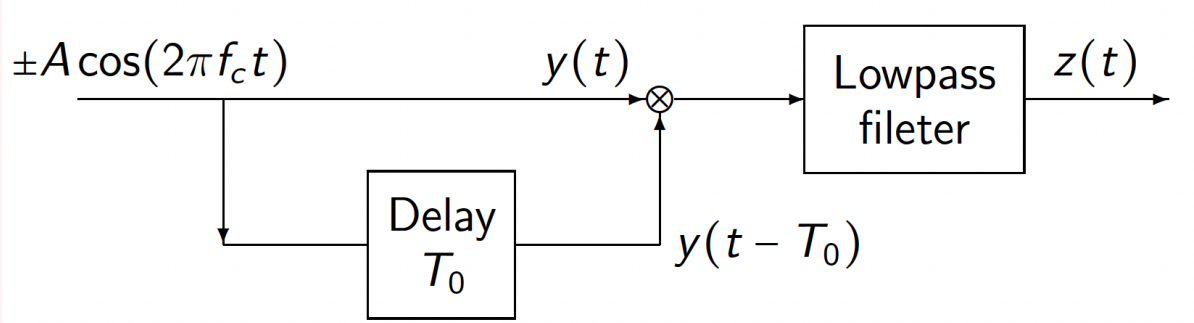
where

◇ $\{a_k\}$ is the binary information sequence into the encoder,

◇ $\{b_k\}$ is the output sequence of the encoder, and

◇ $\oplus$ denotes addition modulo 2.

Example 6. ***Non-coherent Demodulation of Differential BPSK.***



$$z(t) = \begin{cases} A_c^2 \cos^2(2\pi f_c t) = \frac{A_c^2}{2} + \frac{1}{2}\cos(4\pi f_c t) \rightarrow \frac{1}{2}A_c^2, & \text{if } y(t) = y(t - T_0) \\ -A_c^2 \cos^2(2\pi f_c t) = -\frac{A_c^2}{2} - \frac{1}{2}\cos(4\pi f_c t) \rightarrow -\frac{1}{2}A_c^2, & \text{if } y(t) = -y(t - T_0) \end{cases}$$

- **Advantages:**
  - The phase or signs of the received waveforms are not important for detection.
  - What is important is the change in the sign of successive pulses.
  - The sign change can be detected even if the carrier has a phase ambiguity.
  - No need to generate a local carrier (at the receiver side).
  - Use the received signal itself as a carrier.

Why Differential BPSK?

- In **PSK modulation**, digital information is carried by **absolute phase**.
- Synchronization is typically achieved using a **pilot signal** or a **self-synchronization** scheme.
- The demodulator must detect the phase, but **phase ambiguity** can arise due to noise and other constraints.
- **Differential encoding** resolves this issue by encoding data based on **phase changes** rather than absolute phase.
- **Ideal (noiseless) case**

$$\begin{cases} f_{\text{Tx}} = f_c : \text{receive } \cos(2\pi f_c t + \theta) \\ f_{\text{Rx}} = f_c : \text{estimation it based on } f_c \end{cases} \quad \Rightarrow \quad \text{estimate } \hat{\theta} = \theta$$

- **Ambiguous case**

$$\begin{cases} f_{\text{Tx}} = f_c : \text{receives } \cos(2\pi f_c' t + \theta) \\ f_{\text{Rx}} = f_c' \neq f_c : \text{estimate it based on } f_c' \end{cases}$$

Receives

$$\cos\left(2\pi f_c' t + [2\pi(f_c - f_c')t + \theta]\right)$$

estimate it based on f_c'

$$\Rightarrow \quad \text{estimate } \hat{\theta} = [2\pi(f_c - f_c')t] + \theta$$

3.1 Continuous Phase FSK (CPFSK)

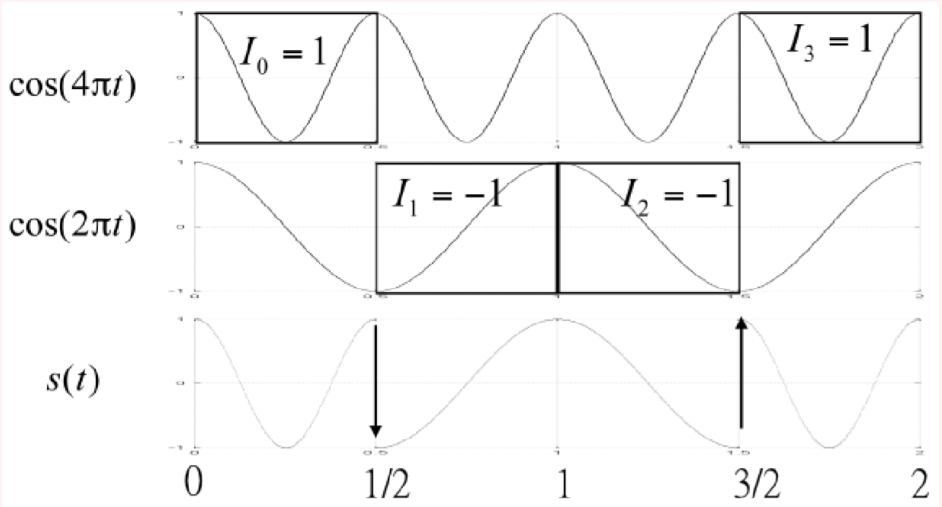
- **Q:** Why CPFSK? What is wrong with the FSK?
- **A:** Phase discontinuity $\Longrightarrow$ larger bandwidth

	FSK	**CPFSK**
Phase	Discontinuous phase, causing abrupt changes.	Continuous phase, leading to smooth transitions.
BW Efficiency	Requires more bandwidth due to abrupt transitions.	More spectrally efficient due to smoother transitions.
Detection	Can be detected non-coherently (simpler)	Requires coherent detection for optimal performance.
Complexity	Simple hardware implementation, widely used in low-power systems.	More complex due to phase memory but improves error performance.
Applications	Low-power RF (Bluetooth, RFID, pagers).	GSM, military/satellite communications, bandwidth-limited or power-efficient systems.

Example 7. *Consider an FSK signaling with the following parameters:* $T = 0.5, \Delta f = 0.5, f_c = 1.5, \mathcal{E} = 0.25$. *Plot* $s(t)$ *and the phase of* $s_\ell(t)$ *when the input sequence is* $\boldsymbol{I}$ = 1,-1,-1,1, ...

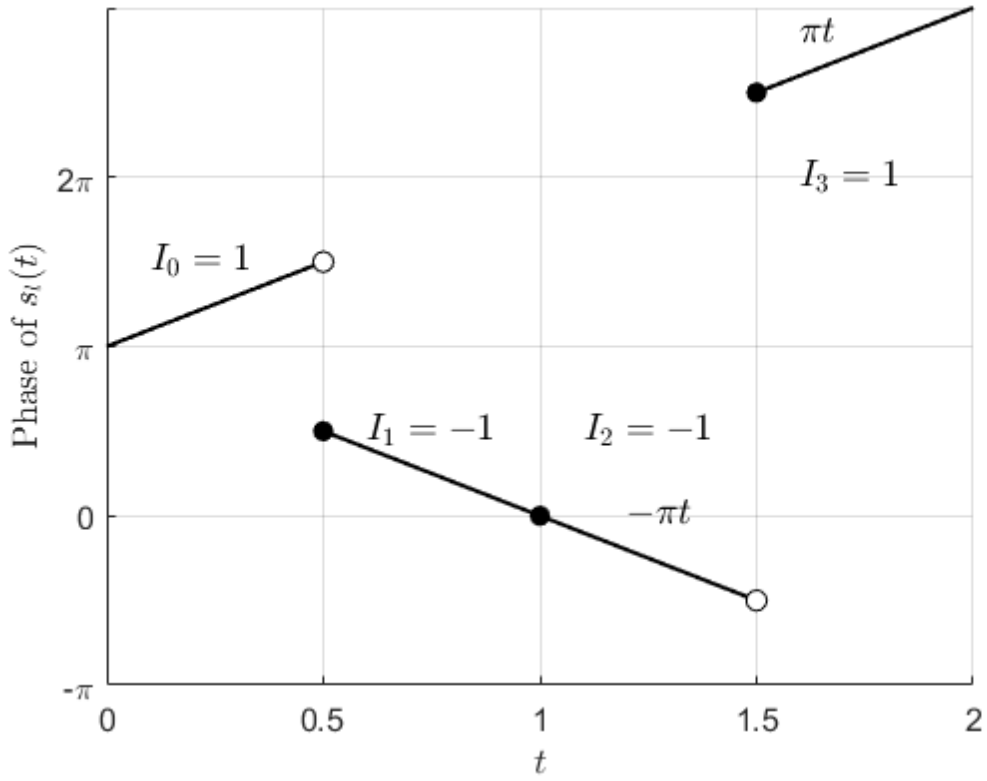
$$s_m(t) = \sqrt{\frac{2\mathcal{E}}{T}} \cos\left(2\pi f_c t + 2\pi(m\Delta f)t\right) \qquad m = \pm 1, \quad 0 \le t \le T$$

$$\Longrightarrow \quad s(t) = \begin{cases} \cos 4\pi t, & I_n = 1 \\ \cos 2\pi t, & I_n = -1 \end{cases} \qquad \text{for } t \in [nT, (n+1)T)$$



- Phase of $s_\ell(t)$ is not continuous

$$\text{Phase of } s_\ell(t) = \begin{cases} \pi t, & I_n = 1 \\ -\pi t, & I_n = -1 \end{cases} \quad \text{for } t \in [nT, (n+1)T)$$



- Consider a **binary** continuous-phase frequency-shift keying (CPFSK) signal, which is defined for the signaling interval $0 \leq t \leq T_b$ as follows:

$$s(t) = \begin{cases} \sqrt{\frac{2\mathcal{E}}{T_b}} \cos(2\pi f_1 t + \theta(0)) & \text{for symbol 1} \\ \sqrt{\frac{2\mathcal{E}}{T_b}} \cos(2\pi f_2 t + \theta(0)) & \text{for symbol 0} \end{cases} \tag{2}$$

  where $\mathcal{E}$ is the transmitted signal energy and T_b is the bit (symbol) duration.

  - This equation distinguishes itself from FSK equation in using the phase $\theta(0)$
  - $\theta(0)$ is the value of the phase at time $t = 0$

- Another useful way of representing the CPFSK signal is to express it as conventional angle-modulated signal:

$$s(t) = \sqrt{\frac{2\mathcal{E}}{T_b}} \cos(2\pi f_c t + \theta(t)) \tag{3}$$

  - $\theta(t)$, the phase at time t, is a continuous function of time

- In a CPFSK signal, assuming a rectangular pulse $g(t)$, the phase $\theta(t)$ increases or decreases linearly with time during each bit duration of T_b seconds, as shown by:

$$\theta(t) = \theta(0) \pm \left(\frac{\pi h}{T_b}\right) t, \quad 0 \le t \le T_b$$

  where
  - the plus sign corresponds to sending symbol 1,
  - the minus sign corresponds to sending symbol 0, and
  - the dimensionless parameter h is called frequency deviation.

- Comparing the angle of the cosine function with that of (2), we get

$$f_c + \frac{h}{2T_b} = f_1$$
$$f_c - \frac{h}{2T_b} = f_2$$

- Solving this pair of equations for f_c and h, we get

$$f_c = \frac{1}{2}(f_1 + f_2), \qquad \text{and} \qquad h = T_b(f_1 - f_2).$$

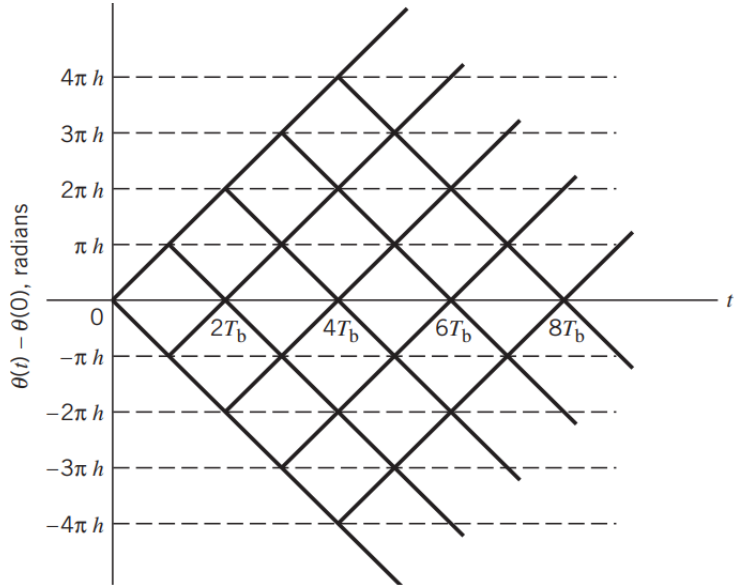
- Then, at time $t = T_b$, we have

$$\theta(T_b) - \theta(0) = \begin{cases} \pi h, & \text{for symbol 1} \\ -\pi h, & \text{for symbol 0} \end{cases}$$

  – sending symbol 1 increases the phase of a CPFSK signal πh radians,

  – sending symbol 0 reduces it by an equal amount.

- The variation of phase $\theta(t)$ with time t follows a path consisting of a sequence of straight lines. The **phase tree** is shown below:



- **Phase Continuity in CPFSK:**
  - The above phase tree demonstrates phase continuity, a key property of CPFSK signals.
  - Sunde's FSK in (1), a form of CPFSK, has a deviation ratio $h = 1$, leading to phase changes of $\pm\pi$ radians per bit interval.
  - Since $+\pi$ and $-\pi$ are equivalent modulo 2π, there is *no memory* in Sunde's FSK, meaning past phase changes provide no information about the current interval.
- **Special Case: Deviation Ratio $h = 1/2$**
  - The phase can take on values
    - $\pm\pi/2$ at odd multiples of T_b, and
    - only 0 or π at even multiples.
  - This results in a **phase trellis** structure (Figure 13), where each path represents a specific binary sequence.

> Phase trellis = Phase tree plotted modulo 2π, say, in the range $(-\pi, \pi)$

– **Example:** The bold path corresponds to the binary sequence 1101000 with $\theta(0) = 0$.

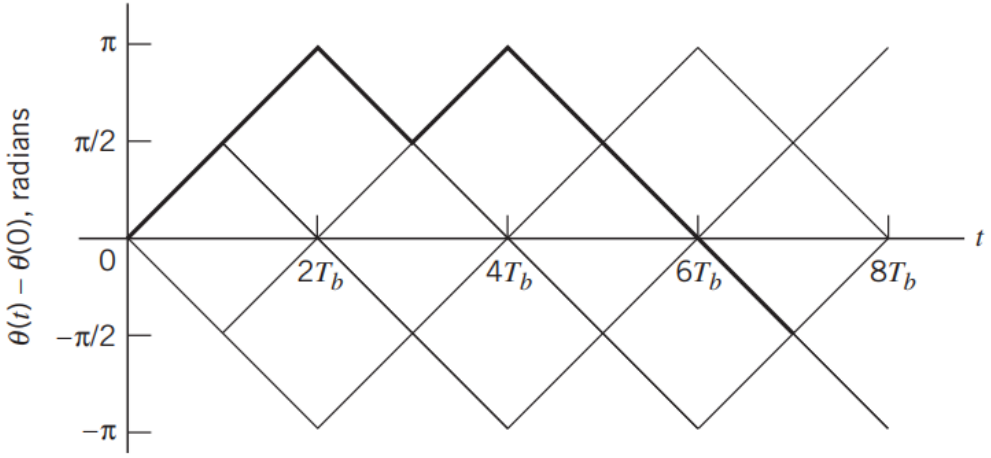


Figure 13: Phase trellis–**boldfaced path** represents the sequence **1101000**.

- CPM can be decoded by Viterbi trellis decoding.

3.2 Minimum Shift-Keying (MSK)

- With $h = 1/2$, the frequency deviation (difference between f_1 and f_2) equals half the bit rate.

$$\begin{cases} f_1 &= f_c + \frac{h}{2T_b} = f_c + \frac{1}{4T_b} \\ f_2 &= f_c - \frac{h}{2T_b} = f_c - \frac{1}{4T_b} \end{cases} \Longrightarrow \Delta f = |f_1 - f_2| = \frac{1}{2T_b}$$

- $h = 1/2$ is the minimum frequency spacing that allows the two FSK signals representing symbols 1 and 0 to be coherently **orthogonal**.
- Such a signaling is called *minimum shift keying (MSK) modulation.*
- MSK has
  - constant envelope,
  - continuous phase transition, and
  - excellent spectral efficiency

3.3 Continuous-Phase Modulation (CPM)

- CPFSK is a speciall case of a general class of continuous-phase modulation (CPM)
- In general, in a CPM signal can be represented as

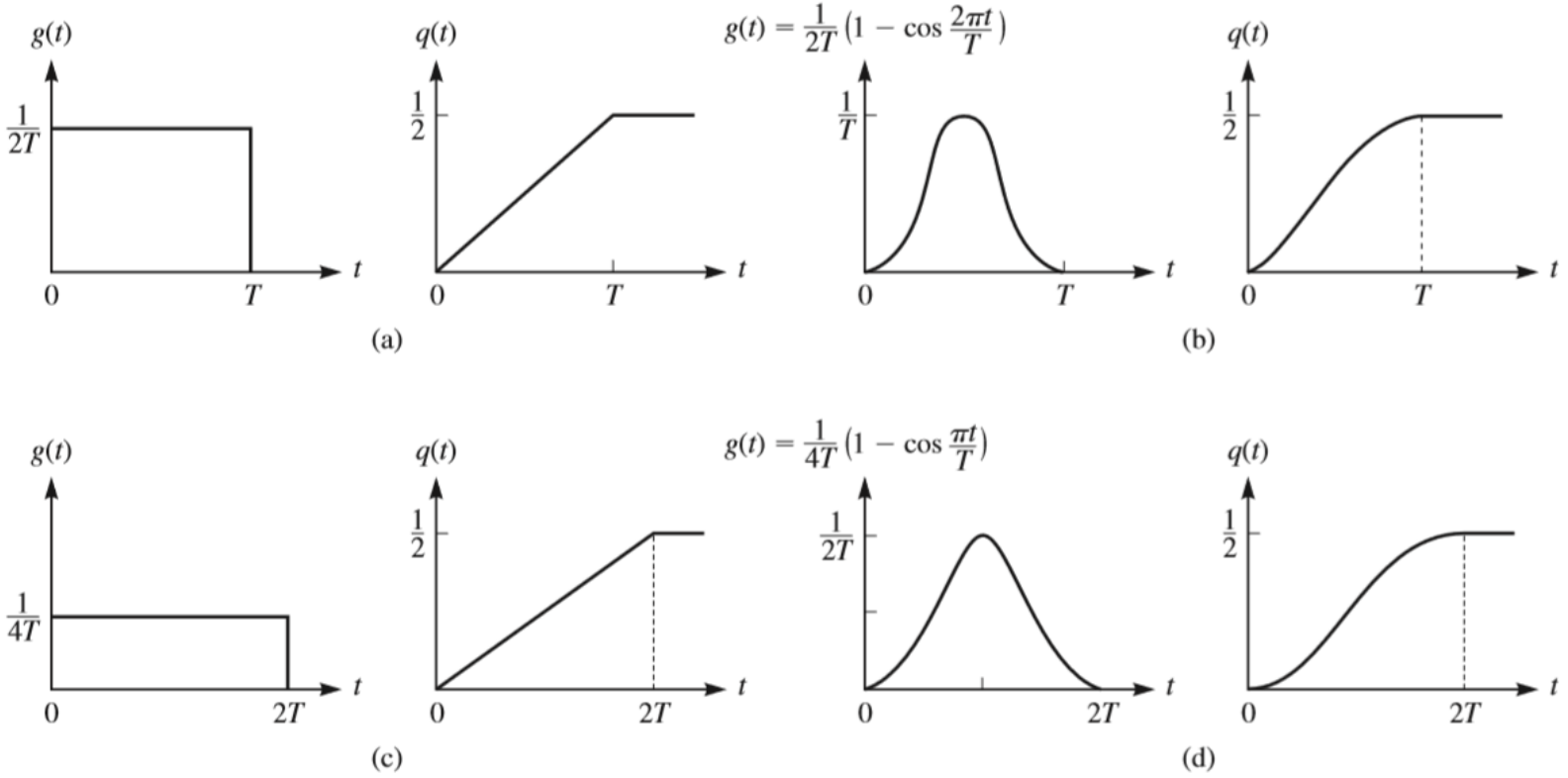
$$s(t) = \sqrt{\frac{2\mathcal{E}}{T}} \cos\left[2\pi f_c t + \phi(t;\mathbf{I}) + \phi_0\right]$$

where $\phi(t;\mathbf{I})$ represents the time-varying phase of the carrier given by

$$\phi(t;\mathbf{I}) = 2\pi \sum_{k=-\infty}^{n} I_k h_k q(t-kT), \quad nT \le t \le (n+1)T \tag{4}$$

  - $\{I_k\}$ is the sequence of M-ary information symbols selected from the alphabet $\pm 1, \pm 3, \ldots, \pm(M-1)$,
  - $\{h_k\}$ is a sequence of modulation indices, and
  - $q(t)$ is some normalized waveform shape, obtained by the integral of pulse $g(t)$, i.e.,

$$q(t) = \int_0^t g(\tau) d\tau.$$



- CPFSK is a spacial case of CPM in which
  - $g(t)$ is a rectangle pulse with duration T and amplitude $1/2T$ (Fig. (a))
  - it has a fixed modulation index h for all symbols. Specifically,

$$h_k = h = 2\Delta f T$$

– Therefore, for CPFSK the instantaneous frequency is

$$\begin{aligned} f_i = \frac{1}{2\pi}\frac{\partial \theta_i}{\partial t} &= \frac{1}{2\pi}\frac{\partial}{\partial t}(2\pi f_c t + \phi(t;\mathbf{I}) + \phi_0) \\ &= f_c + h\sum_{k=-\infty}^{n} I_k g(t-kT) \\ &= f_c + \Delta f \sum_{k=-\infty}^{n} I_k \text{rect}(t-kT) \end{aligned}$$

This gives a clue as to why $\phi(t;\mathbf{I})$ is defined as in (4).

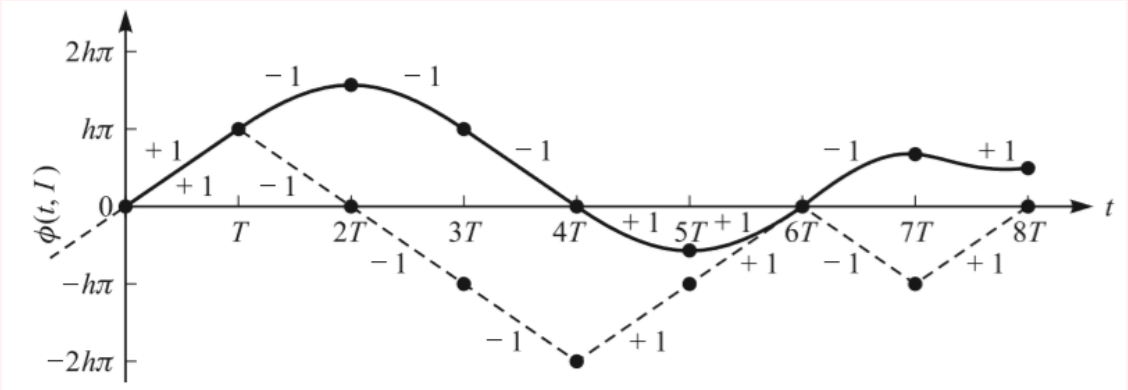
Example 8. *For the sequence of* $\boldsymbol{I}_n$

$$\boldsymbol{I}_n = (+1, -1, -1, -1, +1, +1, -1, +1, \ldots)$$

plot the phase trajectories for CPFSK (rectangular $g(t)$*) as well as CPM with the function* $g(t)$ *is defined as:*

$$g(t) = \frac{1}{6T}\left(1 - \cos\left(\frac{2\pi t}{3T}\right)\right)$$

which is a raised cosine function of length $3T$*.*



*Phase trajectories for binary CPFSK (**dashed**) and binary, CPM based on raised cosine pulse of length* $3T$ *(**solid**)*

- Commonly used CPM pulse shapes
  - **LREC (Rectangular)**:

$$g(t) = \frac{1}{2LT}\left[u(t) - u(t - LT)\right]$$

    LREC with $L = 1$ is CPFSK
  - **LRC (Raised Cosine)**:

$$g(t) = \frac{1}{2LT}\left[u(t) - u(t - LT)\right]\left(1 - \cos\left(\frac{2\pi t}{LT}\right)\right)$$

  - **GMSK Pulse**

$$g(t) = \frac{Q\left(2\pi B\left(t - \frac{T}{2}\right)\right) - Q\left(2\pi B\left(t + \frac{T}{2}\right)\right)}{\sqrt{\ln 2}}$$

3.4 Gaussian Minimum Shift-Keying (GMSK)

- **GMSK**: The function $g(t)$ is given by

$$g(t) = \frac{Q\left(2\pi B\left(t - \frac{T}{2}\right)\right) - Q\left(2\pi B\left(t + \frac{T}{2}\right)\right)}{\sqrt{\ln 2}}$$

  where the $Q(\cdot)$ is the Q-function and B represents the 3 dB bandwidth.

  – GMSK with $BT = 0.3$ is used in the *global system for mobile communications* (GSM), the most successful 2G system.

  – At $BT = 0.3$, the GMSK pulse may be truncated at $t = 1.5T$ with a relatively small error incurred.

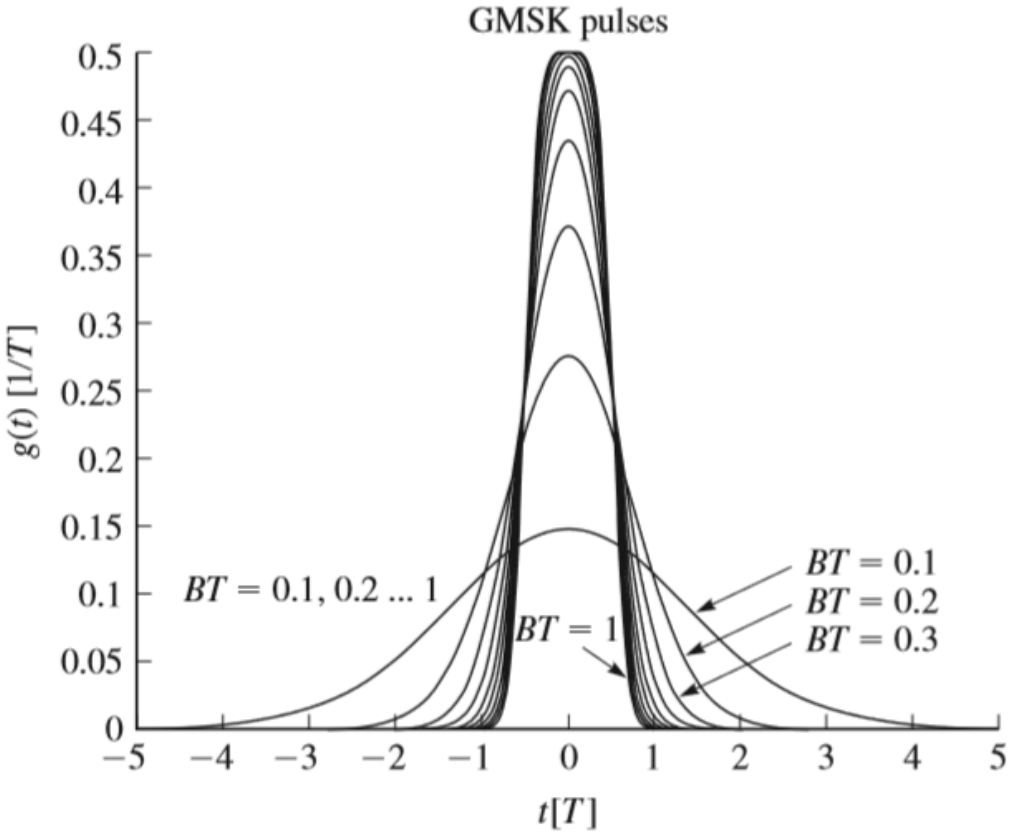


Figure 14: Pulse shapes for GMSK.

3.5 Other Variants of QPSK

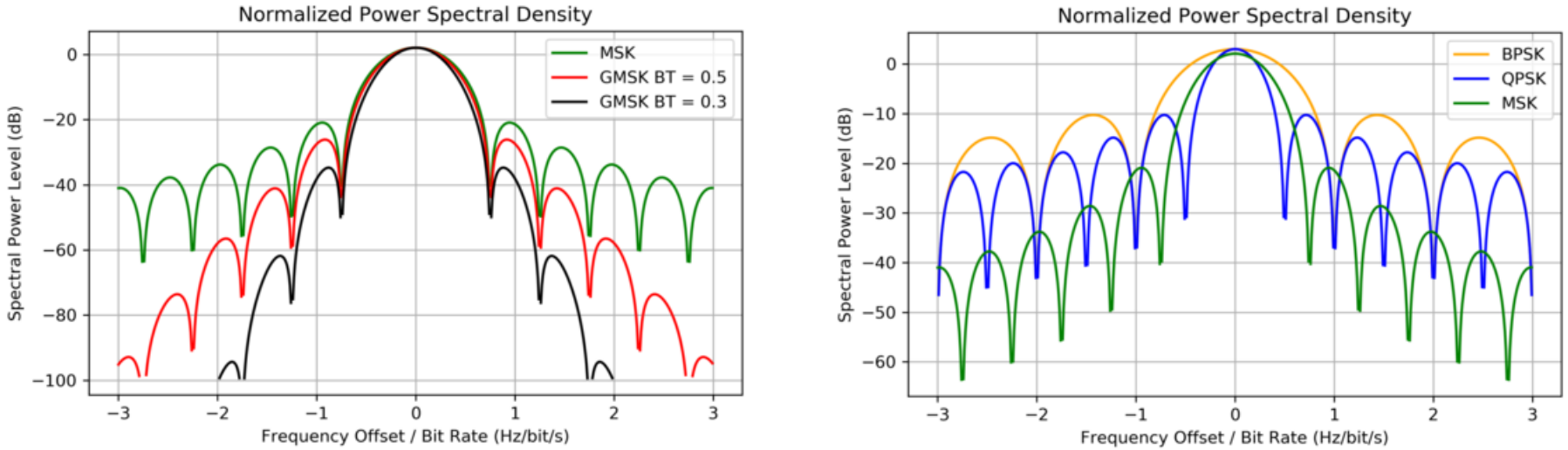


Figure 15: Power spectral densities of MSK and GMSK and other signaling.

3.5.1 QPSK

- Consider a QPSK system with the constellation shown in Figure 16.
- Each pair of 2 information bits is mapped to a constellation point.
- **Example:** The binary sequence **11000111** is split into **11**, **00**, **01**, and **11**, and the corresponding constellation points are transmitted.
- The first bit in each pair determines the I component, and the second bit deter-

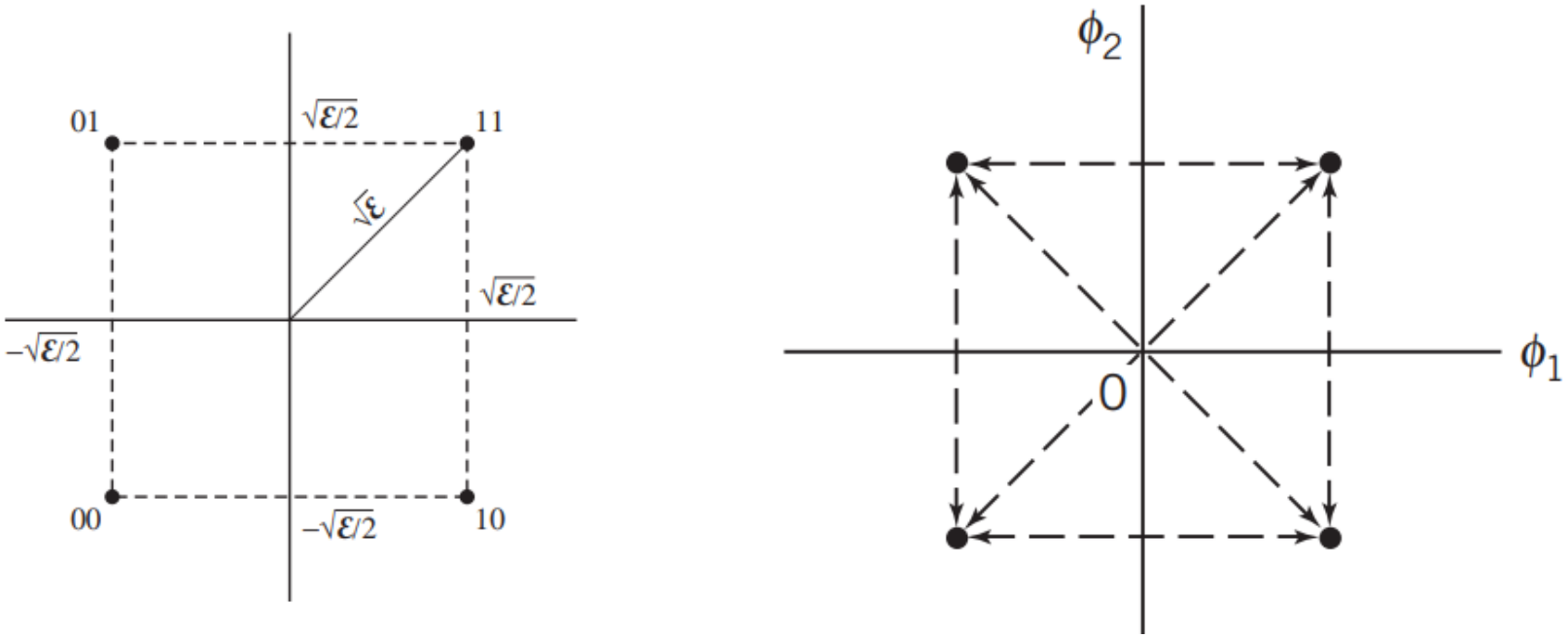


Figure 16: (left) A possible mapping for QPSK signal. (right) All possible paths for switching between the message points in QPSK.

mines the Q component of the baseband signal, each with a duration of $2T_b$.

- Changes in I and Q components occur only at even multiples of T_b.
- Simultaneous changes in I and Q components result in a 180° phase shift (e.g., at $t = 2T_b$ in Figure 17).

- Possible phase transitions for QPSK signals occur only at nT_b, where n is even.

The binary sequence **11000111**

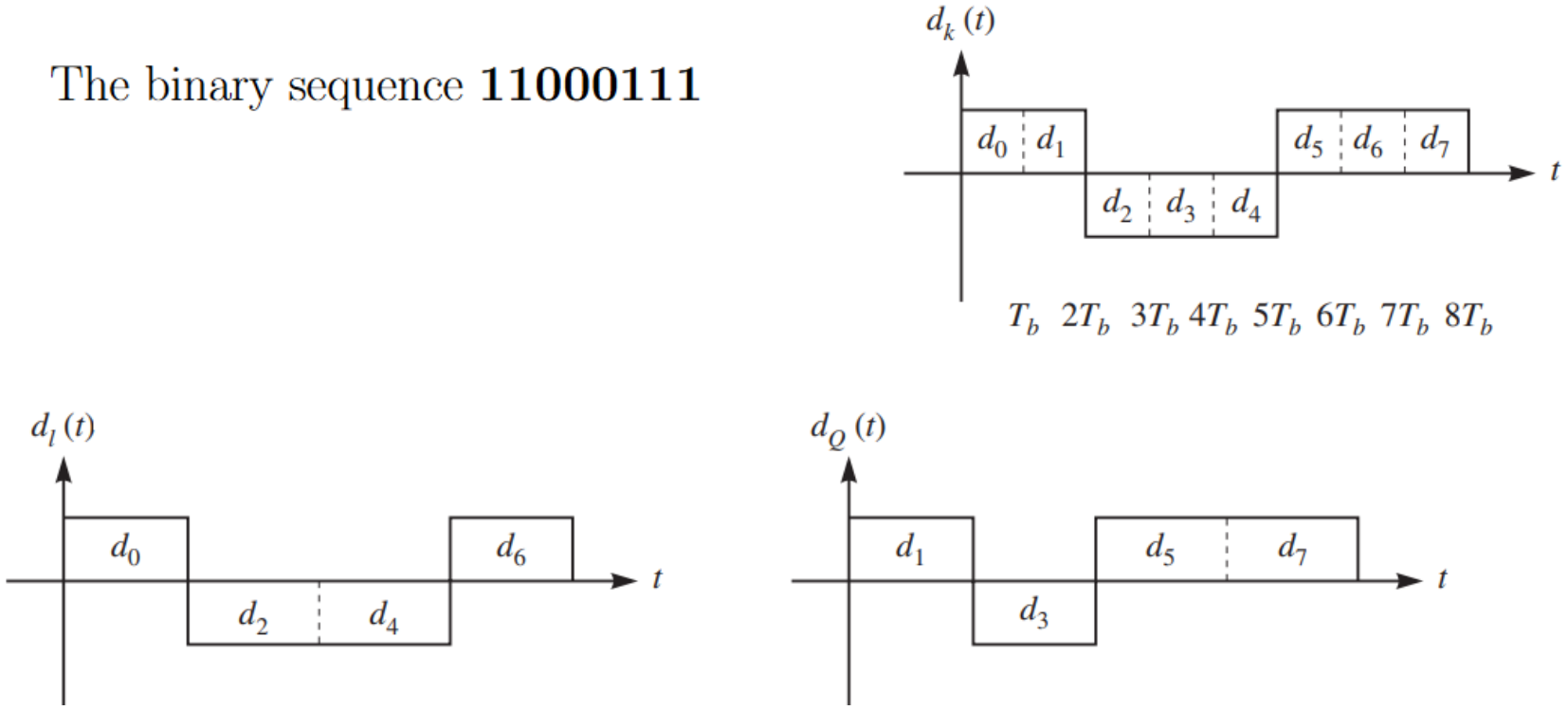


Figure 17: The I and Q components for QPSK for the binary sequence **11000111**.

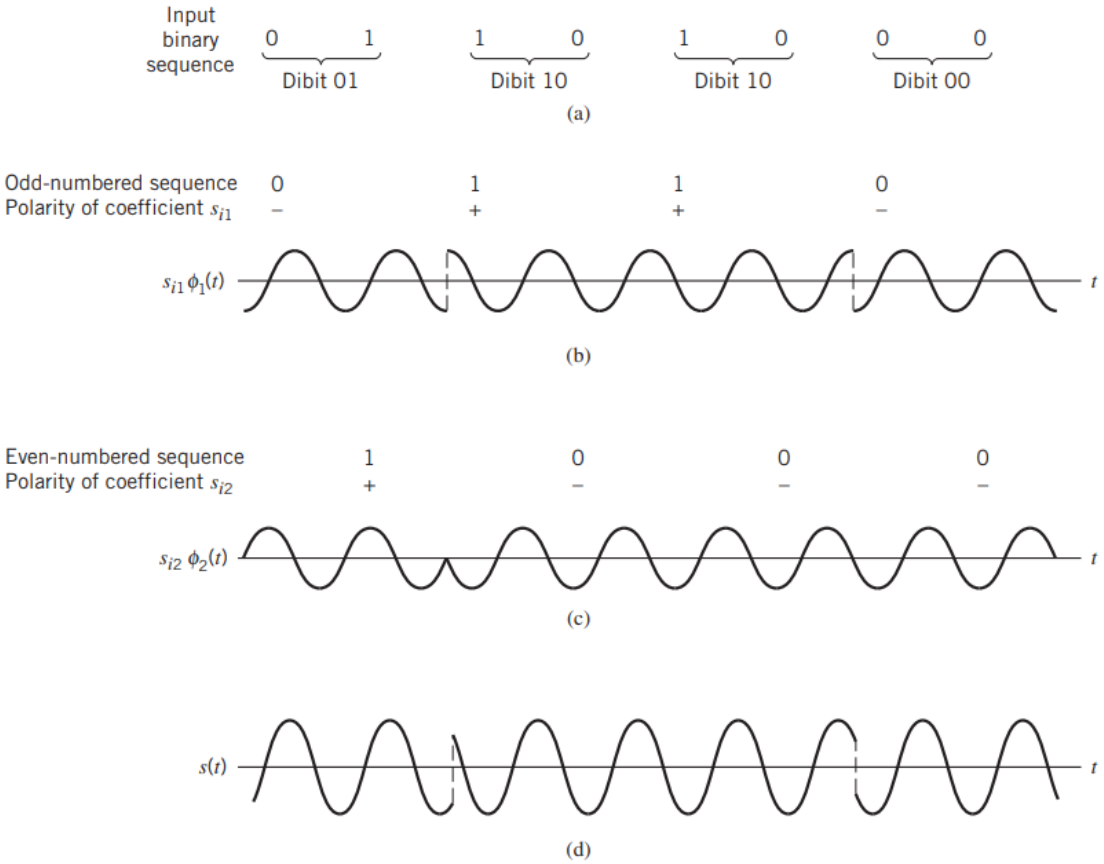


Figure 18: **(a)** Input binary sequence. **(b)** The odd-numbered bits and their associated binary PSK signal. **(c)** The even-numbered bits from the input sequence and their associated binary PSK signal. **(d)** The resulting QPSK signal $s(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$ where s_{i1} and s_{i2} are the I and Q components, respectively, and $\phi_1(t) = \sqrt{2/T}\cos(2\pi f_c t)$ and $\phi_2(t) = \sqrt{2/T}\sin(2\pi f_c t)$ are the basis functions.

3.5.2 Offset QPSK

- Offset QPSK (OQPSK), or staggered QPSK (SQPSK), prevents abrupt 180° phase changes that cause large spectral side lobes.
  - In OQPSK, the I and Q components are misaligned by T_b. (Q component is delayed (i.e., offset) by half a symbol interval with respect to the I component)
  - Specifically, the two basis functions of OQPSK are defined by

$$\text{QPSK basis} \quad \begin{cases} \phi_1(t) &= \sqrt{\frac{2}{T}}\cos(2\pi f_c t), & 0 \le t \le T \\ \phi_2(t) &= \sqrt{\frac{2}{T}}\sin(2\pi f_c t), & 0 \le t \le T \end{cases}$$

$$\text{OQPSK basis} \quad \begin{cases} \phi_1(t) &= \sqrt{\frac{2}{T}}\cos(2\pi f_c t), & 0 \le t \le T \\ \phi_2(t) &= \sqrt{\frac{2}{T}}\sin(2\pi f_c t), & \frac{T}{2} \le t \le \frac{3T}{2} \end{cases}$$

  - This offset ensures that I and Q components do not change simultaneously, avoiding 180° phase transitions.
  - Instead of 180° shifts, OQPSK exhibits more frequent ±90° phase shifts.

– Amplitude fluctuations in offset QPSK due to filtering have a smaller amplitude than in the case of QPSK

– Despite these changes, OQPSK and QPSK have the same power spectral density.

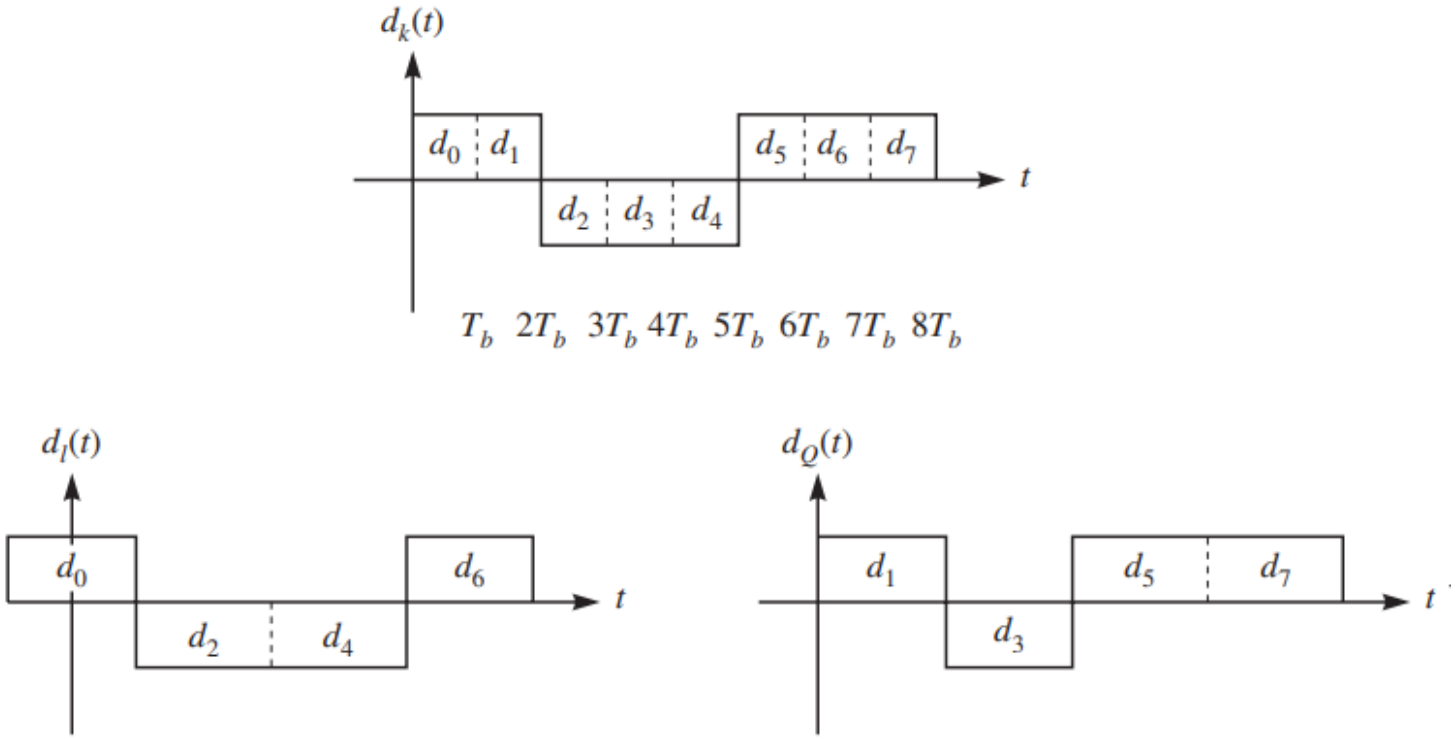


Figure 19: The I and Q components for OQPSK for the binary sequence **11000111**.

– The phase transition diagram for OQPSK is shown in Figure 20.

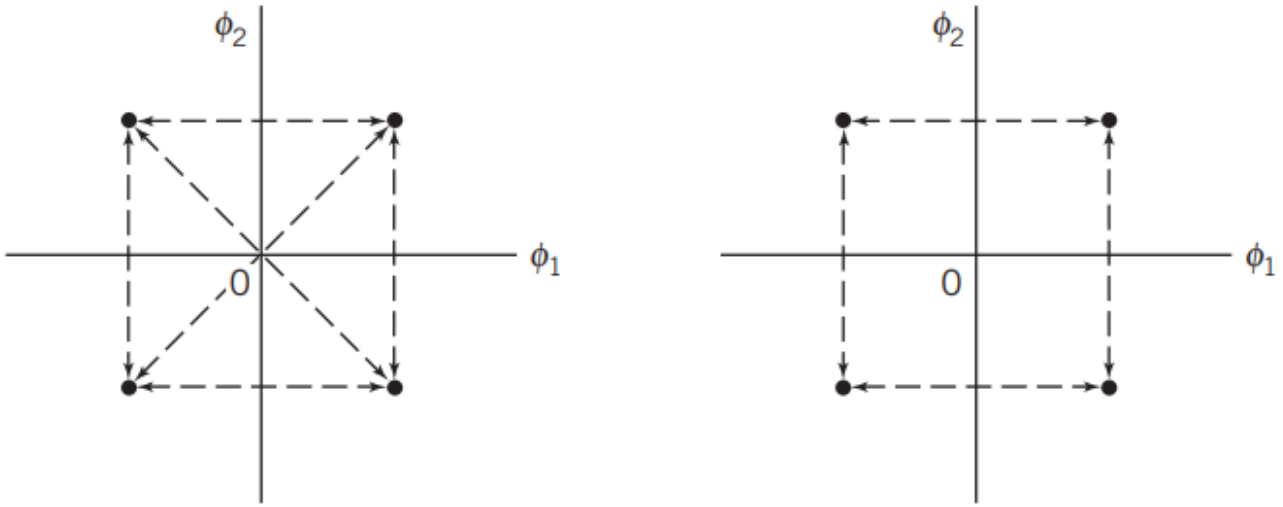


Figure 20: Possible paths for switching between the message points in (a) QPSK and (b) offset QPSK.

- MSK versus OQPSK versus QPSK
  - MSK has a continuous phase (no jumps in the waveform), but its instantaneous frequency has jumps.
  - OQPSK maintains a constant frequency but has jumps in its waveform.
  - Conventional QPSK with rectangular pulses has a constant envelope, but when filtered (e.g., raised cosine), its envelope is no longer constant.

- The $\pm 180°$ phase shifts in QPSK cause the envelope to pass through zero, which is undesirable for nonlinear amplifiers.
- OQPSK is preferred over QPSK when using nonlinear amplifiers such as class C amplifiers.
- In MSK, the phase is continuous, but frequency jumps exist; smoothing these jumps leads to a more compact spectrum.
- GMSK applies filtering before MSK modulation, creating smoother frequency transitions and improving spectral compactness.
- Since shaping occurs before modulation, GMSK maintains a constant envelope, making it suitable for nonlinear amplifiers.

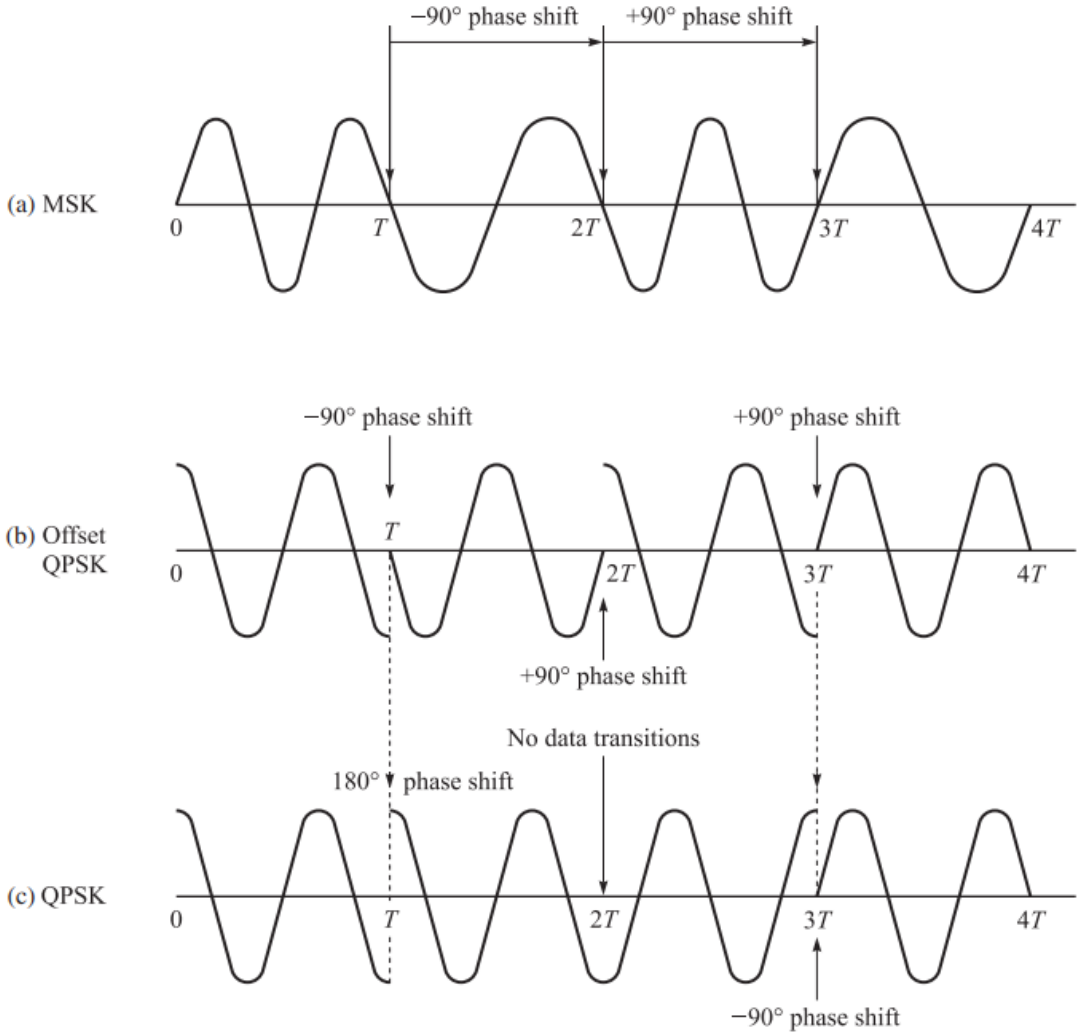
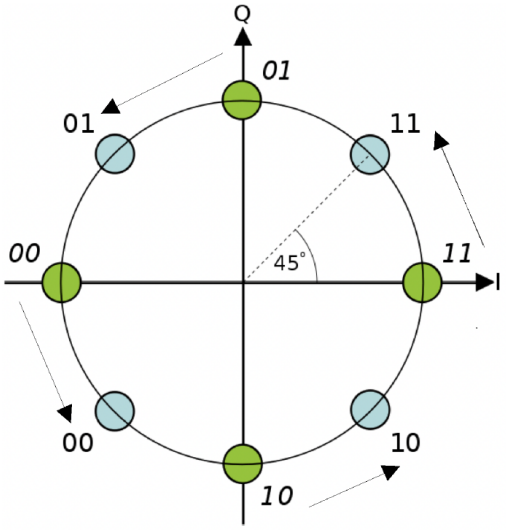


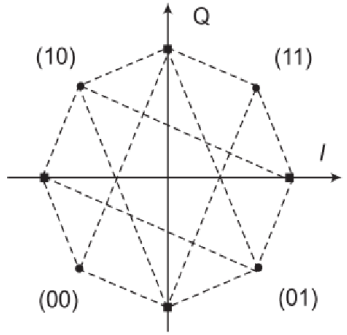
Figure 21: MSK, OQPSK, and QPSK signals

3.5.3 $\frac{\pi}{4}$-QPSK

– Another variant of QPSK, called $\frac{\pi}{4}$-QPSK, is obtained by introducing an additional $\frac{\pi}{4}$ phase shift in the carrier phase in each symbol interval.

– The phase transition in $\frac{\pi}{4}$-QPSK is always limited to $\pm 45°$, $\pm 135°$, avoiding the sharp 180° transitions of QPSK.





Signaling Schemes with Memory

S. No.	Parameter	QPSK	OQPSK	$\pi/4$-QPSK
1.	Maximum phase shift	$\pm 180°$	$\pm 90°$	$\pm 45°$ or $\pm 135°$
2.	Amplitude variations at the instants of abrupt phase changes	Large	Small	Medium
3.	Simultaneous change of in phase and quadrature phase or even and odd bits	Yes	No	No
4.	Offset between in phase and quadrature phase or even and odd bits	No	Yes, by T_b seconds	Yes, by $T_b/2$ seconds
5.	Preferred method of demodulation	Coherent	Coherent	Coherent or non-coherent
6.	Minimum bandwidth	f_b	f_b	f_b
7.	Symbol duration	$2T_b$	$2T_b$	$2T_b$
8.	Receiver design complexity	Yes	Yes	No in case of non-coherent

Credit: The figures are mostly from one of the following reference books.

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[1] John G. Proakis and Masoud Salehi, *Digital Communications*, McGraw-Hill, 2008.

[2] Michael Rice, *Digital Communications: A Discrete-Time Approach*, Pearson-Prentice Hall, 2009.

[3] Simon Haykin, *Digital Communication Systems*, John Wiley & Sons, 2014.